ELECTRICAL ENGINEERING

VOLUME I.

ELECTRICAL ENGINEERING

VOLUME I.

BY

W. TOLMÉ MACCALL, M.Sc., M.I.E.E.

FORMERLY HEAD OF THE DEPARTMENT OF ELECTRICAL ENGINEERING
IN THE TECHNICAL COLLEGE, SUNDERLAND

LONDON

UNIVERSITY TUTORIAL PRESS LTD.

CLIFTON HOUSE, EUSTON ROAD, N.W. 1

BOOK PRODUCTION WAR ECONOMY STANDARD

This book is produced in complete conformity with the authorised economy standards.

Published 1938
Reprinted 1943, 1947

PRINTED IN GREAT BRITAIN BY UNIVERSITY TUTORIAL PRESS LTD., FOXTON, NEAR CAMBRIDGE

PREFACE

THIS book is the first of two volumes which together are intended to cover advanced general Electrical Engineering work in Technical Colleges and Schools. It is founded on a large part of the author's *Continuous Current Electrical Engineering*, first published in 1915, and on a smaller part of the author's *Alternating Current Electrical Engineering*, first published in 1923.

The main reason for this alteration in the order of treatment of the subject is the continued increase in the use of alternating currents. The consequence has been that this part of the subject is now introduced much earlier than before, in both evening and day courses. The change has the advantage of enabling certain items, *e.g.* voltmeters, to be treated together instead of in two separate books. A further advantage is that revision has been much more thorough than can be done conveniently when merely altering an existing book.

The fact that it has been found desirable to retain a great deal of the earlier matter is a proof of the soundness of the two earlier books.

This volume is suitable for the first year of the Advanced Course for the Higher National Certificate; and for the second year of the Degree Course in Universities or University Colleges. It contains more than is included usually in such courses, but by suitable selection it can be adapted readily to the needs of the course in any particular Institution. The mathematical work remains as simple as is possible while providing a concise and sufficient explanation of the subjects.

The symbols agree with the recommendations of the International Electrotechnical Commission. For those not settled by the I.E.C. the choice has been guided by custom. Two special symbols have been adopted again, for "number of turns," and for "number of ampere-turns (or atts)." A list of the symbols used is given on page viii.

The author hopes that the present volume, while offering an improved arrangement, will be found to have retained the good points of the previous books, and thanks those readers and colleagues who have made suggestions for the improvement of the latter.

Thanks are also due to the various manufacturing firms mentioned in the text for the loan of blocks for some of the illustrations. Amongst these may be mentioned: The Cambridge Scientific Instrument Co., W. T. Glover and Co., Ltd., Union Electric Co., Ltd., Ferranti, Ltd., Dorman and Smith, Ltd., and the General Electric Co., Ltd.

W. TOLMÉ MACCALL.

CONTENTS

LIST OF SYMBOLS

A = cross-sectional area.
B = susceptance.
B = flux-density.
C = capacitance.
d, D = diameter.
E = electromotive force, or potential difference.
f = frequency (cycles/sec.).
g = acceleration due to gravity.
G = conductance.
H = magnetic force.
I = current.
l = length.
L = inductance (self).
M = mutual inductance.
n = revolutions per minute.
p = number of pairs of poles.
P = power, or candle-power.
q, Q = quantity of electricity.
R = resistance.
R = reluctance.
t = time, or temperature.
T = torque, or temperature.
v = velocity.
V = potential difference.
W = watts.
X = reactance.
Y = admittance.
Z = impedance.
μ = permeability.
ρ = resistivity (specific resistance).
ϕ = angle of lag or lead.
Φ = total magnetic flux.
$\mathfrak{A}$ = ampere-turns ("atts").
$\mathfrak{T}$ = number of turns in a coil.

ABBREVIATIONS

A.C. = alternating current.
B.S.I. = British Standards Institution.
B.S.S. = British standard specification.
B.Th.U. = British thermal units.
C.G.S. = centimetre-gram-second.
C.P. = candle-power.
D.C. = direct (or continuous) current.
E.M.F. = electromotive force.
F.P.S. = foot-pound-second.
H.P. = horse-power.
I.E.E. = The Institution of Electrical Engineers.
M.H.C.P. = mean horizontal candle-power.
M.S.C.P. = mean spherical candle-power.
M.H.S.C.P. = mean hemispherical candle-power.
M.M.F. = magneto-motive force.
P.D. = potential difference.
r.p.m. = revolutions per minute.
Sp.Gr. = specific gravity.

ABBREVIATIONS AFTER NUMBERS

A. = amperes.
kV. = kilovolts.
kVA. = kilovolt-amperes.
kW. = kilowatts.
kWh. kilowatt-hours.
V. volts.
μF. microfarads.
= cycles per sec.

ABBREVIATIONS AFTER QUESTIONS AND EXAMPLES

C. & G., I. = City and Guilds of London Institute examination in Electrical Engineering Practice, Preliminary Grade.

C. & G., II. = City and Guilds of London Institute examination in Electrical Engineering Practice, Intermediate Grade.

Lond. Univ., El. Eng. = The University of London final examination for the B.Sc. (Engineering), paper on "Generation, Transmission and Utilisation of Electrical Power."

Lond. Univ., El. Mach. = The University of London final examination for the B.Sc. (Engineering), paper on "Electrical Machinery and Design."

Lond. Univ., El. Tech. = The University of London final examination for the B.Sc. (Engineering), paper on "Electrical Technology."

ELECTRICAL ENGINEERING

VOLUME I

CHAPTER I.—INTRODUCTORY

1. Introductory

These volumes give an account of the principles and general practice in connexion with the apparatus for the generation, distribution, and use of electrical energy by means of electric currents. Little is said about the nature of electricity itself, since this, though of great interest, is not of *direct* importance to the electrical engineer. What he requires is a knowledge of the best methods of production, control, and utilisation of electrical energy (see, however, Art. 7).

A general description of a typical electrical supply system is given in this chapter, thus presenting a framework into which the more detailed treatment of the later chapters can be fitted. This description may be postponed if desired, but its perusal should add interest and unity to the study of the rest of these volumes.

2. The Advantages of Electricity

The main uses of electricity may be divided into (*a*) lighting, (*b*) heating and cooking, (*c*) traction, (*d*) other power uses. Every one is familiar with one or more of these ways of employing electricity, and the completion of the electrical "grid" has brought these to the attention of most people in Great Britain.

For lighting, electricity compared with other methods (of which gas is the chief) has the advantages of greater cleanliness, convenience, and adaptability, and less heat, and in many cases it is cheaper. The last is a vexed question, but even when "gas" is cheaper than "electricity" the indirect savings from the use of the latter more than balance the direct difference in price. Much the same considerations apply to the use of electricity for cooking and heating.

For tramway work electricity has become nearly universal. It enables higher speeds and larger cars to be used than were possible with horse traction. The main rival of the electric tram is the petrol motor-bus, whose total running costs (including depreciation

and interest on capital) are usually higher, except possibly on specially good roads or where an infrequent service is sufficient. Even for these cases a good alternative is the electric trolley-bus, which obtains its power from overhead wires but runs on the ordinary road surface.

For power purposes the main advantages of electricity are its adaptability to all purposes, the simplicity of its use, the abolition of much shafting, etc., and its economy (see Art. 3). The electric motor can be started and stopped, and have its speed varied, with ease and safety even by unskilled labour. Extensions can be made readily, or one part of a factory may be run economically alone if this is required.

3. Central Station Supply

"Electricity" is generated by dynamos (or *generators*), which in the United Kingdom are nearly always each driven by a steam (or gas or oil) engine. A number of such "generating sets" are placed in a **central station** together with the boilers for supplying steam to the engines, a switchboard for regulating and measuring the electric currents, and auxiliary apparatus of various sorts. Thence a number of cables run, conveying the current to and from the lamps and motors, etc., where it is required. Water power, when available, is used instead of steam, but the electrical arrangements are not altered by this substitution.

With water power some form of transmission is required, because the power is as a rule some distance from the place where it can be directly used. Electricity is much the best means of effecting the necessary transmission, being more efficient and economical than any other method except for very short distances.

It is not, however, immediately apparent what advantage transmission from steam-driven generators to electric motors has over the use of steam engines to do the work directly. In the former case there are losses in the electrical generators and motors and in transmission, in addition to the steam-engine losses which occur in both cases. The advantages, which more than counterbalance these losses and make electric power cheaper than steam (or gas or oil) power in most cases, are as follows:—

(*a*) Large steam engines are more efficient than small ones. A central station supplying a number of factories can use larger engines than any of the factories, and so generate its power more cheaply.

(*b*) The cost of engines does not increase as fast as their power; or in other words, the cost per horse-power decreases as the size of the engine increases. Since interest must be paid on the capital cost, and depreciation allowed for, this means a further saving by the use of large engines.

(*c*) The cost of attendance is less for a few big engines than for a larger number of smaller ones of equal total power.

(*d*) The maximum power which the central station has to supply is less than the sum of the separate maxima required by the various consumers supplied from the station. If every consumer required his maximum power at the same instant the station would have to supply the sum of all the maxima, but as this is never the case in practice, the station maximum is less than this.

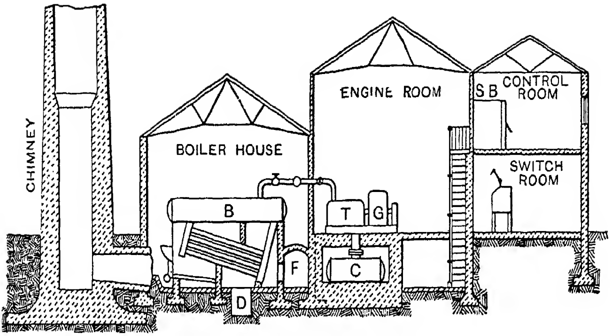
This results in a further saving of capital cost to the station and a diminution in the cost of power. The greater the variety of the load the greater this saving.

In addition to these main advantages there are certain others which often occur, *e.g.* the possibility of placing the station where a good supply of cooling water is available for the condensers, and where coal can be obtained cheaply; or the employment of special apparatus to effect economy of labour, coal, etc., which would not be suitable for work on a small scale.

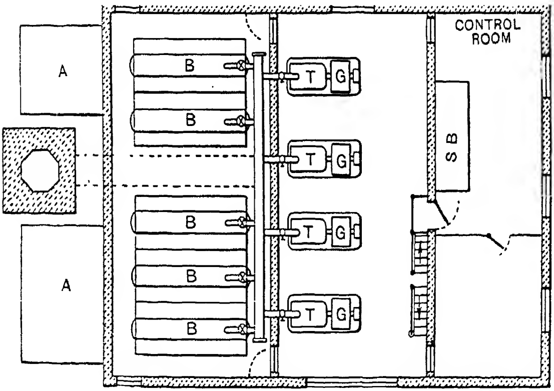
The effect of all these factors is to make it more economical in very many cases to generate mechanical power by steam engines in a central station; to convert this into electrical power by electrical "generators"; to transmit the electrical power some miles; and finally to reconvert it into mechanical power by electric motors, than to generate the mechanical power by steam engines at the place where it is required.

4. Generators and Switchboard

Generators are now always directly coupled to the engines which drive them. A station may contain from two to twenty of them. When the number is large their sizes will differ as a rule. It is then generally possible to arrange matters so that each generator works at an output approaching its full load during most of the time that it is running. This is advantageous because the efficiency of all generators falls off at light loads. As the power taken by the motors and lamps increases or diminishes fresh generating sets are started up or some of those running are stopped, and thus the above condition can be realised nearly always.



SECTIONAL ELEVATION.



PLAN (mainly on Ground-floor Level).

A A, Coal Bunkers.	F, Flue.
B B, Boilers.	G G, Generators.
C, Condenser.	S, B, Switchboard.
T. T. Turbines.	

Fig. 1.01.—GENERAL ARRANGEMENT OF CENTRAL STATION.

Leads are taken from each generator to the switches, which are actuated from the control room. Here the currents from the various generators are measured and can be regulated, and the generators connected to, or disconnected from, the supply mains. From this, too, run the mains to the various points where the electrical power is utilised. The currents in these are measured too, and they can be disconnected from the generators if desired. Other instruments measure the pressure at which the electricity is supplied, the total amount of energy supplied, and so on.

Fig. 1.01 shows the general arrangement of a typical central power station.

5. A.C. Generation

The great majority of large central stations are worked on the alternating current system (usually of the three-phase variety) particularly when the distance of transmission exceeds a few miles. The chief exceptions to this are a number of places in Switzerland where the high pressure D.C. system is used (see Vol. II.). For economical transmission to long distances a high pressure is essential (see Chap. XVIII., Art. 3). The pressure of alternating currents can readily be raised or lowered by means of static transformers which have no moving parts and so require little attention; moreover, their efficiency is very high. This explains the preference given to A.C. for long distance work over direct currents which necessitate rotating machinery for pressure changes.

The methods of generation of A.C. electric power do not differ essentially from those employed in D.C. stations except that alternators (*i.e.* A.C. generators) are employed instead of D.C. generators. For the extra high pressures (20,000 volts to 200,000 volts), used for very long transmission distances, a minor difference is that the alternators are wound for a moderately high pressure which is then increased by transformers to the higher pressure used for transmission.

The differences in the switchboard arrangements are due chiefly to the higher pressures employed and to the differences between the two types of generator.

All the purposes for which D.C. is used can be carried out by A.C. except electro-chemical work, including in this term the charging of accumulators. Even this class of work can be performed by A.C. with the assistance of rectifiers.

For lighting, whether by incandescent or other lamps, each method of supply has its advantages and corresponding disadvantages.

For heating and for cooking there is no difference of importance between the two, except in some special A.C. cookers.

For factory driving and similar power work, again each system has its advantages. For constant speed work the A.C. induction motor is the better, but if the speed has to be varied the D.C. shunt-wound motor is more convenient and more efficient.

For trams the D.C. series motor is supreme. But for trains the monophase A.C. commutator motor is a strong competitor, and for mountain railways the three-phase induction motor has many advantages.

6. The Supply Network

This includes all the electrical conductors by which electrical power is transmitted from the central station to the buildings where it is required; there to be converted into mechanical power by motors, or into light by electric lamps, or into some other useful form of power. These conductors may be divided into three classes—

(*a*) feeding cables or "feeders," (i) primary, (ii) secondary;

(*b*) "distributors";

(*c*) "service mains."

The primary feeders are pairs of cables which run from the central station to sub-stations. Here the pressure is reduced by static transformers, and the secondary feeders connect the sub-stations to various places called feeding points. If distribution is to be by D.C. the sub-stations must contain in addition either rotating machines or some form of rectifier. Each pair of cables consists of a "lead" (called alternatively in the case of D.C. a positive feeder) and a "return" (or for D.C. a negative feeder). No branch is taken from any feeder at any point along its length.

From each feeding point a number of pairs of distributors (a lead and a return) are run along the streets. In many cases a pair of distributors is connected to a feeding point at each end, but in other cases the distributors run from one feeding point into an outlying district. The distributors are branched when necessary, and some of them may be connected directly to the sub-station if power is required in its vicinity. To them (never to a feeder) the service mains are connected. These each consist of a pair of short lengths of cable of smaller size than the distributors, one connected to a positive distributor and the other to the corresponding negative distributor. They end at the main terminals of a factory or other

building. From these main terminals another series of conductors starts carrying the current to and from the motors, lamps, etc., and forming the wiring of the building.

7. Theory of Matter

The atoms of the various chemical elements are built up of *electrons* and *protons*. The former may be called an atom of negative electricity and cannot be subdivided by any known method. One coulomb contains about $6{\cdot}4 \times 10^{18}$ electrons. The mass of an electron is about 1/1800 of that of the lightest known atom, hydrogen, which itself is $1{\cdot}63 \times 10^{-24}$ grm.

The proton is an atom of positive electricity, or of positively electrified matter, of the same quantity as the electron but of a mass nearly equal to that of an atom of hydrogen.

The normal atom of hydrogen consists of one proton with one electron rotating about it at a distance of the order of 10^{-8} cm. This minute distance is, however, about 100,000 times the diameters of the proton and the electron, so that a model of a hydrogen atom might consist of a cricket ball with a soap bubble of equal size rotating round it at a distance of two miles.

Atoms of heavier elements consist of a *nucleus* of several protons and a smaller number of electrons all close together, and a further number of electrons (making them up to the number of protons) rotating about the nucleus.

In addition there has been discovered the existence of uncharged particles of almost exactly the same size and mass as protons, and these are called neutrons. The neutron may be a proton with an electron embedded in it; or the neutron may be the more fundamental of the two. The distinction between these possibilities is immaterial for the purposes of these books.

In some substances two or more different forms (called isotopes) are found. These have the same number of protons in their nucleus, but different numbers of neutrons. Their chemical behaviour depends on the number of protons and so is alike, but their atomic weights depend on the sum of the protons and neutrons and so differ.

In a molecule of two or more atoms the nuclei are separate, but the rotating electrons are shared so that the atoms are bound together to an extent which varies according to the molecule.

Under certain conditions a molecule may lose one or more electrons and so become positively electrified, or may acquire more than its normal number and so become negatively electrified.

8. The Electric Current

This consists of the movement of electricity, but its nature differs in the various types of substance. Those in which continuous movement is possible are called conductors.

Gases normally are insulators, but may have free electrons produced in them, *e.g.* by ultra-violet light, and are then said to be *ionised*. Under electric stress free electrons move from negative to positive, accompanied by a slower counter-movement of positively-electrified molecules.

In conducting liquids the substance is split up into positively- and negatively-electrified *ions*, *e.g.* in copper sulphate ($CuSO_4$) positive ions of copper move with the electric stress, and negative ions of sulphuric radicle (SO_4) move against it.

Probably conducting solids contain a large number of free electrons which cannot leave the body except at high temperatures. Under electric forces these move within the body and give rise to the phenomena summarised by saying that an electric current is flowing in the solid (in the opposite direction to the general motion of the electrons since they are negative electricity). The electrons collide with the molecules of the solid, and so use up electric energy which is converted into heat.

9. Insulators

A gas when not ionised is a perfect insulator, *i.e.* it prevents any continuous movement of electricity through it. A liquid is an insulator when it cannot be split up into ions. None are perfect since they permit a small current to flow, probably due to the presence of a few free electrons.

Similarly, solid insulators permit very little current to flow (see Chap. III., Art. 9) but do not prevent it completely. These small currents are called conduction currents.

In all insulators electric stress produces electric *displacement*. This means that the orbits of the planetary electrons are displaced relative to their nuclei. The result is that the mean positions of the electrons are moved a distance proportional to the electric stress (cf. definition of capacitance, Chap. II., Art. 1).

When the stress is removed the electrons return to their original orbits, *i.e.* the displacement is restored. This may not occur immediately, and thus gives rise to what is usually called dielectric hysteresis.

CHAPTER II

UNITS AND STANDARDS

1. The C.G.S. Electromagnetic System of Units

Since all the practical electrical units are obtained from the centimetre-gramme-second (C.G.S.) units by multiplying or dividing by some power of 10, it is convenient to summarise the latter briefly.

Unit magnetic pole is that which experiences a repulsion of 1 dyne when placed in air 1 cm. from a pole of equal strength.

If a *current* flows in a conductor forming an arc 1 cm. long and of 1 cm. radius its strength in C.G.S. units is numerically equal to the force in dynes which it exerts on unit pole at the centre of the arc

Unit quantity of electricity passes across every cross-section of a circuit in each second when unit current is flowing in the circuit. When the current is constant this is expressed by the formulae—

$$Q = I.t$$

and $$I = \frac{Q}{t}.$$

When the current is variable—

$$Q = \int I.dt$$

and the current at any instant $= \dfrac{dQ}{dt}$.

The *difference of electric potential* (P.D.) between two points, P and Q, is the number of ergs of work done in transferring unit quantity from one point to the other.

If in moving from P to Q work is done against the electric forces, Q is said to be at a higher potential than P: if work is done by the electric forces, Q is said to be at a lower potential than P. Hence positive electricity always tends to move to points of lower potential.

If a unit charge is carried round a complete circuit the total number of ergs of work done is called the *electro-motive force* (E.M.F.) in that circuit.

The relation between E.M.F. and magnetic lines is dealt with in Chapter VIII.

N.B.—E.M.F. and P.D. are measured in the same unit, viz. ergs per unit quantity, but they are not identical (see Chapter VIII., Art. 8, and Chapter XI., Art. 15).

The *electrical power* in a circuit or in part of a circuit is EI,

where E = the E.M.F. in the circuit,

or the P.D. between the ends of the portion considered

and I = current flowing in the circuit.

For the power is the work done per second,

and E = ergs of work done on unit quantity,

I = number of units of electricity passing per second.

Similarly the amount of electrical work done in a circuit during any period = EQ,

where Q = quantity of electricity which has passed each point of the circuit during the period.

The *Resistance* of a body is measured by the P.D. which sends unit current through the body.

Inductance is measured by the number of magnetic linkages produced by unit current. An alternative definition is given in Chapter V., Art. 7.

Capacitance is measured by the quantity contained when charged to unit P.D.

2. The Practical System

The following table gives the names of the chief practical units and their relations to the corresponding C.G.S. units:—

QUANTITY	SYMBOL	NAME OF UNIT	EQUIVALENT IN C.G.S. UNITS
Current	I	Ampere	0·1
Quantity	Q	Coulomb	0·1
Potential	E	Volt	10^8
Power	P	Watt	10^7
Work	W	Joule	10^7
Resistance	R	Ohm	10^9
Inductance	L	Henry	10^9
Capacitance	C	Farad	10^{-9}

Most of the C.G.S. units have no special name, but the C.G.S. unit of work is the erg (or centimetre-dyne). (See Art. 4.) Some writers use the names of the practical units with the prefix ab-; *e.g.* an abampere means the C.G.S. unit of current, *i.e.* 10 amperes.

The factor 10^8 for the volt was chosen in order to make the volt as nearly as possible equal to the E.M.F. of the Daniell cell (used

as a standard in the early days of electricity), while retaining a power of 10 as the multiplier.

Similarly 10^9 C.G.S. units of resistance are nearly equal to the resistance of a column of mercury 1 metre long and of 1 sq. mm. cross-section (see Art. 3), the old Siemens unit of resistance.

The multiplier for the ampere must then be 0·1 $\left(=\frac{10^8}{10^9}\right)$ in order that the relation

$$I = \frac{E}{R}$$

may still hold good without introducing any constant.

The remaining equivalents are obtained in the same way from the relations between these three and the remaining quantities.

3. Prefixes

In addition multiples and submultiples of the above units are employed for convenience, their names being obtained by attaching the following prefixes to the corresponding unit:—

PREFIX	MEANING
Mega-	A million
Kilo-	A thousand
Deci- (unusual)	A tenth
Centi-	A hundredth
Milli-	A thousandth
Micro-	A millionth

Thus a megohm	means	1,000,000 ohms, or 10^6 ohms.
a kilowatt	,,	1000 watts, or 10^3 watts.
a centiampere	,,	·01 ampere, or 10^{-2} A.
a millivolt	,,	·001 volt, or 10^{-3} volt.
a deci-milliampere	,,	·0001 ampere, or 10^{-4} A.
a microfarad	,,	·000001 farad, or 10^{-6} farad.

4. Gravitational Units

The C.G.S. unit of force is the dyne, but the weight of 1 gramme is also used as a unit. The relation between the two differs according to the situation on the earth, because the gravitational pull of the earth on a given body is not the same at all places. If g is the acceleration in cm. per sec. per sec. of a body falling freely at any place, then

the weight of 1 gramme $= g$ dynes (at that place).

The variation in the value of g amounts to about 1 per cent., and its value is approximately 980 cm. per sec.^{2}.

Similarly two units of work are used. The "absolute" unit is the **erg**, which is the work done by a force of 1 dyne in moving a body 1 cm. in the direction in which the force is acting. The "gravitational" unit is the **centimetre-gramme**, which is the work done in raising 1 gramme through a height of 1 cm. against gravity.

Therefore— 1 centimetre-gramme = g ergs.

A body or system of bodies is said to possess energy when it is capable of doing work. Consequently the same units are used for measuring energy as for work.

The units used for measuring power (*i.e.* rate of doing work) are the erg per sec. and the cm.-g. per sec. respectively.

In the F.P.S. system the corresponding units are—

"Absolute" unit of force ..	Poundal.
"Gravitational" unit of force ..	Pound weight.
"Gravitational" unit of work ..	Foot-pound.
" " power ..	Foot-pound per sec.

The weight of 1 pound = g poundals, as above, but in this case g must be expressed in ft. per sec.^{2}, and therefore is about 32.

Other units of power often employed are—

The Horse-power = 33,000 ft.-lb. per minute, or 550 ft.-lb. per sec.

The French (or Metric) Horse-power (*Force à cheval*) = 75 metre-kilogrammes per sec.

The electrical units are (see Art. 2) the joule (= 10 megergs) for work or energy, and the watt for power. The joule is often called the watt-second. For large amounts of energy the kilowatt-hour (also called the Board of Trade unit and sometimes the kelvin) is employed.

Evidently 1 kWh. = $1000 \times 60 \times 60 = 3{\cdot}6 \times 10^6$ joules.

Example 1. *Define one* horse-power. *Having given that the length of one foot equals* 30·48 *centimetres, that the mass of one pound equals that of* 453·6 *grammes, and that the acceleration of gravity* (*at London*) *is* 32·2 *ft. per second* (*per second**), *deduce that one* horse-power *is* (*approximately*) *equal to* 746 *watts.* [C. & G., II.

One horse-power is the doing of work at the rate of 33,000 ft.-lb. per minute, *i.e.* at the rate of 550 ft.-lb. per second;

$\therefore$ 1 H.P. = $550 \times 30{\cdot}48 \times 453{\cdot}6$ centimetre-grammes per second.

* Omitted in original question.

Again, the acceleration due to gravity at London is $(32{\cdot}2 \times 30{\cdot}48) = 981$ centimetres per second per second;

$$\therefore\ 1 \text{ gramme weight} = 981 \text{ dynes};$$

$\therefore$ 1 H.P. $= 550 \times 30{\cdot}48 \times 453{\cdot}6 \times 981$ centimetre-dynes (or ergs) per second

$$= 7{\cdot}46 \times 10^9 \text{ ergs per second}$$
$$= 7{\cdot}46 \times 10^2 \text{ watts} = 746 \text{ watts.}$$

Example 2. *An electric lift, with a cab or body weighing ½ ton and a balance weight of 2 tons, takes a load of 3 tons to a height of 60 feet in 30 seconds, returning empty in the same time. The efficiency of the motor and gearing for the raising of the dead weight is 75 per cent. in either direction. If it makes 20 double journeys per hour, how many kilowatt hours will be consumed in an hour?* [Lond. Univ.

In going up the weight raised is $3 + \frac{1}{2} - 2 = 1\frac{1}{2}$ tons.

In returning the weight raised is $2 - \frac{1}{2} = 1\frac{1}{2}$ tons.

Therefore in each case the work done is $1\frac{1}{2} \times 2240 \times 60 = 201{,}600$ ft.-lb.;

$\therefore$ the energy supplied to the motor in a double journey

$$= 201{,}600 \times 2 \times \frac{100}{75} = 537{,}600 \text{ ft.-lb.}$$

Now 550 ft.-lb. = 1 H.P.-sec. = 746 watt-seconds (or joules);

$$\therefore \text{ energy per double journey} = 537{,}600 \times \frac{746}{550} = 729{,}100 \text{ watt-seconds};$$

$$\therefore \text{ energy consumed per hour} = 729{,}100 \times 20 \text{ watt-seconds}$$

$$= \frac{14{,}582{,}000}{3{,}600{,}000} \text{ kilowatt hours} = 4{\cdot}05 \text{ kWh.}$$

N.B.—This is equivalent to saying that the average power taken is 4·05 kW. Since it is at work during 20 minutes in the hour, the average power taken during working is 12·15 kW.

5. Legal and International Standards

The international ohm is the resistance at 32° F. (0° C.) of a column of pure mercury 106·300 cm. long and weighing 14·4521 grm. (thus having a cross-section of 1 sq. mm.).

The legal ohm is the resistance at 16·4° C. between terminals of a " Standard Ohm " kept at the National Physical Laboratory.

This was verified in 1894, and again in 1909 against a mercury ohm. As a result of the later verification the standard temperature was altered from its previous value of 15·4° C. to its present one.

The international ampere is the current which deposits 1·11800 milligrammes of silver per second when passed through a solution of silver nitrate.

The legal ampere is the current which gives a certain reading on a standard ampere balance kept at the National Physical Laboratory. This balance was, however, standardised by the deposition of silver.

The international volt is the P.D. which sends a current of 1 ampere through a resistance of 1 ohm. It may further be taken as $\frac{1}{1 \cdot 0183}$ of the E.M.F. of the Weston cadmium cell at 20° C. (see Art. 9).

The legal volt is $\frac{1}{100}$th of the P.D. which produces a certain deflexion on a standard electrostatic voltmeter kept at the National Physical Laboratory. As in the case of the ampere balance the voltmeter is intended to give the same result as the international definition.

The limits of accuracy obtainable (in the equalisation of the international and legal units) are: for the ohm, within one-hundredth of 1 per cent.; for the ampere and the volt, within one-tenth of 1 per cent. In comparison the errors can be kept within one-tenth of these amounts.

The remaining units are obtained from the above three.

6. Practical Standards of Resistance

The mercury standard is inconvenient for ordinary use: hence the standards employed are made of wire, or, if they have to carry considerable currents, of metal strips. The metallic alloys usually employed are platinum-silver or manganin for the highest class of work, and German silver, platinoid, constantan, and a number of patented alloys for less exact purposes. The material used should possess—

(*a*) a resistance unaffected by the lapse of time,
(*b*) low temperature-coefficient,
(*c*) strength and ductility,
(*d*) high resistivity.

For details of the above materials see Chapter III., and for examples of standards and their use see Chapter VII.

7. Ampere Balance

The ampere balance mentioned in the legal definition of the ampere (Art. 5) is a special type of the Kelvin ampere balance. These are made in a variety of sizes, and form very accurate standards of current. One pattern is illustrated in Fig. 2.01, this particular size being suitable for currents from $\frac{1}{4}$ amp. to 10 amp. Its action depends on the mutual attractions and repulsions of four fixed and two movable coils (see Fig. 2.01), through all of which

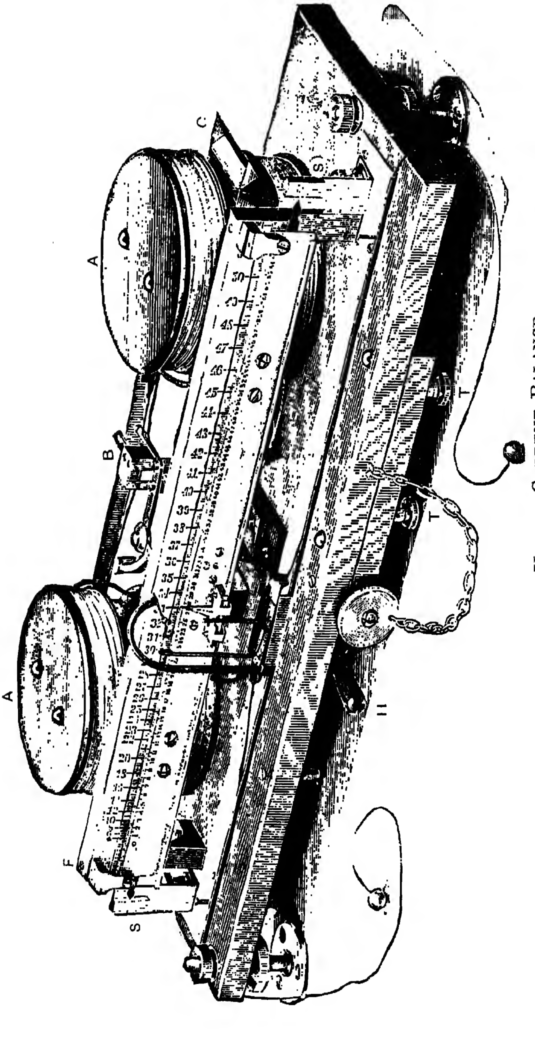


Fig. 2.01.—Kelvin Current Balance.

AA, Upper fixed coils. B, Beam support. C, Counterpoise. F, Fixed "inspectional scale." II, Handle for moving "flag." S,S, Scales for observing balance of beam. T,T, Terminals.

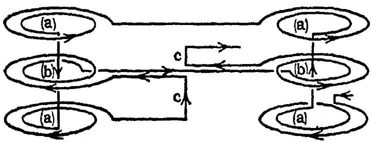


Fig. 2.02.—DIAGRAM OF CONNEXIONS OF THE KELVIN BALANCE.

a a a a, Fixed coils. *b b*, Moving coils. *c c*, Suspension ligaments.

the current passes, its direction being as indicated in Fig. 2.02. Note that the current is in opposite directions in the two moving coils, so as to eliminate the effect of any uniform field due to external causes, such as the earth's magnetism.

The two moving coils are attached to a beam which is supported at its centre by fine wire ligaments. All the electric forces on the moving coils tend to make the right-hand end of the beam rise and the left-hand end fall: it is brought back to its zero position by means of a sliding weight. The current is then calculated from the movement of the weight which is read on the uniformly divided movable scale.

Since the current passes through both fixed and movable coils and there is no iron in the instrument, the deflecting couple is proportional to (current)^2. The restoring couple due to gravity is proportional to (weight $\times$ its displacement).

Hence when balance is obtained

$$\text{current} \propto \sqrt{\{\text{weight} \times \text{displacement}\}}$$

or

$$\text{current} = \text{constant} \times \sqrt{\text{displacement}},$$

the constant depending on the dimensions of the coils, and on the square root of the weight. The weights supplied are adjusted so

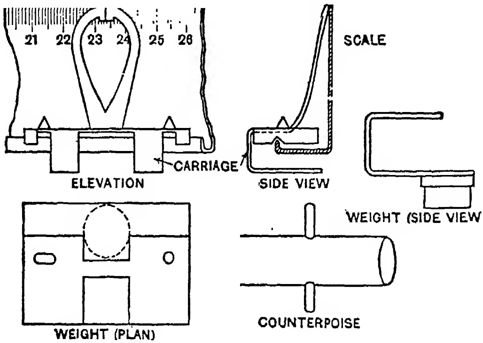


Fig. 2.03.—DETAILS OF KELVIN BALANCE.

that the constant is a simple number, and with each balance four sliding weights and four counterpoise weights are supplied. These give four ranges of current, with constants in the ratios 1 : 2 : 4 : 8. The sliding weights (see Fig. 2.03) are arranged to fit on to the carriage (which itself forms the first sliding weight) by means of a hole and a slot fitting on to conical pins. Their weights must be respectively 3, 15, and 63 times that of the carriage, while the counterpoises are 1, 4, 16, and 64 times the weight of the carriage: these latter consist of brass cylinders with pins through (Fig. 2.03), and are carried to an aluminium trough attached to the right-hand end of the movable system. For fine adjustment of the zero there is a small metal flag attached near the centre of the beam. This can be moved by a fork actuated by a handle (Fig. 2.01, H) outside the instrument case, and thus the beam is caused to balance at its zero (or "sighted") position with no current passing and with the sliding weight at zero.

For approximate readings there is a fixed scale, called the "inspectional" scale, behind the movable one (Fig. 2.01). The numbers on this are each twice the square root of the corresponding number on the movable scale, and so are directly proportional to the amperes. For more accurate results the reading is taken on the movable scale, and the doubled square root obtained from the table supplied with the instrument: this gives the reading on the fixed scale without any possibility of parallax error.

In Rayleigh's form of primary standard current balance two (not four) fixed coils are used, of equal size and at a distance apart equal to their radius. This makes the field near the midway plane almost uniform. The moving coil is half the radius of the fixed ones, which results in making the constant of the balance depend mainly on the *ratio* of the radii. It thus becomes possible to calculate with great accuracy from the dimensions the weight required to restore equilibrium when 1 ampere in the fixed coils is reversed.

8. The Daniell Cell

For specially accurate work the volt is obtained from the standards of resistance and current; but for ordinary laboratory purposes standard cells are employed (see Art. 5). The Daniell cell was the earliest of these, and still is used occasionally. The cathode, *i.e.* the portion to which the positive terminal is connected, consists of copper, and is in contact with a saturated solution of copper sulphate, to which 1 per cent. of sulphuric acid is added.

The anode consists of zinc, in a solution of zinc sulphate or dilute sulphuric acid or a mixture of the two. The two liquids are kept separate by a porous partition. In one pattern (see Fig. 2.04) the copper cathode forms the outer cylindrical vessel containing the copper sulphate, while the zinc and its solution are contained in an inner porous pot. There is a shelf inside the copper vessel on which copper sulphate crystals may be placed so as to keep the solution saturated. The chemical actions are as follows:—

In the porous pot:

$$Zn + H_2SO_4 = ZnSO_4 + H_2$$

Zinc + Sulphuric acid = Zinc sulphate + Hydrogen.

Fig. 2.04.—DANIELL CELL.

In the outer vessel:

$$H_2 + CuSO_4 = H_2SO_4 + Cu$$

Hydrogen + Copper sulphate = Sulphuric acid + Copper.

The copper is deposited on the inner surface of the copper vessel so that there is always a freshly formed surface present. The hydrogen does not appear, combining with the $CuSO_4$ as fast as it is set free. The amount of sulphuric acid present is constant.

The copper sulphate is decomposed, but is replaced by the dissolving of the crystals on the shelf. Zinc sulphate is formed and crystallises out if sufficient electricity passes through the cell. For this reason, and because the liquids gradually diffuse into each other, it is advisable to set up the cell afresh each time it is required for standard purposes.

The E.M.F. of this cell is from 1·04 volt to 1·10 volt at 15° C. (59° F.) according to the strengths of the solutions used. The E.M.F. is increased by strengthening the copper sulphate solution or by weakening the zinc sulphate solution. If the former is of Sp. Gr. 1·100 and the latter of Sp. Gr. 1·200 (which are the strengths recommended by Fleming for standard purposes), the E.M.F. is 1·072 volt. This E.M.F. increases by about ·00004 volt for each 1° C. rise of temperature, *i.e.* the temperature coefficient is negligible for all ordinary temperatures.

9. The Clark and Weston Cells

Owing to the drawbacks of the Daniell cell as a standard its place has been taken almost entirely by the *Clark* and *Weston* cells.

The Clark cell is on the same principle as the Daniell, but the copper is replaced by mercury. Consequently the copper sulphate is replaced by mercurous sulphate ($HgSO_4$). This has the advantage that it is much easier to ensure the purity of mercury than of copper.

The E.M.F. of the Clark cell is 1·433 volt at 15° C., and it decreases by about ·0011 volt per ° C. rise of temperature. The

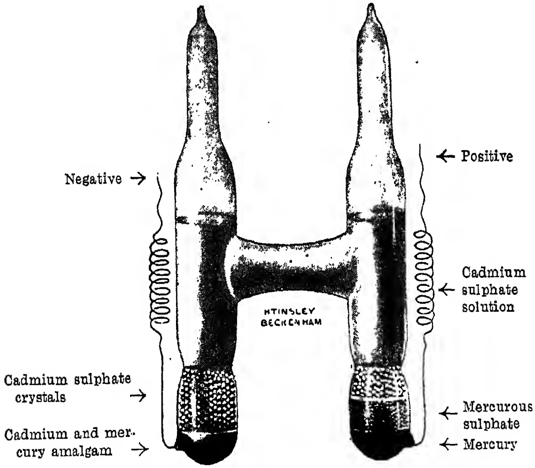


Fig. 2.05.—WESTON STANDARD CELL.

figure given in the Board of Trade specification of 1897 was 1·434 volt at 15° C., but later comparisons with the ohm and ampere standards have shown this value to be too high.

The Weston or Cadmium cell differs from the Clark in the use of cadmium in the place of zinc, and of cadmium sulphate ($CdSO_4$) in the place of $ZnSO_4$. The H-form, due to Lord Rayleigh, is shown in Fig. 2.05. The one shown is made by H. Tinsley and Co., and has the advantage of being hermetically sealed.

Its E.M.F. is 1·0183 volt at 20° C., and it has the advantage of a much smaller temperature coefficient than the Clark cell. The

value of this for the Weston cell is under ·00004 volt decrease per ° C. rise, which is negligible for ordinary purposes.

Weston cells, when made carefully, agree in their E.M.F.s within about 1 part in 40,000.

For examples of the use of standard cells see Chapter VII.

10. Standards of Inductance

Primary standards of self-inductance consist of single layer coils whose inductance can be calculated accurately from their dimensions. They are made of hard-drawn bare copper wire wound in a screw thread cut on a marble cylinder. Standards of mutual inductance are used more often. They consist of a primary coil of the above construction with a secondary coil of short axial length placed over the centre of the primary. From the dimensions and numbers of turns in the coils it is possible to calculate the magnetic linkages produced in the secondary by unit current in the primary (cf. Chapter IV., Art. 30).

For secondary standards, whether of self- or of mutual inductance, the dimensions need not be known but should be permanent. Consequently multilayer coils can be used, the wires being fixed in position by paraffin wax or other means.

Laboratory standards are made adjustable over a range by altering the relative position of two coils.

11. Standards of Capacitance

Primary standards are air condensers of accurately known dimensions, whose capacitance in electrostatic units is

$$4\pi \times \text{(total area of each plate)}/\text{(distance between plates)}.$$

Air is used because it is the only insulator whose permittivity (see Chapter III., Art. 10) is known with sufficient accuracy. Primary standards have capacitances of the order of 100 microfarads.

For secondary standards the dimensions need not be accurately known provided they remain constant. Hence each electrode can be made of a number of plates connected together, and capacitances up to 0·02 microfarad can be constructed conveniently.

For laboratory standards condensers with mica as the dielectric are used. The higher permittivity of mica reduces the bulk of a given capacitance; moreover for high pressure work its higher dielectric strength permits of a shorter distance between plates, which reduces the bulk further.

QUESTIONS ON CHAPTER II.

1. Define the units commercially used for current, electromotive force, resistance, power, and energy. What is the electrical equivalent of a mechanical horse-power? [C. & G., II.

2. Define: Watt, joule, kilowatt-hour, calorie. Prove that 1 H.P. = 746 watts, and find the distance that one kWh. could raise 1 ton, with 100 per cent. efficiency.

3. A steady current is passed through a silver voltameter for half an hour and deposits 3.162 grammes of silver. What is the value of the current?

4. Describe the method of action of the Kelvin ampere balance. Explain why it is suitable for use as a standard, and discuss its advantages and disadvantages for this purpose compared with the silver voltameter.

5. Give a full description of one form of the Weston standard cell. Why is it to some extent preferable to the Clark cell?

CHAPTER III

CONDUCTORS AND INSULATORS

1. Resistivity and Conductivity

The object of **Conductors** is either (*a*) to guide electrical energy to the place where it is to be utilised, or (*b*) to control and regulate electric currents and pressures.

In the former case resistance is a drawback as it entails waste of part of the energy. In the latter case, *i.e.* in *rheostats* or *resistors*, resistance is necessary and useful since the desired control is obtained by varying the resistance. In both cases the **specific resistance** (or **resistivity**) of the material employed is of importance. This may be stated in a variety of ways.

Since $R \propto \frac{l}{A}$, where $l =$ length of conductor, $A =$ cross-sectional area of conductor:

$$\therefore\ R = \frac{\rho l}{A},$$

where ρ is constant for any material, at a constant temperature (see Art. 3). This constant ρ is the resistivity of the material. It is sometimes called the volume-resistivity, to distinguish it from a second method of stating this quality (see below).

The name is not a good one, since the resistance of a given volume is not constant, but depends on the form of the conductor as well as on its material.

The numerical value of the resistivity (ρ) of a particular material depends on the units employed for l and A. Thus if l is in centimetres and A is in sq. cm. ρ must be the resistance of a conductor 1 cm. long and 1 sq. cm. cross-section, and is called the resistivity per centimetre cube* (*not* per cubic centimetre). Frequently it is expressed in microhms instead of in ohms so as to avoid a string of 0's following the decimal point (see Table A, page 60). In Europe the general method is to state ρ as the resistance of a wire 1 m. long and of 1 sq. mm. cross-section, the values thus being 10^4 times as large as those "per cm. cube," and so better expressed in ohms.

* A better name is ohm-centimetre (or microhm-cm.).

If inches are used instead of centimetres ρ is stated "per inch cube," and its value is ·3937 (or 1/2·54) of its value as above (see Table A).

Another way, largely used in America, and particularly useful in calculations for round wires, is to state ρ "per mil-foot," *i.e.* the resistance of a wire 1 ft. long and 1 mil (·001 in.) in diameter. This avoids the introduction of the multiplier $\pi/4$ in calculating the area of round wires, for the unit of area in this case is the circular mil (a circle of 1 mil diameter) and the area of a wire of d mils diameter is d^2 circular mils.

The second method, referred to above, is to state the resistance of a wire of the material of unit length and uniform cross-section and of unit mass, *e.g.* of 1 m. length and weighing 1 grm.

This is called the mass-resistivity, and the relation between this and the volume-resistivity depends on the density of the material as well as on the units employed; for the greater the density the smaller the cross-section of a wire of fixed length and weight. (See Example 1 and Art. 2.)

The conductivity of a material is the reciprocal of its resistivity. Only relative conductivities are used in practice, either annealed silver or standard copper (see Art. 2) being taken as having a conductivity of 100.

Example 1. *The volume-resistivity of a certain metal is* 11·6 *microhms per cm. cube at* 60° *F. and its specific gravity is* 21; *the corresponding figures for a certain alloy are* 21 *microhms per cm. cube and* 8·8 *Sp. Gr. Find the mass-resistivity (per metre-gramme) in each case and the ratios of the two volume-resistivities and of the two mass-resistivities.*

In the first case the volume of 1 grm. of the metal is $\frac{1}{21}$ of a c.cm. Therefore the cross-section of a wire 1 m. long weighing 1 grm. is $\frac{1}{2100}$ sq. cm.

The resistance of 1 cm. length of 1 sq. cm. cross-section is 11·6 microhms;

$$\therefore \text{ resistance per metre-gramme} = \frac{11{\cdot}6 \times 100}{\frac{1}{2100}} \text{ microhms}$$

$$= \frac{11{\cdot}6 \times 100 \times 2100}{10^6} \text{ ohms} = 2{\cdot}43 \text{ ohms.}$$

Similarly the cross-section of a 1 m. wire of the alloy of 1 grm. weight is $\frac{1}{880}$ sq. cm.;

$$\therefore \text{ resistance per metre-gramme} = \frac{21 \times 100 \times 880}{10^6} = 1{\cdot}85 \text{ ohms.}$$

$$\frac{\text{Volume-resistivity of alloy}}{\text{Volume-resistivity of metal}} = \frac{21}{11{\cdot}6} = 1{\cdot}81.$$

$$\frac{\text{Mass-resistivity of alloy}}{\text{Mass-resistivity of metal}} = \frac{1{\cdot}85}{2{\cdot}43} = 0{\cdot}76.$$

Example 2. *Find the relative conductivities of the two materials in Example 1 compared with copper (annealed).*

The specific resistance of copper at 60° F. is 1·696 microhms per cm. cube (see Table A);

∴ the volume-conductivity of the metal in Ex. (1) relative to copper (= 100) is

$$\frac{1\cdot696}{11\cdot6} \times 100 = 14\cdot6,$$

and the per cent. volume-conductivity of the alloy is

$$\frac{1\cdot696}{\text{[illegible]}} \times 100 = 8\cdot1.$$

2. Copper and Aluminium

Copper and aluminium are the two chief materials employed for transmitting currents, for which purpose a low resistivity is advantageous. As a reference to Table A will show, copper is excelled in this respect by silver only, and the difference is so small that the much greater cost of the latter makes it commercially impossible except in very special cases. For dynamo windings and other applications where the available space is limited copper has no rival. When the space is unimportant the higher resistivity of aluminium is compensated for by its low density, so that its mass-resistivity is only about half that of copper, for

$$\text{Mass-resistivity of Al at } 15^\circ\text{ C.} = 2\cdot78 \times 10^4 \times 2\cdot70/10^6$$
$$= \cdot0751 \text{ ohm per metre-gramme,}$$
$$\text{Mass-resistivity of annealed Cu at } 15^\circ\text{ C.} = 1\cdot692 \times 10^4 \times 8\cdot89/10^6$$
$$= \cdot1504 \text{ ohm per metre-gramme.}$$ (See Table A, page 53.)

Thus, when no other considerations intervene, aluminium is a cheaper conductor than copper if its price per ton is less than 2.00 $\left\{= \frac{1504}{751}\right\}$ times as great. It has, however, the disadvantage that satisfactory soldering is difficult. (See further Art. 14.)

Steel rails are used often as conductors in electric traction, both as the return uninsulated conductor and, in the third rail system, as the supplying conductor. The conductivity of the steel used is about one-sixth that of copper, but may be much less if the composition of the steel is altered slightly.

For overhead transmission wires hard-drawn copper is employed. The effect of the drawing is to raise the tensile strength to 29 tons per sq. in., compared with $12\frac{1}{2}$ tons per sq. in. for annealed copper. The resistivity is raised at the same time, but only by under 2 per cent. An alternative is steel-cored aluminium, *i.e.* a stranded

conductor with the central wires of steel for strength, and the outer ones of aluminium for conductivity.

Certain standard values have been adopted for the resistivity of copper and aluminium. It is, however, possible to obtain them of higher conductivity than this, *i.e.* with over 100 per cent. conductivity compared with the standard (see Table A).

3. Effect of Temperature

The effect of a rise in temperature is to increase the resistivity of pure metals. The amount of increase per degree rise of temperature is nearly constant. For most practical purposes it is sufficiently

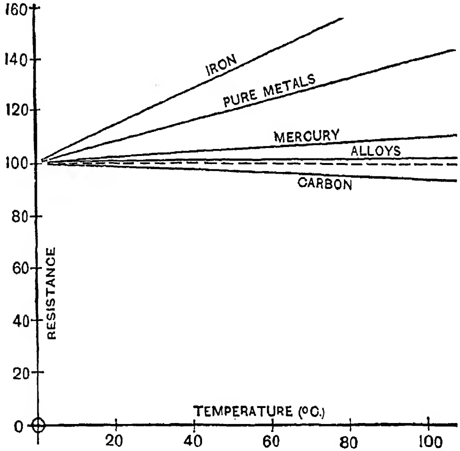


Fig. 3.01.—VARIATION OF RESISTANCE WITH TEMPERATURE.
Resistance at 0° C. taken as 100 in each case. The broken line is horizontal.

accurate to assume that it is constant for temperatures between 0° C. and 100° C. or even higher.

The assumption that it is constant is equivalent to the statement that the graph connecting the resistance of a conductor with its temperature is a straight line. This leads to the formula—

$$R_t = R_s\{1 + k(t - s)\},$$

where R_s = resistance of a conductor at the standard temperature $s°$,

R_t = resistance of the same conductor at another temperature $t°$;

so that $(t - s)^\circ$ is the rise of temperature, and k is a constant for each metal and is called the **temperature coefficient** of that metal. Strictly this is the "constant-mass temp. coeff. of resistance," *i.e.* between points rigidly fixed to the conductor.

From this it may be seen that k is the increase (in ohms) of resistance *per degree rise of temperature per ohm of original resistance.*

For most of the pure metals k has a value of about ·004 (or 0·4 per cent.) if the temperatures are measured on the Centigrade scale (for details see Table A), the main exceptions being wrought iron (higher), and cast iron and mercury (lower).

The value of k depends on the temperature adopted as a standard: this is often 0° C. (32° F.), but it is more convenient in practice to take 15° C. or 60° F., *i.e.* about the average air temperature. The connexion between the two values of k can be seen by a simple example—

Conductor of 100 ohms resistance at 0° C. and temperature coefficient of ·004—

Temperature	0,	1,	2,	3,	15,	16	° C.
Resistance	100,	100·4,	100·8,	101·2,	106·0,	106·4	ohms.

∴ Temperature coefficient at 15° C.

$$= \frac{106{\cdot}4 - 106{\cdot}0}{106{\cdot}0} \times 100 \text{ per cent. per } ^\circ\text{C.}$$

$$= {\cdot}377 \text{ per cent. per } ^\circ\text{C. or } {\cdot}00377 \text{ per } ^\circ\text{C.}$$

Thus the general relation is

$$k_s = \frac{k_0}{1 + s.k_0} = \frac{1}{\frac{1}{k_0} + s},$$

i.e. $k_s = 1/(\text{constant} + s)$,

where $k_0 = \text{temp. coeff. at } 0^\circ \text{C.},$

$k_s = $ „ „ at s° C.

Similarly on the Fahrenheit scale

$$k_s = \frac{k_{32}}{1 + (s - 32)\,k_{32}},$$

and $k_s = 1/(\text{constant} + s)$, as before.

(See Examples 3 and 4, and Art. 6.)

For large ranges of temperature, or when greater accuracy is required, more complicated formulae must be used to represent the effect of change of temperature (see Art. 7).

Example 3. *The temperature coefficient of lead is 0·411 per cent. per ° C. at 0° C. Find its value at 15° C. and at 30° C.*

A lead conductor of 100 ohms resistance at 0° C. would have

at 15° C. a resistance or ($100 + 15 \times \cdot 411 =$) 106·165 ohms

and at 16° C. „ „ ($100 + 16 \times \cdot 411 =$) 106·576 „

Difference = ·411 „

$\therefore$ temperature coefficient at 15° C. $= \dfrac{\cdot 411}{106\cdot 165} \times 100$ per cent. $= 0\cdot 387$ per cent.

Similarly at 30° C. its resistance would be 112·330 ohms,

and at 31° C. „ „ „ 0·411 ohm more;

$\therefore$ temperature coefficient at 30° C. $= \dfrac{\cdot 411}{112\cdot 33} \times 100$ per cent. $= 0\cdot 366$ per cent.

Or using the formula of Art. 3:—

$$\frac{1}{k_0} = \frac{1}{\cdot 00411} = 243\cdot 3$$

$$k_{15} = 1/(243\cdot 3 + 15) = 1/258\cdot 3 = \cdot 00387$$

and

$$k_{30} = 1/(243\cdot 3 + 30) = \cdot 00366,$$

which agree with the above results.

Example 4. *The temperature coefficient of a certain iron wire is found to be 0·537 per cent. of its resistance at 68° F. Find the value at 32° F.*

Denote the resistance at 68° F. by 100.

Then „ „ 32° F. $= 100 - (68 - 32) \times \cdot 537 = 80\cdot 67$.

And the resistance at 33° F. is 0·537 more;

$\therefore$ temperature coefficient at 32° F. $= \dfrac{\cdot 537}{80\cdot 67} = \cdot 00666$ or 0·666 per cent.

4. Alloys

The resistivity of *alloys* is always greater than the average of the constituent metals, and their temperature coefficients are much lower, in some cases negative. For instance, manganin, of which more than five-sixths is copper, has a resistivity about 26 times that of copper, while its temperature coefficient is less than 1/100th as great.

The main uses of alloys are—

(*a*) For standard resistances, where a considerable resistivity and a small temperature coefficient are advantages.

(*b*) For regulating resistances.

(*c*) For overhead transmission lines, and more especially for telegraph and telephone lines, where a high tensile strength is desirable, particularly for long spans.

For (*a*) the alloys mainly used are German silver, platinoid, platinum-silver, and manganin; the last is now generally employed for high class standards.

For (*b*) there are a number of alloys to which trade names have been given, such as Eureka, Beacon, Ferry, Nichrome, etc. Some of these are used for standards also.

When a greater tensile strength than that of hard-drawn copper is desired bronzes are used. These are composed mainly of copper and tin, but contain small percentages of other elements, *e.g.* silicon, manganese, etc. They have tensile strength up to 50 tons per sq. in. combined with high conductivity. Another alloy useful for this purpose is cadmium-copper, which is 44 per cent. stronger than hard-drawn copper though its resistivity is only 19 per cent. higher.

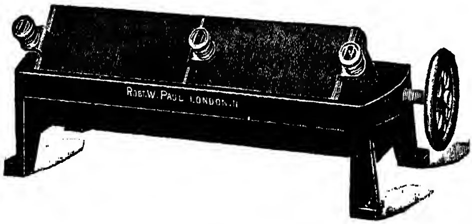


Fig. 3.02.—CARBON RHEOSTAT.

5. Carbon

Carbon is used as a conductor in incandescent lamps (Chapter XVI.), arc lamps, brushes for dynamos (Chapter IX.), and continuously variable rheostats. In this last case it is in the form of plates or discs of hard gas-retort carbon, which are pressed together when the resistance is to be diminished (see also Chapter XVI.). Fig. 3.02 shows a rheostat of this sort made by R. W. Paul. In all cases the resistivity varies considerably according to the methods of manufacture, as noted in the above-mentioned chapters.

The temperature coefficient of carbon is negative, *i.e.* its resistance diminishes as the temperature rises, but the formula of Art. 3 applies with the sign of k changed from positive to negative.

6. Measurement of Temperature Rise of Magnet Coils

The increase of resistance with temperature is applied usefully in measuring temperature. Thus the resistance of the field coils of

a dynamo can be obtained from ammeter and voltmeter readings while the machine is running, and the rise of temperature at any time deduced from the increase of resistance. The formula recommended by the British Standards Institution* for this purpose is equivalent to—

$$\text{Temp. rise in degrees Cent.} = (234{\cdot}5 + t)\left\{\frac{\text{Resistance hot}}{\text{Resistance cold}} - 1\right\},$$

where t = temp. in ° C. at which cold resistance is measured.

This is equivalent to assuming for the temperature coefficient of copper the value $\frac{1}{234{\cdot}5}$ (= ·00427) per ° C., with a standard temp. of 0° C., and consequently the value $\frac{1}{234{\cdot}5 + s}$ per ° C. at any other standard temp. s° C. (cf. Art. 3).

The formula for the Fahrenheit scale corresponding to the above is—

$$\text{Temp. rise in } ^\circ\text{F.} = (390 + t)\left\{\frac{\text{Resistance hot}}{\text{Resistance cold}} - 1\right\},$$

where t = temp. in ° F. at which cold resistance is measured.

The same method can be used for the armature at the end of a test.

7. Resistance Thermometers

In *resistance thermometers*, or pyrometers, the increase of resistance of platinum is utilised for the accurate measurement of temperatures up to 1400° C.

Platinum is employed since it can stand a high temperature without deterioration, and in addition its properties are well known and, with proper treatment, constant.

The linear law of Art. 3 is not sufficiently accurate in this case, but Callendar has shown that the relation between resistance and temperature for platinum can always be expressed in the form—

$$R_t = R_0 (1 + a.t + bt^2),$$

where a and b are constants for any particular wire, but vary somewhat for different wires. Their approximate values are—

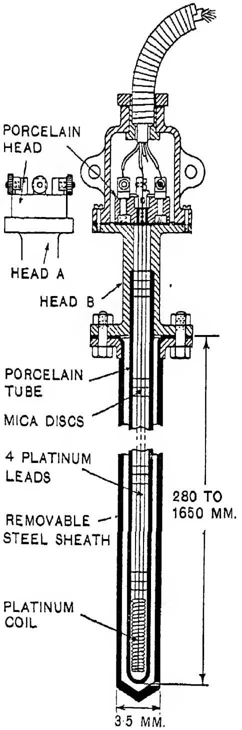
$$a = 37 \times 10^{-4},$$
$$b = -57 \times 10^{-8}.$$

* See B. S. S., No. 169, 1925, "Electrical Performance of Large Electric Generators, etc."

Their values for a particular wire can be obtained by measuring its resistance at three temperatures, usually those of melting ice (0° C.), boiling water (100° C.), and boiling sulphur (444½° C.).

In practice the calculation is simplified as follows:—

From the 0° C. and 100° C. measurements the value of k is obtained using the ordinary linear law $R_t = R_0\{1 + k.t\}$. This formula and the value of k obtained are then used in the measurement of other temperatures, the values thus found being called the **platinum temperatures**, or the temperatures on the platinum scale. To obtain the true (nitrogen gas scale) temperature the amount $\delta.t\,(t - 100) \times 10^{-4}$° C. is added to the platinum temperature. δ has a value of about 1·5, the exact value for any particular wire being obtainable from the measurement of its resistance in boiling sulphur (see Example 5).



g. 3.03.—PLATINUM THERMOMETER.

A standard type of platinum thermometer is shown in Fig. 3.03.

It consists of a coil of fine platinum wire of a few ohms resistance wound on a mica bobbin.

To prevent the resistance of the leads affecting the measurements "compensating leads" are provided. These are similar to the main leads, but are united inside the thermometer tube by a short platinum wire of small resistance.

The difference between the resistances of the two circuits is then measured directly by a Wheatstone Bridge (Chapter VII., Art. 30) arrangement (see Fig. 3·04).

In practice the variable resistance may be marked directly in ° C. or ° F. instead of in ohms. It can then, however, only be used with thermometers with one particular value for the difference of the main and compensating resistances. An indicator of this type is shown in Fig. 3.05.

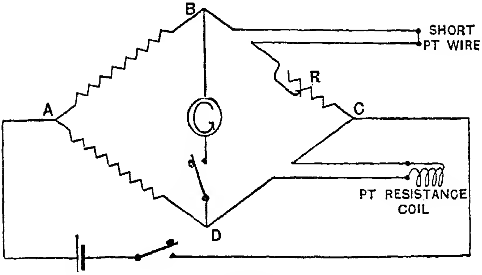


Fig. 3.04.—CONNEXIONS OF PLATINUM THERMOMETER AND INDICATOR.

AB, AD, Equal resistances. R, Variable resistance.

By turning the handle H until balance is shown by the galvanometer pointer at B, the temperature can be read directly at A.

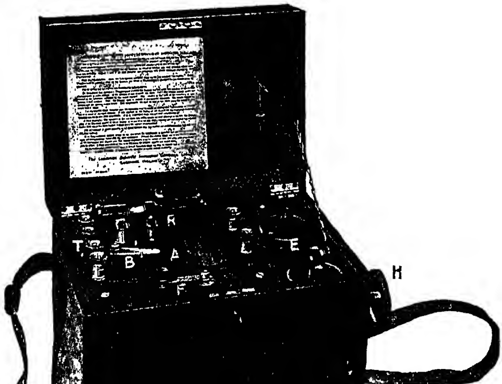


Fig. 3.05.—INDICATOR FOR PLATINUM THERMOMETER.

Example 5. *The resistance of a platinum thermometer is* 4·945 *ohms at* 0 ° *C.,* 6·772 *ohms at* 100° *C., and* 12·652 *ohms at* 444½° *C. Determine its constants, and the temperature when its resistance is* 9·438 *ohms.*

From the first two resistances the value of k in the formula

$$R_t = R_0 \{1 + k.t\}$$

is given by

$$k = \frac{6{\cdot}772 - 4{\cdot}945}{4{\cdot}94 \times 100} = \frac{1{\cdot}827^*}{494} = {\cdot}00370;$$

$\therefore$ platinum temperature $= \dfrac{R_t - R_0}{{\cdot}00370 \times R_0}$, from the above formula:

$\therefore$ when the resistance is 12·652 ohms

$$\text{platinum temperature} = \frac{12{\cdot}652 - 4{\cdot}945}{{\cdot}00370 \times 4{\cdot}945} = 421{\cdot}2^\circ \text{ C.}$$

But the true temperature is 444½° C., a difference of 23·3° C.;

$\therefore$ from the expression $\delta t\,(t - 100) \times 10^{-4}$

$$\delta \times 444{\cdot}5 \times 344{\cdot}5 = 23{\cdot}3 \times 10^4;$$

$$\therefore\ \delta = 1{\cdot}52.$$

When the resistance is 9·438 ohms

$$\text{platinum temperature} = \frac{9{\cdot}438 - 4{\cdot}945}{{\cdot}00370 \times 4{\cdot}945} = 246^\circ.$$

The true temperature will therefore be about 250° C.

Assuming this value the correction is

$$1{\cdot}52 \times 250 \times 150 \times 10^{-4} = 5{\cdot}7^\circ \text{ C.},$$

i.e. the true temperature is 252° C. nearly.

With this value the correction becomes

$$1{\cdot}52 \times 252 \times 152 \times 10^{-4} = 5{\cdot}8^\circ \text{ C.},$$

i.e. the true temperature is 252° C. to the nearest degree.

N.B.—The first estimate of the true temperature was in this case so near the value obtained with the first correction that it was unnecessary to calculate the correction afresh. If there were a considerable difference, or if $\frac{1}{10}$ths of degrees were required a second, and sometimes further, calculations of the corrections become necessary.

8. Thermo-Electric Pyrometers

Another electrical method of measuring temperature is by means of *thermo-electric pyrometers.* Their action depends on the fact that, in a circuit composed of two different metals or alloys, when there is a difference of temperature between the two points where the two materials join an E.M.F. is produced dependent on the temperatures and on the materials of the circuit.

If DAC and DBC (in Fig. 3.06) are wires of two different materials a small E.M.F. is set up in the circuit when D is at a temperature differing from that of C.

If it is in the direction indicated by the arrows when D is at the higher temperature, then DAC (*i.e.* the material towards which the E.M.F. is at the hot junction) is said to be thermally electropositive to DBC.

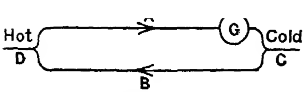


Fig. 3.06.

* In practice this is adjusted to an exact figure, *e.g.* 1 ohm, or 1·8 ohms.

The E.M.F. depends on the materials and on the temperatures of the two junctions. Thus if a high resistance galvanometer is placed in the circuit at any point, and the temperature of one of the junctions is known, that of the other can be obtained from the deflection of the instrument.

The materials employed for the "couples" are copper and iron or copper and constantan for moderate temperatures, and platinum and platinum-iridium (90 per cent. Pt., 10 per cent. Ir.) up to 1000° C., and platinum and platinum-rhodium (90 per cent. Pt., 10 per cent. Rh.) up to 1600° C.

Usually a millivoltmeter is employed with a scale graduated in degrees for the particular couple used, thus rendering the arrangement direct reading. Some patterns have a scale of millivolts as well as of ° C.

9. Insulators

These substances are used to prevent, as far as possible, an electric current from flowing where it is not required. Their resistivities are enormously greater than those of the metals, for instance india-rubber at 15° C. has a resistivity 10^{22} (ten thousand million million million) times that of copper.

The meaning of this may be better appreciated by considering that nearly the whole of the current sent into a Transatlantic cable travels several thousand miles through a copper wire in preference to passing through a quarter of an inch or so of gutta-percha, whose resistivity is less than one-tenth of that of india-rubber.

The resistivity of insulators decreases rapidly with rise of temperature, and the graph connecting resistance and temperature is no longer even approximately a straight line as it is with conductors.

The graph in most cases is a "logarithmic" curve (such as the one given for india-rubber in Fig. 3.07), *i.e.* one in which if a rise of $t°$ reduces the resistance to a half $\left(\text{or to } \frac{1}{m}\right)$ of its previous value, double this rise of temperature will reduce the resistance to one quarter $\left(\text{or } \frac{1}{m^2}\right)$ of the original value, *e.g.* if 2° C. rise reduces the resistance by 10 per cent. the following are corresponding values:—

Temp.	0,	2,	4,	6,	8,	10,	12 °C.
Res.	100,	90,	81,	72·9,	65·6,	59·0,	53·1 per cent.

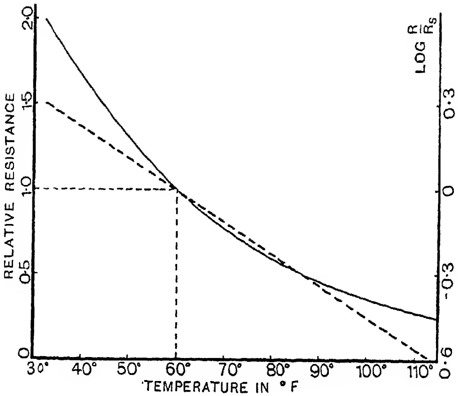


Fig. 3.07.—EFFECT OF TEMPERATURE ON THE RESISTANCE OF INDIA-RUBBER.

each value of the resistance being 90 per cent. of the previous one.

This may be expressed by the formula

$$R_t = R_s / a^{(t-s)}$$

or—

$$\log R_t = \log R_s - k\,(t - s),$$

where R_s = resistance at standard temp. $s°$ } ∴ $(t - s)$ = rise of
R_t = ,, ,, any other temp. $t°$ } temperature,

and a and k are constants for any particular insulator, and

$$k = \frac{\log 2}{t'},$$

where t' is the rise of temperature required to halve the insulation resistance.

For rubber t' is 15° C. (27° F.); see Fig. 3.07.

When the temperature rises so high that the insulator softens or oxidises, or is changed in some other physical or chemical way, the above law does not hold. Usually the change makes the insulator unfit for its intended purpose whatever the value of its resistance, and this temperature is an important factor in the choice of an insulator.

Example 6. *The insulation resistance of a certain length of cable is 422 megohms at 75° F. Calculate the value at 60° F. if the insulating material is such that a rise of 12° F. halves its resistance.*

Using the formula— $\log R_t = \log R_s - k(t - s)$

since $R_{s+12} = \frac{1}{2}R_s$;

$$\therefore k \times 12 = \log R_s - \log(\tfrac{1}{2}R_s) = \log \frac{R_s}{\frac{1}{2}R_s} = \log 2;$$

$$\therefore k = \frac{\log 2}{12} = \frac{\cdot 3010}{12} = \cdot 0251;$$

$$\begin{aligned}\therefore \log R_{60} &= \log R_{75} + (75 - 60) \times \cdot 0251 \\ &= \log 422 + 15 \times \cdot 0251 = 2{\cdot}6454 + \cdot 3765 \\ &= 3{\cdot}0219 = \log 1052;\end{aligned}$$

$\therefore$ resistance of insulation at 60° F. = 1052 megohms.

10. Dielectric Strength and Permittivity

Another property of an insulator is its ability to withstand electric pressure. This is known as its *dielectric strength*, and is usually stated in volts per mil (·001″) or in kilovolts per mm., *i.e.* in terms of the pressure necessary to break through the stated thickness of the substance.

A large insulation resistivity does not necessarily mean great dielectric strength, and for high pressure work the latter is more necessary.

The above method of stating dielectric strength implies that the voltage to produce breakdown is directly proportional to the thickness of the insulating sheet. Though this view is held by some authorities there is considerable doubt on the question.

Accurate experiments on the point are difficult owing to the variability in quality of many insulators, and the large effect of the conditions on the result. The main conditions to be considered are: (*a*) the shapes of the conductors between which the pressure is applied, and the consequent extent to which the stress in the material is uniform; (*b*) the temperature; (*c*) the presence or absence of moisture; (*d*) the length of time during which the pressure is applied; (*e*) the facilities for the escape of heat generated in the insulator; (*f*) the uniformity of the material.

Among the theories advanced as to breakdown are: (i) that it is a thermal effect; (ii) that it is a combined electrical and thermal effect; (iii) that it depends on a slow increase of electric displacement following that caused at once by the stress; (iv) that it is a purely electron phenomenon.

All of these afford some explanation of the effect of the time factor, and the bearing of some of the factors stated above depends

on the theory favoured. They lead to various formulae for the effect of thickness.

Graphs of three such relations are plotted in Fig. 3.08. A fourth probable one is $t = pE + qE^2$. This coincides almost exactly with No. 3 (if suitable constants are taken).

A third property is the *permittivity* (or "specific inductive capacity"). This is the ratio in which any capacitance is increased by substituting the material in question for air. It is thus a measure of the relative ease with which electric displacement can be produced, and so is analagous to magnetic permeability.

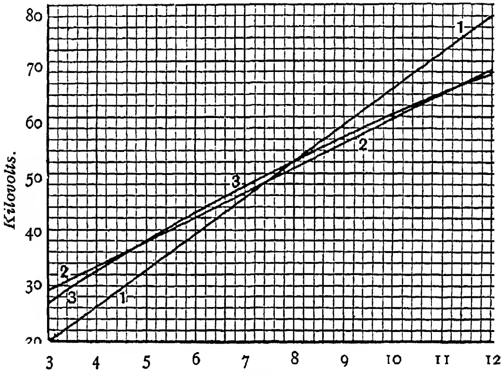


Fig. 3.08.—COMPARISON OF LAWS OF DIELECTRIC STRENGTH.
(1) $E \propto t$. (2) $E = a + bt$. (3) $E = c\sqrt[3]{t^2}$.

11. Dielectric Losses

When an insulator is subjected to alternating electric stress the loss due to the small conduction current is usually negligible in comparison with another source of loss. The latter is due to the fact that the electric displacement under steady stress takes an appreciable time to reach its full value. Consequently with a given stress the displacement is less when it is increasing than when it is decreasing.

Owing to resemblances to magnetic hysteresis (Chapter IV., Art. 20) this phenomenon is frequently called dielectric hysteresis. As, however, the extent of it depends on the rate at which the stress is changing (unlike the magnetic case) it is more of the nature of viscosity.

Like magnetic hysteresis it results in a loss of energy and consequent development of heat, and its value is of great importance in high pressure work (see Vol. II.).

12. India-rubber

One of the most useful insulators is *india-rubber*. Pure rubber is insoluble in water and is not acted on by alcohol, alkalies, or dilute acids. It is decomposed by strong sulphuric or nitric acid or by chlorine, and is gradually acted on by oil and more slowly by Portland cement. It can be dissolved in carbon bisulphide, turpentine, benzene, or coal-tar naphtha. It is almost useless when exposed to the air owing to its becoming oxidised, especially in the presence of light and moisture. It melts at about 250° F.

Vulcanised india-rubber is made by mixing pure rubber with about five per cent. of sulphur and adding various other ingredients (zinc oxide, litharge, calcium carbonate, etc.), varying according to the purpose for which it is intended. The mixture is then heated, often by steam, to a temperature of 250° F. to 300° F. It is more durable and stronger than pure rubber, and may be much cheaper, the price depending mainly on the percentage of pure rubber used and the quality of the latter. Correspondingly its dielectric strength varies from 4 *k*V/mm. to 30 *k*V/mm., and its resistivity from $1{\cdot}5 \times 10^{15}$ to 16×10^{15} ohm-cm.

Ebonite (vulcanite, or hard rubber) is made by mixing two parts of pure rubber and one of sulphur, and heating the mixture to a temperature of about 170° F. under a pressure of 60 lb. per sq. in. to 70 lb. per sq. in. for several hours. It may have a strength of 50 *k*V/mm., and a resistivity of 4×10^{15} ohm-cm.

Gutta-percha resembles india-rubber but becomes soft at 150° F. It rapidly oxidises when exposed to light in the air, but appears to be quite permanent if kept in water and screened from light. Its main use is for submarine telegraph cables.

13. Fibrous Insulators

These have the advantages of flexibility if desired, and of fair mechanical strength, but are hygroscopic (*i.e.* liable to absorb moisture) unless impregnated with varnish or wax, cannot long withstand temperatures much above 100° C., and have only moderate dielectric strength.

Cotton and *silk* are used for covering wires for use in dynamo windings, etc. Cotton is the cheaper, but silk is less hygroscopic,

also a thinner covering of it can be applied satisfactorily, which is of special advantage where fine wires are required, *e.g.* in voltmeter windings.

The usual form of covering consists of a number of threads wrapped continuously round the wire to form a complete covering (single cotton covered, S.C.C.), often followed by a second similar wrapping in the reverse direction (double cotton covered, D.C.C.). For large conductors either a braid or a tape is used instead. The dielectric strength depends mainly on the impregnating substance.

Paper is used for cables (Art. 16), condensers, and induction coils. For the last two purposes it is soaked in paraffin wax.

Many special forms of paper and cardboard, such as *presspahn, fullerboard, leatheroid,* are used for moderate pressures (up to 600 volts). *Vulcanised fibre* is also a vegetable fibre, consolidated under pressure. It is used in place of ebonite where cheapness is more important than very good insulation. They have dielectric strengths ranging from 5 *k*V/mm. to 20 *k*V/mm.

14. Other Insulators

Mica is a transparent mineral which splits easily into thin plates. It has a high dielectric strength (50 *k*V/mm. to 140 *k*V/mm.) and is fireproof, but mechanically weak. It is used mainly in various built-up forms.

Micanite consists of sheets of mica cemented together with shellac varnish, and is used nearly always in commutators.

Mica cloths, papers, and *tapes* are made by cementing mica flakes on to a backing of paper, silk, etc., sometimes with a further covering of paper. Micarta is built up of a number of sheets of mica paper, varnished, and solidified under pressure.

The mineral *asbestos* is used for fireproof construction. For wires it is applied as tape or braid or as a wet pulp ironed on. Its dielectric strength is low and it is hygroscopic, but the latter weakness is not of importance in most of its applications. By mixing with suitable materials asbestos board is made which is very suitable for switch barriers. In some forms of this the hygroscopic weakness has been overcome and the mechanical strength increased, so that switchboards can be made of it. *E.g.* " Sindanyo " has a resistivity of at least 2×10^{12} ohm-cm., which shows no reduction after 48 hours immersion in water. It has a dielectric strength of 2 *k*V/mm. in thick (1 in.) sheets, and higher in thinner ones, and its mechanical strength is such that smaller thicknesses can be used than when slate is employed.

Slate has a resistivity of $1{\cdot}2 \times 10^9$ ohm-cm., and a strength of 0·2 *k*V/mm., and sometimes has metallic veins in it.

Marble has a resistivity of 15×10^9, measured after prolonged exposure to moist air, and a strength of 6 *k*V/mm., but is more costly than slate.

The figures for *porcelain* are 2×10^{15} ohm-cm., and 10 *k*V/mm. to 15 *k*V/mm., so that it is very suitable as an insulating support (see Vol. II.).

A large number of synthetic resin products are used under a variety of names, of which "bakelite" is the best known. They can be moulded and do not deteriorate below 150° C. They are very suitable for the covers and bases of small switches, ceiling roses, etc. Mechanically they are somewhat weak, so that metal inserts are needed when thin mouldings are used.

Paper impregnated with bakelite varnish is unaffected by hot oil, and so is useful in oil-immersed transformers.

15. Cables

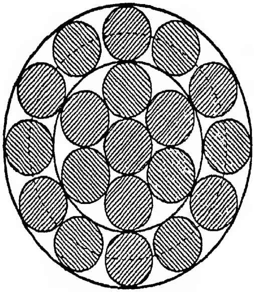


Fig. 3.9.—SECTION OF 19-STRAND CABLE.
------- Pitch circle.

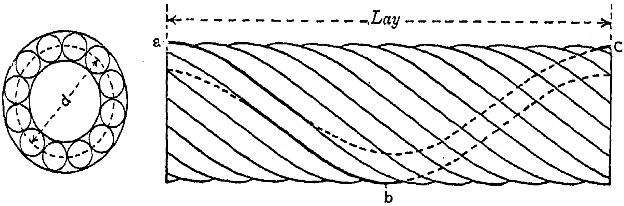
The *conductor* is nearly always of copper, though aluminium is used occasionally. The figures given in Art. 2 require modifying in favour of copper, because the 28 per cent. greater diameter of an aluminium cable for equal resistance causes an increase in the volume of the insulation and of the protective coverings.

For small conductors a single wire is used, but above 16 S.W.G. (·064 in. dia.) a number in parallel are substituted for greater flexibility. Three is the smallest number employed, and then seven, *i.e.* a central one with six surrounding it. The next step is to nineteen by winding a ring of 12 outside a 7-strand, and then in succession 37 (19 + 18), 61, 91, and 127 strands. Cables are denoted by two numbers, *e.g.* either 19/16, meaning 19 wires each of 16 S.W.G. (see Fig. 3.09) or 19/·064″, meaning 19 wires each ·064 in. dia. The wires are laid on spirally, successive layers in alternate directions, so as to make a firm cable. This twist increases the weight of the cable, because the twisted wires are longer than the cable or than the centre strand (if present) which is straight.

Since the current flows almost exclusively along the wires and not parallel to the axis of the cable from wire to wire, the resistance per unit length of cable is increased very nearly in the same proportion as the weight. Thus the actual cross-section of the cable is increased by the twist, but the *equivalent cross-section* is reduced inversely. The latter is the cross-section of a solid bar of the same material and of equal resistance per mile (or other unit of length). The amount of these increases depends on the "lay," viz. the distance along the cable for a complete turn of the wires (see Fig. 3.10).

The B.S.I. has adopted as standard a lay of 15·6 times the pitch diameter (*i.e.* the diameter of the circle on which the *centres* of the wires lie; marked ...... in Fig. 3.09 for the outer layer of wires).

This causes an increase of resistance (and of weight) of 2 per cent. on all but the central wire.



End view. Side view.

Fig. 3.10.—LAY AND PITCH DIAMETER OF CABLE.
d = Pitch diameter.

The diameters of stranded cables compared with that of the separate strands are given in the table opposite, the values (except for 3-strand) are obtained readily from a diagram such as Fig. 3.09.

Example 7. *Calculate the diameter, resistance per* 1000 *yards, and equivalent cross-section of* 3/·064″ *and* 7/·064″ *cable. A single wire has a resistance of* 7·48 *ohms per* 1000 *yd. at* 60° *F.* (*N.B.—The former is not a standard size.*)

Diam. of 3/·064″ cable $= 2\cdot155 \times 64 = 138$ mils.

" " 7/·064″ " $=\quad 3 \times 64 = 192$ mils.

Neglecting the effect of twisting the wires:—

Resistance of 3/·064″ cable per 1000 yd. $= \dfrac{7\cdot48}{3} = 2\cdot493$ ohms at 60° F.

" " 7/·064″ " " " " $= \dfrac{7\cdot48}{7} = 1\cdot069$ " "

No. of Strands	$\left(\frac{\text{Diam. of Cable}}{\text{Diam. of 1 Strand}}\right)$	$\left(\frac{\text{Sectional Area of Cable}}{\text{Area of Solid Wire of Same Diam.}}\right)$ (a)	(b)
3	2·155	64·6	63·3
7	3	77·8	76·5
19	5	76·0	74·6
37	7	75·5	74·0
61	9	75·3	73·8
91	11	75·2	73·7
127	13	75·15	73·65

(*a*) Without allowance for increased resistance due to twisting.
(*b*) " Equivalent " sectional area with B.S.I. standard lay.

With the B.S.I. standard lay the twist increases the resistance of 3/·064 in. cable by 2 per cent.;

$$\therefore \text{ res. of } 3/\cdot064'' \text{ cable per 1000 yd.} = 2\cdot493 \times 1\cdot02$$
$$= 2\cdot54 \text{ ohms at } 60^\circ \text{ F.}$$

With a 7-strand cable the increase is 2 per cent. on 6 of the 7 wires, *i.e.* an average increase of $\frac{6}{7}$ of 2 per cent. = 1·71 per cent.;

$$\therefore \text{ res. of } 7/\cdot064'' \text{ cable per 1000 yd.} = 1\cdot069 \times 1\cdot017$$
$$= 1\cdot087 \text{ ohms at } 60^\circ \text{ F.}$$

Neglecting twist of wires:—

$$\text{Cross-section of } 3/\cdot064'' \text{ cable} = 3 \times 0\cdot785 \times (\cdot064)^2$$
$$= \cdot00965 \text{ sq. in.}$$
$$\text{Cross-section of } 7/\cdot064'' \text{ cable} = 7 \times 0\cdot785 \times (\cdot064)^2$$
$$= \cdot02252 \text{ sq. in.}$$

Since twist increases resistance it diminishes the effective cross-section (because $R \propto 1/A$);

$$\therefore \text{ effective cross-section of } 3/\cdot064'' \text{ cable} = \frac{\cdot00965}{1\cdot02}$$
$$= \cdot00946 \text{ sq. in.}$$

and

$$\text{effective cross-section of } 7/\cdot064'' \text{ cable} = \frac{\cdot02252}{1\cdot017}$$
$$= \cdot0221 \text{ sq. in.}$$

16. Cables (*cont.*)

The insulation used is either (*a*) Vulcanised rubber; (*b*) Impregnated paper; (*c*) Varnished cambric; or (*d*) Vulcanised bitumen.

In class (*a*) to protect the copper from sulphur the wires are tinned, then covered with a double layer of pure rubber. The

vulcanised rubber is usually in two layers. The inner (the so-called "separator") is light drab and contains chiefly zinc oxide in addition to the sulphur of vulcanisation, which latter is kept small in quantity so as to avoid much excess being left. The outer layer is grey and contains a larger proportion of sulphur, and various minerals are employed (cf. Art. 12).

The quality of the insulation, especially in respect of durability, depends mainly on the percentage of rubber in the mixtures, and on whether fine Para or an inferior quality is used. This percentage should be not less than 40 per cent. for 500 volt cables, and rubber substitutes should not be used on any account. The cable is finished off with a waterproofed cotton tape. It is protected by a tarred jute braid and by soaking in an insulating compound (see Fig. 3.11), or else by a lead sheath.

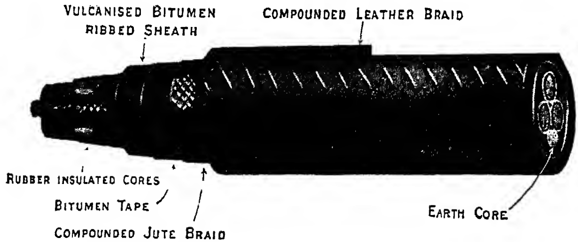


Fig. 3.11.—RUBBER AND BITUMEN INSULATED, BRAIDED CABLE.

In class (*b*) the paper is soaked in an insulating compound whose basis is resin oil, but which varies with different makers. The paper should be of Manila fibre only, as this gives the highest dielectric strength and the greatest durability. A series of paper strips are wrapped spirally round the cable till the required thickness is obtained, each wrapping being about five mils thick. The paper may be impregnated either before or after application, the latter method being more general.

Being hygroscopic it must be protected by a waterproof covering. A pure lead sheath is the usual form (see Fig. 3.12), but occasionally something of the vulcanised bitumen class is used.

The change of resistance with temperature is very rapid, about 8° F. rise being sufficient to halve the resistance.

Varnished cambric has a higher dielectric strength than impregnated paper, so that thinner insulation may be used, and it is less

hygroscopic. Owing to its greater cost it is used only in very high pressure cables.

In class (*d*) the insulator is produced by vulcanising a distillation product of certain oils, not bitumen itself as the name implies. It is waterproof and so a lead sheath is unnecessary, and it is cheaper than vulcanised rubber. The main disadvantage is liability to decentralisation of the conductor. Attempts to prevent this are apt to increase the dangers of oxidation and of damage by other chemical influences, and introduce a possibility of cracking in cold weather.

When cables may be exposed to mechanical injury, *e.g.* when laid direct in the ground, they should be armoured. This armouring consists either of two steel tapes wrapped on so that the second covers the gaps in the first, or of galvanised steel wires applied over a layer of tarred jute.

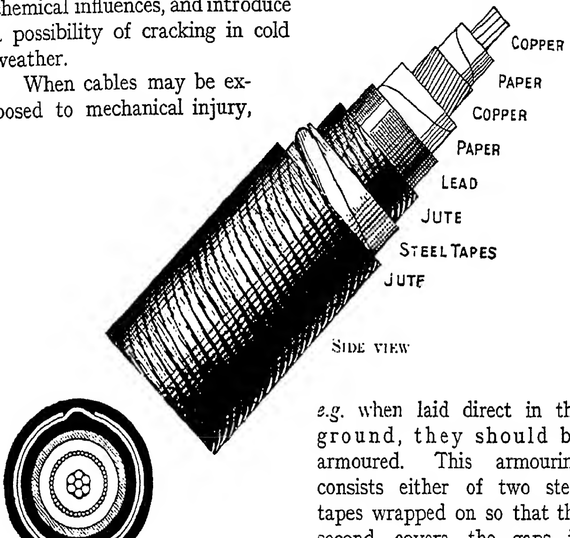


Fig. 3.12.—CONCENTRIC CABLE, PAPER-INSULATED, LEAD-COVERED, AND ARMOURED.

17. Cable Insulation Tests

The direct deflexion test is the usual method employed at the makers' works. The cable is immersed in water for twenty-four hours previous to the tests. The ends are trimmed and protected from moisture by wax or by rubber tape.

A 500-volt battery is then connected through a highly sensitive galvanometer (see Fig. 3.13) to the conductor ends. The other end is connected to the sides of the tank, if metallic, or to a metal plate

in the tank, or to the lead sheath of the cable (if it has one). The short-circuiting key of the galvanometer must be closed before completing the circuit, to avoid damage to it from the first rush of current which charges up the capacitance of the cable. After a few seconds this key may be opened, and the gradually decreasing deflexion of the galvanometer watched and noted down one minute after closing the circuit (" after one minute's electrification ").

The same battery and galvanometer (but shunted) are then

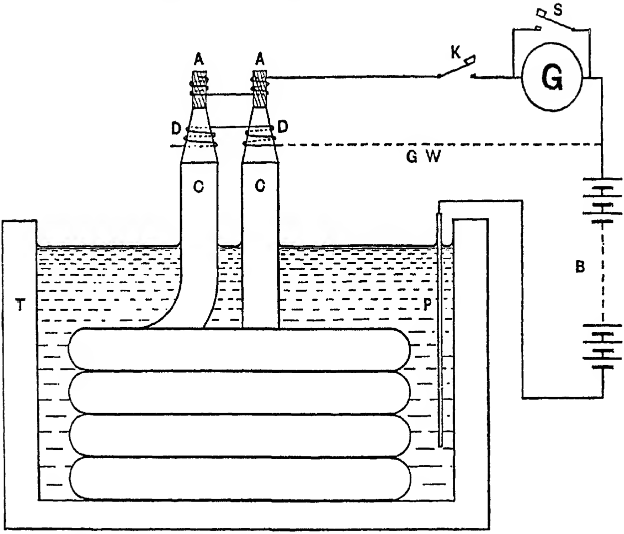


Fig. 3.13.—DIRECT DEFLEXION CABLE TEST, AND GUARD WIRE.

A A, Ends of cable core. B, Battery. C C, Cable sheath. G, Galvanometer. G W, Guard wire. K, Circuit Key. S, Galvanometer short-circuiting key. P, Metal plate. T. Water tank.

connected to a standard high resistance (usually one megohm) and the deflexion again noted. The insulation resistance can then be obtained since the currents sent through two resistances by the same voltage are inversely proportional to the resistances. *E.g.*—

RESISTANCE	DEFLEXION OF GALVANOMETER	SHUNT POWER	DEFLEXION × SHUNT POWER
Insulation of cable	33	1	33
1 megohm ..	192	1000	192,000

Then insulation resistance of cable $= \frac{192000}{33} \times 1 = 5{,}800$ megohms.

A slightly different method is to use a definite fraction of the battery voltage in the second test instead of shunting the galvanometer. This fractional voltage is obtained by a potential divider which acts on the same principle as a potentiometer volt-box (see Chapter VII., Art. 26).

Price's guard wire is used sometimes to prevent errors in the results owing to leakage over the surface of the ends of the insulation. A few turns of wire are wound on the end trims between the conductor and the lead sheath but touching neither, and are connected as shown by the dotted line in Fig. 3.13. Then if leakage occurs it takes place from this wire to the sheath, and so does not affect the galvanometer reading. If the guard wire is omitted all leakage from the conductors to the sheath over the end trims passes through the galvanometer, and so lowers the measured resistance.

18. Insulation Resistance of Cables

The insulation resistance of a cable depends on—

(*a*) the length of the cable,

(*b*) the thickness of the insulation,

(*c*) the material used for insulation.

(*a*) The insulation resistance varies *inversely* as the length, whereas the conductor resistance varies directly as the length. This is not because insulators differ from conductors in the way in which resistance depends on form, but because the length of the insulator whose resistance is in question is the thickness of the dielectric. And the longer the cable the greater the breadth of the insulator, and thus the less the insulation resistance; in accordance with the ordinary law, R varies inversely as sectional area.

Consequently, though the quality of the insulation resistance of a cable is usually stated in "megohms per mile," a better term is "megohm-miles": for the value is obtained by multiplying (not dividing) the insulation resistance in megohms by the length in miles (see Example 7).

(*b*) It might appear at first sight that doubling the thickness of the insulation would double its resistance, but the increase is actually less. The reason will be seen by noting that though current has to go through twice the thickness, the average width of its path is increased by the increasing diameter. Thus the

second layer of insulation of given thickness is less effective than the first in preventing a leakage of current.

Let d = diameter of conductor.

t = thickness of insulation.

D = diameter of insulation = $d + 2t$.

l = length of cable.

ρ = resistivity of insulator.

Then the insulation resistance is that of a body of length t, breadth l, and width varying from πd to πD.

The assumption that this is equivalent to one whose uniform width is the mean of these two gives the result—

$$\text{Insulation resistance} = \frac{\rho t}{l\frac{\pi(D+d)}{2}} = \frac{2\rho t}{\pi l(D+d)}$$

$$= \frac{\rho t}{\pi l(d+t)}.$$

[N.B.—Any unit of length may be used so long as it is kept to throughout, and the corresponding value of ρ is taken.]

The above assumption is not true, but gives a fair approximation for moderate thicknesses. The exact law requires the calculus for its proof, and is—

$$\text{Insulation resistance} = \frac{\rho}{2\pi l}\log_{10}\frac{D}{d} = \frac{0{\cdot}366\,\rho}{l}\log_{10}\frac{d+2t}{d}.$$

where ρ = resistivity of dielectric in ohm-cm.,

l = length of cable in cm.

If D, d, and t are in inches (or other convenient unit), l in miles,

and ρ in megohms per inch cube,

then— $$\text{I.R.} = \frac{\rho}{1{\cdot}726 \times 10^5 \times l}\;\frac{D}{d}\text{ megohms.}$$

Fig. 3·14 shows the way in which the insulation resistance varies with increasing thickness of insulation according to the exact and approximate formulae.

It is worth noting that to maintain constant insulation resistance per mile the thickness of the insulation would have to be increased in proportion to the diameter of the cable. This is not done, hence small cables should have a higher insulation resistance per mile than large ones with the same quality of insulation.

Example 8. *In what manner do (a) the* conductor resistance, (*b*) *the* insulation resistance *of uniform electric light cable depend upon the length of the cable? A 55-yard length of such a cable when tested gave* 0·11 *ohm and* 10,000 *megohms respectively, for (a) and (b); find the corresponding values for a mile length of similar cable.* [C. & G., I.

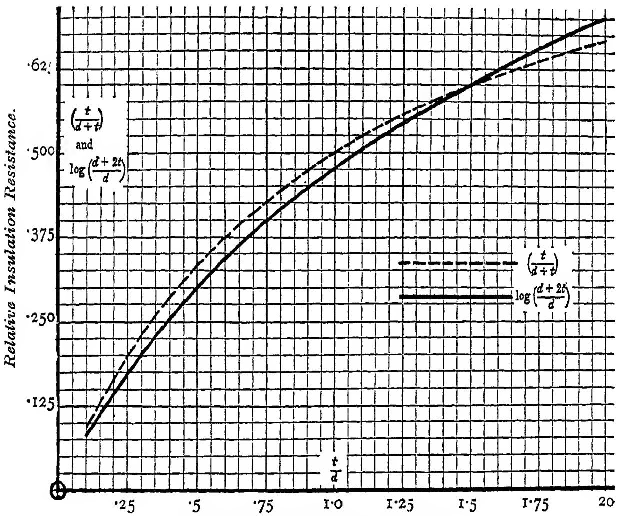
Conductor resistance varies directly as the length;

$$\therefore \text{ conductor resistance of 1 mile} = 0{\cdot}11 \times \frac{1760}{55} = 3{\cdot}52 \text{ ohms.}$$

Insulation resistance of a cable varies inversely as length;

$$\text{insulation resistance of 1 mile} = 10{,}000 \times \frac{\quad}{1760} \quad 312{\cdot}5 \text{ megohms,}$$

i.e. the insulation resistance is $312\frac{1}{2}$ megohm-miles.



Ratio of insulation thickness to core diameter.

Fig. 3.14.—INSULATION THICKNESS AND RESISTANCE.

Example 9. *If a* 37/·103″ *cable with vulcanised rubber insulation* 102 *mils thick has an insulation resistance of* 600 *megohms per mile; what will be the I.R. per mile of a* 7/·036″ *cable with similar insulation* 41 *mils thick? (These are standard sizes for* 0·3 *sq. in. and* 0·007 *sq. in.)*

The diameter of a 37/·103″ core = 7 × 103 = 721 mils.

" " " 7/·036″ " = 3 × 36 = 108 mils;

$$\text{approx. } \frac{\text{I.R. of } 7/{\cdot}036'' \text{ cable per mile}}{\text{cable per mile}} = \frac{\frac{41}{108 + 41}}{721 + 102} = \frac{41}{} \cdot \frac{823}{}.$$

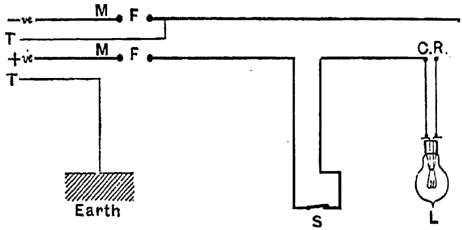
$$\text{I.R. of } 7/{\cdot}036'' \text{ cable} \quad \frac{41 \times 823}{149 \times 102} \times 600 \quad 1330 \text{ megohms/mile.}$$

By the more accurate logarithmic rule

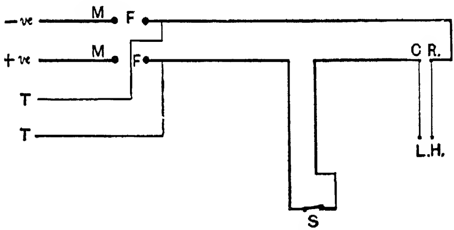
$$\frac{\text{I.R. of } 7/{\cdot}036'' \text{ cable}}{\text{I.R. of } 37/{\cdot}103'' \text{ cable}} = \frac{\log \dfrac{108 + 82}{108}}{\log \dfrac{721 + 204}{721}} = \frac{2{\cdot}2787 - 2{\cdot}0334}{2{\cdot}9661 - 2{\cdot}8579}$$

$$= \frac{{\cdot}2453}{{\cdot}1082} = 2{\cdot}27;$$

$\therefore$ I.R. of $7/{\cdot}036''$ cable $= 600 \times 2{\cdot}27 = 1360$ megohms/mile.



(*a*) Insulation to earth.



(*b*) Insulation between mains.

Fig. 3.15.—TESTS OF INSULATION OF WIRING.

C R, Ceiling rose. F F, Position for fuses, which are removed. L, Lamp in holder.
L H, Lampholder with lamp removed. M M, Mains. S, Switch.
T T, Test leads to ohm-meter.

19. Insulation Tests of Wiring

When a house or factory is wired for electric lighting, heating, or power the insulation resistance of the system depends but little on the cable used, provided there are no faults in it.

Most of the leakage takes place from switches, ceiling roses, distribution boxes, and over the ends of the cables where they are connected to these. The total insulation resistance of the installation may therefore be expected to be lower the more switches and

lights are installed. An allowance is made for this in the rules regarding installations.

Thus the I.E.E. rule is that the insulation resistance of the wiring shall not be less than 100 megohms divided by the number of outlets, and that of the wiring, with all lamps and their fittings connected, not less than 50 megohms divided by the number of outlets. In the latter case the test is made in two ways, "to earth" and "between conductors." The former test is made, with all lamps connected and all fuses and switches on, one test lead being connected to any point of the wiring and the other to a water pipe or other convenient "earth." If faulty, further tests with switches open, and with lamps removed, aid in locating the fault.

The second test is made with the lamps and other appliances removed, and the test leads connected to the two mains. These connexions are shown diagrammatically in Fig. 3.15 for a single lamp installation.

20. The Megger and Ohm-meter

Wiring installation tests are readily made by means of some form of *megger* or *ohm-meter*, *i.e.* an instrument which gives the resistance directly without any calculation. The principle of these instruments is shown in Fig. 3.16.

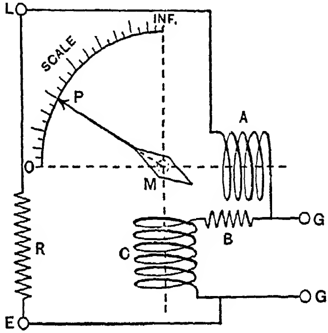


Fig. 3.16.—PRINCIPLE OF OHM-METER.

A, Current or series coil. B, High resistance in series with C, The pressure coil. G G, Terminals connected to generator. L, E, "Line" and "earth" terminals of instrument for connecting to R. M, Pivoted magnet. P, Pointer. R, Insulation resistance under test.

Two coils, A and B, at right angles are connected as shown. Thus A carries the current which flows through the insulation under test, and B carries a current proportional to the voltage applied to it. A pivoted magnet is acted on by both coils, and since there is no controlling force it takes up its position in the direction of the resultant field of the two coils. This direction depends on the *ratio* of the ampere-turns of the two coils and not on their actual amounts, for a change of the two in the same proportion will alter the strength, but not the direction, of the resultant field.

Now $\frac{\text{ampere-turns in B}}{\text{ampere-turns in A}}$ is proportional to $\frac{\text{voltage}}{\text{current}}$, *i.e.* to the resistance under test. Thus the position of a pointer attached to the magnet gives this resistance directly, the scale ranging from 0 when the magnet points along the axis of A, to Inf. (*i.e.* infinity or enormously large) when it points along the axis of B. The sort of scale obtained in between depends on the shapes of the coils, and on the numbers of turns in A and B.

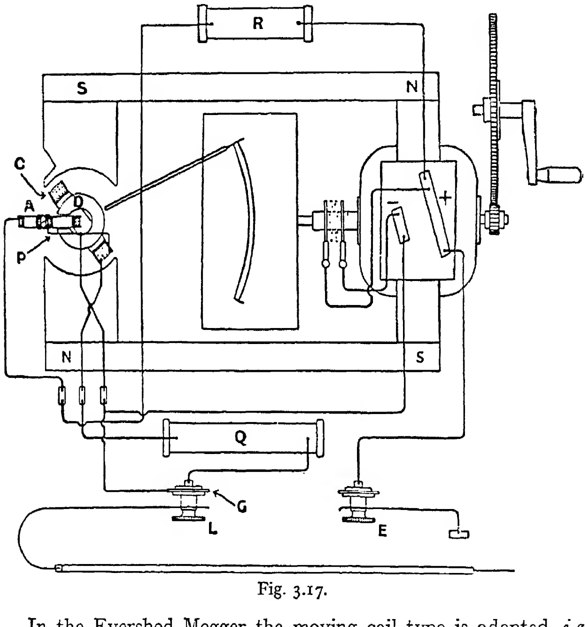


Fig. 3.17.

In the Evershed Megger the moving coil type is adopted, *i.e.* the magnet is fixed and the two coils are pivoted. The effect of stray fields is diminished greatly by this arrangement, as in other moving coil instruments.

The general arrangement is shown in Fig. 3.17. SN, NS are two straight bar magnets which supply the fields for the generator and for the ohm-meter itself.

C is the current coil. P is the voltage coil.
A is a compensating coil. D is an iron core.

E and L are the terminals (earth and line) for connecting to the insulation resistance under test.

I is the pointer.

The armature is driven by hand through gearing and generates from 100 to 2500 volts (5000 volts with twin generator) when the handle makes 160 revolutions per minute.

The current goes from the +ve terminal by terminal E through the resistance under test, back by terminal L through a resistance Q (to protect the instrument on short circuit) and coil C to the −ve terminal. There is also a shunt circuit through the high resistance R and the coils A and P. The three coils A, P, C, and the pointer all move together and the directions of the currents in the coils are shown in Fig. 3.18. The object of coil A, the special shaping of the S pole, and the form adopted for the core D is to obtain a better divided scale than could otherwise be done.

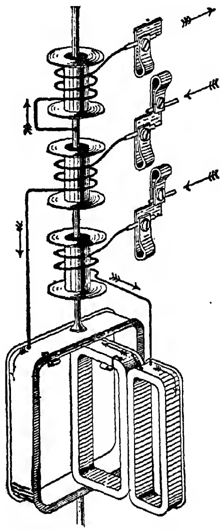


Fig. 3.18.

21. Heating of Conductors

The heat produced in a conductor of R ohms resistance through which a current of I amperes flows for t seconds is—

$$I^2 Rt \text{ watt-seconds.}$$

The *temperature rise* of the conductor depends upon this and upon (*a*) the rate of loss of heat by radiation, convection, and conduction; and (*b*) the specific heat capacity of the material. The latter affects the temperature only in the initial stages. If the current is kept constant till the temperature has reached a steady state, the rise of temperature is determined by the first two alone. In this case the heat produced per second in the conductor must be equal to the heat per second leaving it by convection, etc.

The latter depends upon

(*a*) the presence or absence of insulation,

(if bare) (*b*) the extent and nature of the surface of the wire,

(if insulated)

(*c*) the thickness and heat conductivity of the insulation,

(*d*) the extent and nature of the surface of the insulation,

(*e*) the surroundings, *i.e.* the freedom of heated air to rise from the wire, the presence of other bodies in contact with or close to the wire, etc.

The determination of the rate for any particular case is therefore a complicated problem, but certain general results can be stated.

I. If two wires, one polished, the other blackened or rough, but the same in all other respects, carry equal currents, the polished one will be heated to the higher temperature. For it is the poorer radiator, and so becomes hotter before it radiates heat as rapidly as the other.

II. An insulated wire *may* be cooler than a similar bare wire carrying an equal current. The nature and thickness of the insulation determine whether the increased surface is more or less than sufficient to compensate for the drop of temperature from the wire to the surface of the insulation. See further, Arts. 22, 25.

22. Carrying Capacity

The *carrying capacity* of a conductor is the maximum current which can flow through it without causing undue heating.

An empirical rule for this in the case of insulated copper cables is to take 1000 amperes per square inch of cross-section, *i.e.* a constant *current density*. While satisfactory for large cables this is too low for small ones, in which the current density can be raised much higher without causing any greater rise of temperature.

In 1911, as the result of a series of experiments at the National Physical Laboratory, the Institution of Electrical Engineers issued a table of carrying capacities in connexion with their revised wiring rules.

This table is on the basis of a rise of temperature of 20° F. for rubber-insulated cables and of 50° F. for paper- or fibre-insulated ones, with a margin to allow for contingencies.

A few examples are given below for single cables run in pairs (up to four may be bunched in the smallest size given).*

No. of Wires	Diam.	Carrying Capacity (amp.)				Current Density, amp./sq. in.	
		a	*b*	*c*	*d*	*c*	*d*
3	·036″	3·0	6·4	10	10	3 340	3 340
7	·044″	10·5	18	31	42	2 970	4 000
19	·064″	60	75	83	135	1 370	2 240
37	·083″	196	197	184	296	920	1 480
127	·103″	1040	774	595	932	575	900

a. At 1000 amp. per sq. in. *b*. By old rule $I = 2{\cdot}6\ A^{0\cdot82}$.
c. I.E.E. table for rubber-insulated cables.
d. I.E.E. table for paper-insulated cables.

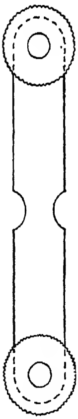


Fig. 3.19. FUSE STRIP.

Fig. 3.20.—PORCELAIN FUSE-HOLDER.

* For complete tables see the " Regulations for the electrical equipment of buildings " of the Institution of Electrical Engineers, Tenth Edition.

23. Fuses

Fuses are short pieces of metal which melt when an excessive current flows through them for a sufficient time. The circuit is thereby interrupted and so damage by overheating is prevented at the expense of the fuse. The greater the current the shorter the time needed to melt the fuse.

For small currents (up to 20 A.) tin wires are used, or an alloy of tin and lead. The B.S.I. standard alloy consists of 63% tin and

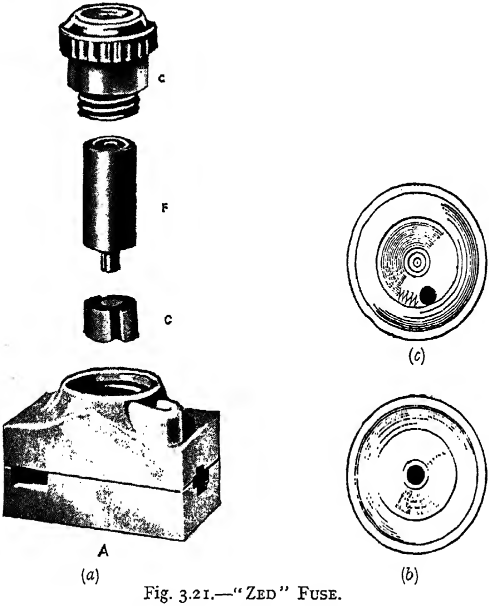


Fig. 3.21.—"ZED" FUSE.

B, Porcelain body. C, Screw cap. F, Fuse. G, Guage ring to prevent insertion of larger fuse.
(a) Side view of parts. (b) Front view, no mal. Front view, fuse blown.

lead. For larger currents copper is usual, or occasionally silver. Aluminium is not very satisfactory, as its fusing current is variable owing to a skin of oxide sometimes forming and holding up the molten metal within. Zinc (in strip form only) is good if a fuse with a considerable time-lag is required (*i.e.* one which does not melt very quickly with a small overload). For still larger currents copper strip is used in place of wire, the width of the

strip being reduced at the centre to ensure melting occurring there (see Fig. 3.19). Copper strip or wire should be tinned to protect it from oxidation.

The fuse may be freely exposed to the air, but is generally supported on a porcelain holder such as the one shown in Fig. 3.20.

To diminish the liability to damage by the splashing of molten metal when a short circuit occurs fuses may be *protected*, *i.e.* enclosed in a porcelain or fibre tube with open ends, or *enclosed*, *i.e.* completely shut in such a tube. In the latter case the tube is filled with a refractory non-conducting powder (*e.g.* sand, french chalk, etc.) to prevent the formation of an arc; and sometimes the centre portion of the tube is not filled, so as to ensure the fuse breaking there.

A circular type of porcelain fuse-holder has been widely adopted in Europe and the U.S.A. An example of this is the " Zed " fuse of Siemens (see Fig. 3.21).

24. The Carrying Capacity of Fuses

This depends mainly on the metal used and the cross-section, but is affected also by the length, the state of the surface, and the surroundings of the fuse. As stated in Art. 21, when the temperature is steady—

Heat produced per sec. = heat lost per sec. by convection, radiation, and conduction.

If it is assumed that the heat lost per second is proportional to the surface, then for round wires

$$I^2R = \text{surface} \times \text{constant} = d \times l \times \text{const.}$$

where d = diameter of wire

and l = length " "

Further
$$R = \frac{\rho l}{A} = \frac{\rho l}{\frac{\pi}{4}d^2};$$

$$\therefore\ I^2 \frac{\rho l}{\frac{\pi}{4}d^2} = d \times l \times \text{const.};$$

$$\therefore\ I^2 = \frac{d^3}{\rho} \times \text{const.}$$

or for any particular metal

$$I^2 = d^3 \times \text{const.}$$

i.e.
$$I = a\sqrt{d^3}.$$

This is the ordinary "fuse law," and Preece has given the values of a shown in Table C.

The truth of this law is doubtful.

TABLE C.—FUSE CONSTANTS (Formula $I = ad^{\frac{3}{2}}$).

(Sir W. H. Preece.)

MATERIAL	VALUE OF CONSTANT			
	d in inches	d in mils	d in cm.	d in mm.
Copper	10244	0·324	2530	80
Aluminium ..	7585	·230	1873	59
Iron	3148	·0996	777	24·6
Tin	1642	·0520	405·5	12·8
Lead	1379	·0436	340·6	10·8

A. Russell (Proc. Phys. Soc., Vol. 22, p. 432) shows that a more probable law is $I \propto \sqrt[4]{d^5}$, because the heat dissipated by convection varies as $\sqrt{d}$. He supports this by the results of experiments by Schwartz and James.*

All the above apply to the "normal fusing current" (N.F.C.), *i.e.* the least current which will melt the fuse if sufficient time is allowed for heating up (cf. Art. 21).

The length of the fuse affects the fusing current, since some of the heat will be conducted to the terminals and dissipated from them. Hence the shorter the fuse the greater the fusing current. For a fixed length the percentage increase of current is greater the larger the fuse. The reason for this is that the heat conducted varies nearly as d^2, and so increases faster with size than does the heat dissipated in the other ways (see further Art. 25).

The size of the terminals has some effect on the fusing current for the same reason. This effect is slight unless the terminals are so small that they become very hot.

In any case the *effects of the holder* in which fuses are usually placed modify the fusing current. In the type in which the fuse wire is in contact with porcelain the fusing current is increased, the increase being most marked with small fuses. In the type in which the fuse runs down the centre of a tube the fusing current is diminished.

* Journal Inst. E. E., Vol. 35, p. 364.

Experiments on tin by the author* lead to the approximate result

$$I \propto d$$

for tin wires in a given fuse-holder.

The *state of the surface* evidently may cause considerable differences between two fuses similar in all other respects (cf. Art. 21). The same applies to the powder used for packing enclosed fuses. A *stranded fuse* will fuse with less current than the product of the fusing current multiplied by the number of strands, as the following table shows:—

No. of wires:		3,	4,	7.
$\dfrac{\text{Fusing current}}{\text{Fusing current for 1 wire}}$	·3,	·4,	·4	4.

The above ratios are, however, considerably affected by the form of fuse-holder used.†

If the strands are twisted together the fusing current will be reduced further.

25. Fuses of Rectangular Section

Fuses of this type will carry currents depending on their shape as well as on their cross-section and material. *E.g.* by doubling the breadth and halving the thickness the surface area is increased and the escape of heat facilitated, thus the fusing current is increased without changing the cross-section.

The following approximate rules, due to Schwartz and James‡ take this fact into account.

For copper $5\frac{1}{2}$ in. long:—

$$\text{Fusing current} = 36{,}500 \times b \times (t + \cdot 00355) \text{ amperes.}$$

If placed horizontal the N.F.C. is 10 per cent. more.

For lead:—

$$\text{Fusing current} = 2060 \times (b + \cdot 016) \times t^{0 \cdot 47} \text{ amperes.}$$

For zinc:—

$$\text{Fusing current} = \text{constant} \times (b + \cdot 04) \times (t + \cdot 004) \text{ A.}$$

where b = breadth of strip in inches

and t = thickness of strip in inches.

* Journal Inst. E. E., Vol. 44, p. 162.

† See further Schwartz and James, and Maccall, *loc cit.*

‡ *El. Rev.*, Vol. 57, p. 792.

The constant multiplier varies with the length of fuse employed; *e.g.* for zinc this constant is 11,000 for fuses 3 in. long, and 9,500 for 4 in. fuses in a vertical position. If the fuse is placed horizontally the constants are increased by about 300. The formula holds for thicknesses from ·004 in. to ·04 in., and for breadths from 0·16 in. to 0·6 in. It is purely empirical and the following formula (also empirical) obtained by the author fits the experimental results at least equally well and is more rational:—

$$\text{Fusing current for zinc strips} = \text{const.} \times A^{0\cdot 78} \times P^{0\cdot 14} \text{ amperes,}$$

where
$$A = \text{cross-section in sq. in.}$$
$$P = \text{periphery} = 2\,(b + t),$$

and the constant is 5200 for a fuse 3 in. long placed vertically, and 4500 for a 4 in. vertical fuse.

This has the advantage of showing more clearly the effect of changes of shape apart from change of cross-section and of being applicable over a wider range, though it is not quite so convenient to use.

For aluminium the formula is similar to that for zinc, being fusing current = constant $\times$ $(b + \cdot 035)$ $\times$ $(t + \cdot 0024)$, amperes.

For fuses 4 in. long, in horizontal position, the constant is 20,000; increasing to 24,000 for 3 in. fuses, and 30,000 for 2 in. In the vertical position the currents are 10 per cent. less.

26. Electric Heaters

Electrical energy is converted into heat easily, and the efficiency of conversion is 100 per cent. in all forms (with the exception of a small percentage turned into light in some forms), but the heat may not be all developed where it is required.

The forms of heaters used may be classified as—

(1) Radiators. (2) Convectors.

In the first class the heating element sometimes consists of carbon filament lamps (see Chapter XIII.). These are heated to a bright red, and owing to this lower temperature they last longer than similar lamps used for lighting. Each lamp consumes about 250 watts of electrical power and produces a corresponding amount of heat per second. The bulbs are 9 in. long and 2¼ in. diam., and are obscured. The heat is given off partly by radiation and partly by air convection.

A variant was the Bastian heater, which consisted of wires of Nichrome inside quartz tubes. When current passed through the wire the quartz tube became red hot. Consequently a larger proportion of the heat was radiated than from a lamp radiator.

The usual type consists of resistance wire spirals with a refractory support.

Frequently polished metal reflectors are used to direct the heat in desired directions. These do not affect the total heat produced, but may increase greatly the *useful* heat, just as a reflector may increase the useful light (Chapter XIV.)

An example of a convector is the Prometheus heater. The heating element consists of very thin metallic films deposited on mica. The working temperature is comparatively low, so that nearly the whole of the heat is given off by convection.

A more recent pattern is the Thermovent. In this a ribbed heating element heats the air, which flows out horizontally through a series of ducts. There are three of these, one inside another; and as they are heat-insulated from each other, the outer casing remains at a low temperature.

The volume of air passed per kilowatt of loading is 2600 cubic feet an hour: the velocity of the air stream from the convector is about 140 feet a minute. Consequently there is considerable circulation of air in the space to be heated, and the whole of it becomes warmed in an hour or less.

The low temperature of the casing prevents any fire risk and has advantages for portable patterns. The heat is distributed widely, and more uniformly than with radiators; this is preferable in many applications.

If so desired, the cool air may be drawn in from outside; this furnishing ventilation as well as heating. In other cases the cool air comes from low down in the room, giving air circulation but not true ventilation.

The advantage of radiant heat is that persons or objects can be warmed by it with little heating of the intervening air. Moreover the heat can be reflected in any desired direction, whereas in convection the heated air always tends to rise. Therefore radiators are better for heating particular objects (or persons) and for intermittent use, while convectors are better for warming the whole of a room fairly uniformly, and for continuous use.

TABLE A.—RESISTIVITIES, ETC.

MATERIAL.	Resistivity at 59° F. (= 15° C.) Microhm.	Microhm-inches.	Ohms per mil-foot.	Temperature coefficient. %	%	Density. Sp. Gr. Water = 1.	lb. per cu. ft.	lb. per cu. inch.
METALS.								
Copper (pure) ..	1·66	·654	10·0	·41	·23			
„ (annealed) ..	1·692*	·666*	10·18*	·401*	·223*	8·90*	555*	·321*
„ (hard drawn)	1·724*	·679*	10·37*	·393	·218			
Silver (annealed) ..	1·557	·613	9·36	·377	·210	10·5	655	·38
Aluminium (hard drawn)	2·788*	1·098*	16·77*	·408*	·227*	2·70*	169*	·098*
Iron, Telegraph Wire	12·2*	4·80*	73·4*	·57	·32	7·86	490	·28
Steel rails	10 upwd.	4 upwd.	60 upwd.	·4	·22	8	500	·29
Lead	21·7	8·55	130	·39	·22	11·4	710	·41
Mercury	95·4	37·56	573	·09	·05	13·57	850	·49
Nickel (commercial)	11·8	4·6	71	·27	·15	8·9	555	·321
„ (electrolytic)	7·6	3·0	45·5	·62	·35			
Platinum	11·54	4·54	69·4	·36	·20	21·5	1340	·78
Tin	11	4·3	66	·4	·22	7·3	455	·26
Tungsten	4·8	1·9	29	·47	·26	18·8	1170	.68
Zinc	6	2·4	36	·38	·21	7·1	400	·25
ALLOYS.								
Cadmium-Copper ..	2·054	0·809	12·4	·40	·22	8·9	556	·32
Constantan (Eureka) 60 % Cu, 40 % Ni	46	18·3	279	·002	·001	8·88	555	·32
German Silver—								
(*a*) 50 % Cu, 30 % Ni, 20 % Zn ..	39	15	235	·02	·01	8·8	530	·32
(*b*) 62 % Cu, 15 % Ni, 22 % Zn ..	27	10·6	160	·04	·02	8·9	555	·32
(*c*) 6 Cu, 1 Ni, 3 Zn	21	8·3	126	·07	·04	8·9		
Manganin— 84 % Cu, 12 % Mn, 4 % Ni	44	17·3	260	·00	·00	8·5	550	·31
Nickel Chrome ..	93·5	37	560	·04	·02	—	—	—
Nickel Steel	84	33	505	·08	·04	—	—	—
Platinoid— German Silver—								
(*a*) + 1 % Tungsten	42	16·5	250	·02	·01	8·8	550	·32
(*b*) „ „	34	13·4	200	·025	·015	8·9	555	·32
Platinum Silver— 67 % Pt, 33 % Ag	24·2	9·53	145	·03	·015	18	1120	·65
Carbon—								
Graphite	3000	1200	18,000			2·3	145	·08
Gas retort or filament ..	4000 to 7000	1600 to 2800	24,000 to 42,000	− ·05	− ·03	1·9	120	·07

of th… :graph wire are derived from those
N.B.… the best experimental results.
…mall).

TABLE B.—ANNEALED COPPER WIRE.

S.W.G. No.	DIA. Inch	DIA. Mm.	Sectional Area Sq. Inch	Sectional Area Sq. Mm.	Lb. per 1000 Yards	Ohms per 1000 Yards at 15° C.	Current Capacity I.E.E. Standard
0	·324	·230	08245	53·19	953·4	·2909	97
1	·300	·620	·07069	45·60	817·6	·3393	85
2	·276	·010	·05983	38·60	692·0	·4009	75
3	·252	·401	·04988	32·18	576·7	·4809	64
4	·232	·893	·04227	27·27	488·8	·567	57
5	·212	·385	·03530	22·77	408·2	·679	48
	·192	4·877	02895	18·68	334·7	·828	42
	·176	4·470	·02433	15·70	281·3	·985	36
	·160	4·064	·02011	12·97	232·5	1·192	
9	·144	3·658	·01629	10·51	188·4	1·471	27
10	·128	3·251	·01287	8·303	148·8	1·862	21
11	·116	2·946	·01057	6·819	122·2	2·268	18
12	·104	2·642	·008495	5·480	98·24	2·821	
13	·092	2·337	·006648	4·289	76·88	3·605	12·4
14	·080	2·032	·005027	3·243	58·13	4·768	9·8
15	·072	1·829	·004072	2·627	47·09	5·89	8·2
16	·064	1·626	·003217	2·075	37·20	7·45	6·8
17	·056	1·422	·002463	1·589	28·48	9·73	5·4
18	·048	1·219	·001810	1·168	20·93	13·24	4·2
19	·040	1·016	·001257	·8109	14·53	19·07	3·2
20	·036	·9144	·001018	·6567	11·77	23·54	2·6
21	·032	·8128	·000804	·5188		29·80	2·2
22	·028	·7112	·000616	·3973	7·120	38·92	1·7
23	·024	·6096	·000452	·2919	5·233	53·0	1·35
24	·022	·5588	·000380	·2453	4·392	63·0	1·2
25	·020	·5080	·000314	·2027	3·633	76·3	1·0
26	·018	·457	·000254	·1642	2·942	94·2	
27	·0164	·4166	·000211	·1363	2·440	113·4	
28	·014	·3759	·000172	·1110	1·989	139·3	
29	·0136	·345	·000145	·09372	1·676	165·0	
30	·0124	·3149	·000121	·07791	1·399	198·4	
31	·0116	·2946	·000106	·06818	1·222	226·8	
32	·0108	·2743	·0000916	·05910	1·059	261·6	
33	·0100	·2540	·0000785	·05067	·9085	305·1	
34	·0092	·2337	·0000665	·04289	·7688	360·5	
35	·0084	·2134	·0000554	·03575	·6406	432·4	
36	·0076	·193	·0000454	·02927	·5249	528	
37	·0068	·172	·0000363	·02343	·4197	660	
38	·0060	·1524	·0000283	·01824	3272	848	

To obtain external diameter of Cotton Covered Wires.

	D.C.C. (fine)	D.C.C. (ordinary)	Braiding.
No. 34 to No. 18	..add 7 mils...	add 10 mils......	add 15 mils
No. 17 to No. 9....	„ 10 „ ...	12 „	„ 18 „
No. 8 and larger	.. „ 11 „ ...	14 „	„ 22 „

QUESTIONS ON CHAPTER III

1. Define the specific resistance of a metal. A wire made of a certain alloy 0·09 inch diameter has a resistance of 0·047 ohm per yard. Find the Sp. R. of the alloy.

How many yards of the same material, drawn to 0·018 inch diameter, would have a resistance of 100 ohms? [C. & G., I.

2. Find the rise in the mean temperature of a shunt field winding the resistance of which increases 20 per cent. on load.

3. A coil of wire has a resistance of 11·73 ohms at 15° C., and of 15·67 ohms at 100° C. What is its temperature coefficient, and what would its resistance be at 0° C.?

4. The resistance of a sample of pure copper is 16·3 ohms, whilst that of an exactly similar piece of commercial copper is 16·55 ohms. What is the percentage conductivity of this commercial copper?

If the temperature at which it was measured was 16° C., what would the resistance be at 0° C. taking the temperature coefficient as 0·00428 per ° C. at 0° C.

5. Calculate the diameter and equivalent cross-section of 3/18, 7/18, 19/18, and 37/18 cable.

What effect has stranding on (*a*) the resistance, (*b*) the weight, of a cable?

6. Calculate the resistance per mile of a /·072 inch cable given resistivity of copper = $\frac{2}{3}$ microhm per inch cube. Make no allowance for the stranding of the wires.

7. If the resistance of a cable 585 yards long is 0·627 ohm, and its insulation resistance 1054 megohms, what are the respective values per mile?

8. What is the effect of temperature on resistance?

If the resistances in Question 7 were measured at 60° F., what would they become if the temperature rose to 75° F.? (Temperature coeff. for copper at 32° F. = ·00238 per ° F. Insulation material has its resistance halved by a rise of 12° F.)

9. Describe the direct deflection method of measuring the insulation resistance of a cable, and the use of a "guard-wire."

10. Explain why the increase of thickness of insulation of a cable does not cause a corresponding increase of the insulation resistance per mile: and why the smaller of two cables, insulated with the same thickness of the same insulation, has a higher insulation resistance per mile.

Why is the expression "insulation resistance per mile" somewhat misleading?

11. A 37/15 cable with rubber insulation 80 mils thick has an insulation resistance of 600 megohm-miles. What will be the I.R. per mile of a 7/18 cable with similar insulation 44 mils thick?

12. Describe the action and method of use of any one type of ohm-meter or megger.

13. Calculate the carrying capacity of a 7/·064 inch cable by the rule $I = 2{\cdot}6A^{0{\cdot}82}$. What is the current density in this case?

14. Calculate the currents carried by 3/18 and 7/18 cables (*a*) at 1000 amp. per square inch, (*b*) by the rule $I = 2{\cdot}6A^{0{\cdot}82}$. Calculate the ratio of the currents also.

Why would you expect the larger cable to carry less than $\frac{7}{3}$ of the current carried by the smaller?

15. If 5·2 amp. fuses a 24 S.W.G. (22 mils diameter) tin wire, what current will fuse a 19 S.W.G. (40 mils diameter) tin wire?

Calculate the current density in each case, and explain why the values are different.

16. In what way and to what extent are the fusing currents of wires affected by the fuse-holders?

17. If a zinc strip ·4 inch by ·01 inch fuses with 67 amp., find the fusing current for a zinc strip of the same length, half the width and double the thickness.

18. From the particulars given in the previous question calculate the fusing current of a zinc strip of the same length, double the width, and half the thickness.

19. Assuming that the heat dissipated by conduction varies inversely as fuse length, find the values of the constants for zinc strips, (*a*) 6″ long; (*b*) $2\frac{1}{2}''$ long.

CHAPTER IV

ELECTROMAGNETISM

1. Electromagnetism

When an electric current is passing through a conductor the space in its vicinity possesses certain magnetic properties, or, in other words, a *magnetic field* is produced. The subject of Electromagnetism deals with the dependence of these fields on the disposition of the conductors, the strengths of the currents in them, and the nature of the materials in their neighbourhood. Certain subsidiary problems arise in dealing with these, and may be classed under the same heading.

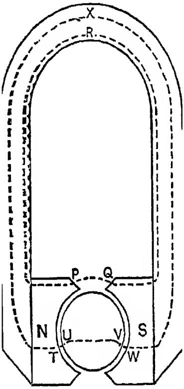


Fig. 4.01.—Magnetic Circuit of a Permanent Magnet.

2. Magnetic Fields: Lines of Force and of Induction

Magnetic fields, whether due to current-carrying conductors or to magnets, are thought of as consisting of magnetic *lines of force*. These are lines so drawn that the tangent at any point is in the direction of the magnetic force at that point. The direction of the lines is taken as that in which a N. pole would be urged. The lines may further be made to represent the strength of the field at every point by choosing their number and distribution in such a way that the number of lines per sq. cm. of area, taken perpendicular to the lines, is equal to the strength of the field at that point, *i.e.* the force in dynes exerted on unit magnetic pole (see Chapter II.).

Under these conditions each line is continuous from a point of N. polarity to one of S. polarity, and the circuit is completed by a *line of induction* through the iron or other magnetised body. An example is shown in Fig. 4.01, which represents the steel permanent magnet and soft iron core of a moving coil ammeter. The line of force PQ starts from the point P on the N. pole and finishes at the point Q on the S. pole, and its circuit is completed by the line of induction QRP. Similarly we have a circuit composed of the lines

of force TU and VW, and the lines of induction UV and WXT: in every case the circuit is a complete one. The lines of induction may be considered as strings of the small magnets of which a magnet (permanent or not) consists, according to the molecular theory of magnetism.

3. Field Strength

The strength of magnetic field at a point P in air, due to a current of I amperes in a short piece of a conductor, is $\frac{I . \delta l}{10 r^2}$,

where δl = length of conductor in centimetres, measured perpendicular to the line PQ joining P to the conductor (see Fig. 4.02),

and r = length of PQ in centimetres.

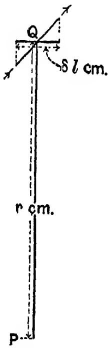


Fig. 4.02

The direction of the field is perpendicular to PQ and to the short piece of conductor. The 10 in the denominator is due to the fact that an ampere is 1/10 C.G.S. unit of current (see Chapter II.).

Hence the field at the *centre* of a coil of $\mho$ turns of radius r cm.

$$\frac{I}{10r^2} \times \quad = \quad \frac{2\pi . \mathit{AT}}{10r}$$

where AT = ampere-turns.

Note that the respective values of the current and of the number of turns makes no difference as long as their product is unaltered. The symbol AT is used to emphasise this fact. It may be pronounced " atts " in reading, or given the full name " ampere-turns." Similarly the symbol $\mho$ for the number of turns may be called " turns " in reading.

The strength of field at any point due to any arrangement of current-carrying conductors can be obtained by the use of the first formula by adding vectorially the forces due to each small portion of the conductors. For instance, it can be shown that the strength of field due to a long straight conductor at a point r cm. from it is

$$\frac{2 . I}{10r}.$$

In the case of a circle the field increases in strength as the conductor is approached: this increase is less than 1 per cent. for points whose distance from the centre is less than $\frac{1}{4}r$.

4. Right-Hand Screw Rule

Reversal of the current causes reversal of the direction of the field it produces, and the relation between the two is conveniently remembered by the *right-handed screw rule,* which may be used in either of the following two forms:—

I. If a right-handed (*i.e.* ordinary) screw is turned in the direction in which the current flows, it will advance in the direction of the magnetic field due to that current [see Fig. 4.03 (*a*)].

II. If a right-handed screw is made to advance in the direction of the current, its direction of turning will be that of the field due to this current, as is indicated by the curved lines in Fig. 4.03 (*b*).

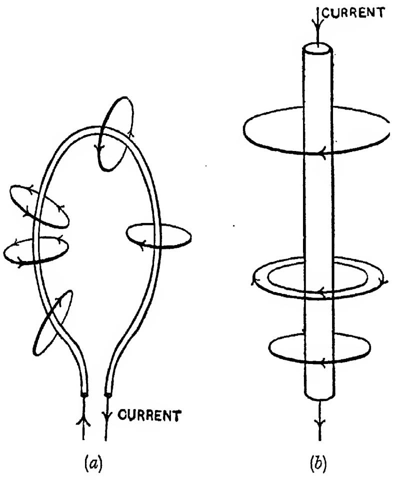


Fig. 4.03.—RELATIVE DIRECTIONS OF CURRENT AND MAGNETIC FIELD.

The former is more convenient for circular coils, and the latter for straight conductors. Their agreement can be tested readily by applying both to the same case.

5. Solenoid

Another important case is that of a solenoid, viz. a cylindrical coil of constant radius, and usually uniformly wound (see Fig. 4.04).

If the length of the solenoid is large compared with its radius, the strength of the field at the central point of the axis is

$\frac{4\pi}{10} \times$ ampere-turns per centimetre, or in symbols

$$\mathbf{H} = \frac{\quad}{10} \cdot \frac{\quad}{l},$$

where l = length of solenoid in centimetres. (For proof see Art. 12.)

The strength of field at other points on the axis inside the coil remains nearly constant at this value till the ends are approached. At the mouth of the coil the strength is just half the above, and outside it falls rapidly to very small values [see Fig. 4.04 (*b*)].

6. The Effect of Magnetic Material

The presence of iron, or other magnetic material, inside or near to a current-carrying coil causes an increase in the number of magnetic lines produced. The amount of the increase depends on the shape, position, and quality of the material. For instance, the placing of a soft-iron core in a solenoid will increase the flux, possibly a thousand-fold or more if the core fills the coil, and by a less amount in proportion to the cross-section of the core. Similarly, a shorter core produces less increase, though not in direct proportion to its length. Again, if the core is placed only partially within the solenoid the increase is less than before.

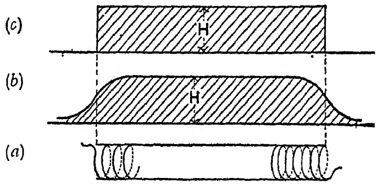


Fig. 4.04.—MAGNETIC FORCE OF SOLENOID.

A cast-iron or a hard steel core will cause an increase of flux, but the increase is less than with a geometrically similar soft-iron core.

7. Permeability

The above-mentioned difference in the magnetic behaviour of different materials is taken into account by means of what is known as the permeability (μ) of the material, which is defined as follows:—

The *permeability* of a material is the ratio of the strength of the magnetism produced in it to that produced in air under the same conditions.

Or:—Let $\mathbf{H}$ = the magnetic force at any point in the material due to current-carrying conductors and magnet poles, *including those of the piece of material under consideration* (see Art. 9),

and let **B** = the flux-density (lines of induction per sq. cm.) at the same point,

Then the permeability (μ) $= \dfrac{\mathbf{B}}{\mathbf{H}}$.

In air (or more strictly in a vacuum) **B** and **H** are equal (cf. Art. 2), since the permeability of a vacuum is taken as unity.

The permeability of nearly all materials is almost exactly unity. The most important exceptions are the various forms of iron and steel (see Table D, page 93, for values of μ for the most usual varieties). The only other exceptions are nickel, cobalt, and certain special alloys. The most interesting of these last is Hensler's Alloy (1 part of aluminium, 2 of manganese, and 4 of copper) which has a permeability about a third of that of the best pure iron.

8. Circular Ring

The case of a ring of uniform cross-section, and with a uniform winding, is a convenient one to consider. The section may be circular (the ring is then called a *toroid* or *anchor-ring*) or of some other shape, provided only that it is the same all round. No poles are then formed, and the magnetic state is the same at all cross-sections. Moreover, if the radius of the ring is large compared with the dimensions of the section, the magnetic force and magnetisation are very nearly the same at all points of the section.

Under these conditions and with a core of non-magnetic material—

The magnetic force at any point inside the winding is $\dfrac{4\pi}{10}$ $\times$ ampere-turns per cm. length of coil, and the magnetic induction or flux-density has the same value,

or
$$\mathbf{B} = \mathbf{H} = \frac{4\pi}{10} \cdot \frac{AT}{l}.$$

If a complete iron core is substituted and no other change made in the coil or the current, **H** is unchanged, while **B** is increased in the ratio of the permeability of the iron,

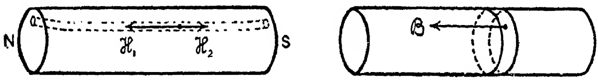
or
$$\mathbf{B} = \mu . \mathbf{H} = \frac{4\pi}{10} . \mu . \frac{AT}{l}.$$

Exactly the same applies to a very long solenoid; in fact, the ring is almost equivalent to an endless solenoid.

9. Demagnetising Effect

If an incomplete iron ring, or a solenoid with a short iron core is employed, the magnetic force (**H**) is diminished by the poles produced in the iron. This may be called the *self-demagnetising effect,* and cannot be calculated except in certain cases, *e.g.* for an ellipsoid. The values for this case and for the practical case of a straight bar of uniform circular section are given in Table E (page 94).

This demagnetising effect may be more evident if it is noticed that (**H**) the true magnetic force at a point is the force on unit pole placed at that point in a small cavity whose sides are everywhere in the direction of the lines of induction. The walls of such a cavity show no polarity and hence produce no force on the pole, its only object being to enable the (imaginary) unit pole to be placed within the metal. This unit pole will be acted on by a force of $\mathbf{H}_1$ dynes due to current-carrying conductors and magnets in its neighbourhood: this is called the *apparent magnetic force.* It will also be acted on by a force of $\mathbf{H}_2$ dynes, due to the poles produced in the



(a) (b)

Fig. 4.05. MAGNETIC FORCE AND FLUX-DENSITY.

piece of metal under consideration: this is called the *self-demagnetising force.* The true magnetic force is the resultant (vector sum) of these two. Usually $\mathbf{H}_1$ and $\mathbf{H}_2$ are directly opposed, so that

$$\mathbf{H} = \mathbf{H}_1 - \mathbf{H}_2.$$

10. Flux-Density

A more concrete idea of the meaning of flux-density can be obtained in a somewhat similar manner.

Imagine an air-gap perpendicular to the lines of induction and of *infinitesimal* breadth. This will not alter the number or distribution of the lines, but allows them to issue into the air and then re-enter the iron. The force in dynes on a unit pole placed at a point in this imaginary air-gap measures the value of the flux-density (magnetic induction) at this point [see Fig. 4.05 (*b*)].

11. Magneto-Motive Force

Except in the simple cases already dealt with, viz. a uniform ring and a uniform straight bar, the notion of the magnetic circuit

requires amplification. Two new conceptions are introduced, the first of which is that of magneto-motive force (M.M.F.).

> The **magneto-motive force** in any magnetic circuit is measured by the work (in ergs) done on unit magnetic pole moved round this circuit.
>
> The **difference of magnetic potential** between two points is the work done on unit magnetic pole moved from one to the other: the point in moving *towards* which work is done being at the higher potential.

It will be seen on comparing these with the definitions of electromotive force and difference of electric potential (Chapter II.) that the only change is the substitution of unit pole for unit quantity. All that applies to the latter applies equally to the magnetic quantities, with the necessary changes.

12. Relation of Magneto-Motive Force and Magnetic Force

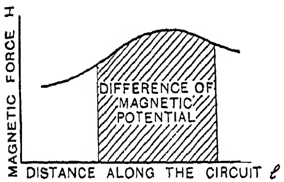


Fig. 4.06.—Relation of Magnetic Potential to Magnetic Force.

Since **H** is the force on unit pole, the above definitions may be written in the form

$$\text{M.M.F.} = \int \mathbf{H}.dl,$$

the integration being extended round the circuit considered, and l being the distance in cm. from some fixed point in the circuit.

And similarly,

$$\text{difference of magnetic potential} = \int \mathbf{H}.dl,$$

taken between the limits corresponding to the two points.

Expressing this graphically:—If the magnetic force at different points of a circuit is plotted to scale (see Fig. 4.06), the difference of magnetic potential between any two points is the area between the curve and the l-axis cut off by the ordinates corresponding to these two points. Similarly the M.M.F. is obtained by taking the whole area when the **H**-curve is continued completely round the circuit. If **H** is negative at any point the curve comes below the l-axis and the corresponding portion of the area is reckoned negative.

Take the case of a long solenoid (see Fig. 4.04). As stated in Art. 5, **H** is nearly constant for points along the axis inside the coil, and has there the value $\frac{4\pi}{10} \times$ (ampere-turns per cm.).

Hence the difference of magnetic potential between two points l cm. apart on the axis

$$= \mathbf{H} \times l = \frac{4\pi}{10} \times \text{ampere-turns per cm.} \times l$$

$$= \frac{4\pi}{10} \times \text{ampere-turns in length } l.$$

Since $\mathbf{H}$ rapidly falls to very small values on moving outside the coil, the actual curve of $\mathbf{H}$ may be replaced by a rectangle of a length equal to that of the coil [see Fig. 4.04 (*c*)]. It follows that the total magneto-motive force of the coil is equal to the area of this rectangle, *i.e.* to

$$\frac{4\pi}{10} \times \text{ampere-turns per cm.} \times \text{length of coil}$$

$$= \frac{4\pi}{10} \times \text{total ampere-turns.}$$

13. Relation of Magneto-Motive Force and Ampere-Turns

Magneto-motive force may be due to the presence of permanent magnets or of current-carrying conductors, or to both. In any magnetic circuit in which it is due to electric currents only the following important relation is true,

$$\text{M.M.F.} = 4\pi \times \frac{\text{I}}{10} \times \mathfrak{T} = \frac{4\pi}{10} \times \mathfrak{A} = 1{\cdot}257 \times \mathfrak{A},$$

where I = current in coil, in amperes,

$\mathfrak{T}$ = number of turns in coil,

$\mathfrak{A} = \text{I} \times \mathfrak{T}$ = ampere-turns.

Or, in words, the magneto-motive force in any magnetic circuit due to a coil is equal to $\frac{4\pi}{10}$ times the ampere-turns of the coil.

The result for a long solenoid (Art. 12) is a particular case of this general relation, and is exact in spite of the approximations employed in obtaining it.

14. Reluctance

This is the second new conception referred to in Art. 11. It depends on two quantities already defined, and is itself defined as follows:—

The reluctance ($\mathbf{R}$) of a magnetic circuit (or portion of a circuit) is the ratio of the M.M.F. applied to the circuit, to the flux produced by this M.M.F.,

or
$$\mathbf{R} = \frac{\text{M.M.F.}}{\Phi}$$

where Φ = total magnetic flux.

Reluctance depends on the form and material of the circuit, and *on the flux in the circuit.*

Thus the reluctance of a circuit of uniform cross-section is proportional to the length of the circuit, or $\mathbf{R} \propto l$.

Again the reluctances of circuits of equal length are inversely proportional to their cross-section, or $\mathbf{R} \propto \frac{1}{A}$.

Combining the above in one formula, $\mathbf{R} \propto \frac{l}{A}$.

Further it depends on the material of the circuit, for

$$\mathbf{R} = \frac{l}{\mu A},$$

μ being the permeability of the material as already defined.

Thus a circuit of high permeability has correspondingly *less* reluctance than one of low permeability.

15. Relations of Magnetic Quantities

This last relation follows from applying the definition of reluctance to any uniform electromagnetic circuit, uniformly wound, for

$$\mathbf{R} = \frac{\text{M.M.F.}}{\text{Flux}} = \frac{\frac{4\pi}{10}\mathcal{T}}{\Phi} = \frac{\mathbf{H}.l}{\mathbf{B}.A} = \frac{l}{\mu.A}.$$

Since the permeability of magnetic materials is variable (see Table D, page 93) the reluctance of a given circuit is not constant, but depends on the flux-density in the circuit.

From the above relation can be obtained the useful formulae—

$$\mathbf{B} = \left[\frac{\Phi}{A} = \frac{\frac{\text{M.M.F.}}{\mathbf{R}}}{A} = \frac{\frac{4\pi}{10}\mathcal{T}}{\frac{l}{\mu A} \times A} = \right] \mu . \frac{4\pi}{10} . \frac{\mathcal{T}}{l} = 1{\cdot}257 \frac{\mu . \mathcal{T}}{l}$$

and
$$\mathcal{T} = \frac{10}{4\pi} . \frac{\mathbf{B}l}{\mu} = 0{\cdot}8 \frac{\mathbf{B}l}{\mu}.$$

Example 1. *Determine the reluctance, the M.M.F., the magnetic force, and the ampere-turns necessary to produce flux-densities of* (a) 5000, (b) 10,000, *and* (c) 14,000 *lines per sq. cm. respectively in a transformer core with a mean length of magnetic circuit of* 60 *cm. and cross-section* 40 *sq. cm. Assume that the permeability has the values* 2500, 2000, *and* 1000 *at the above flux-densities.*

$$\text{Reluctance} = \frac{l}{\mu A} : \ (a) \ \mathsf{R} = \frac{60}{2500 \times 40} = \cdot 00060.$$

$$(b) \ \mathsf{R} = \frac{60}{2000 \times 40} = \cdot 00075.$$

$$(c) \ \mathsf{R} = \frac{60}{1000 \times 40} = \cdot 00150.$$

$$\text{M.M.F.} = \text{Flux} \times \text{Reluctance} = \frac{\Phi l}{\mu A} = \frac{\mathsf{B}l}{\mu}.$$

$$(a) \ \text{M.M.F.} = \frac{5000 \times 60}{2500} = 120.$$

$$(b) \ \text{M.M.F.} = \frac{10000 \times 60}{2000} = 300.$$

$$(c) \ \text{M.M.F.} = \frac{14000 \times 60}{1000} = 840.$$

$$\text{Magnetic force } (\mathsf{H}) = \frac{\mathsf{B}}{\mu} = \frac{\text{M.M.F.}}{l} : = (a) \ 2 : (b) \ 5 \ (c) \ 14.$$

$$\text{Ampere-turns} = \frac{10}{4\pi} \times \text{M.M.F.} : = (a) \ 96 \ (b) \ 240 \ (c) \ 670.$$

16. Comparison of Magnetic and Electric Circuits

There are many points of resemblance between these two forms of circuit. Just as current $= \frac{\text{E.M.F.}}{\text{resistance}}$, so flux $= \frac{\text{M.M.F.}}{\text{reluctance}}$, and there are thus three magnetic quantities corresponding to the three electric quantities. Current and flux correspond, and both magnetic flux and electric flow must take place in a closed circuit. The electro-motive and magneto-motive forces correspond, and, as pointed out in Art. 11, they are defined in similar ways. Finally resistance and reluctance correspond, and both of them vary as length $\div$ area of the circuit, and are dependent on the material of the circuit (Art. 14).

In comparing the electric resistance formula $R = \frac{\rho l}{A}$ with the magnetic reluctance one, $\mathsf{R} = \frac{l}{\mu A}$, it is seen that μ may be called the relative magnetic conductivity of a substance, air being the standard with $\mu = 1$.

There are, however, two very important differences between the magnetic and electric circuits.

Firstly μ (see Art. 15) is not constant for a given magnetic substance, unlike ρ, which remains constant provided the temperature of the substance is unaltered (see Chapter III.). In other words, Ohm's Law does not hold for magnetic circuits (except when no magnetic substances are present), the flux not being *proportional* to the M.M.F. applied, though increasing with its increase. A reference to Table D (or Fig. 4.09) will illustrate this.

Another important difference is that there are no magnetic insulators. The ratio of the highest to the lowest permeability is about 3,000; whereas for electric conductivity the ratio is enormously great (cf. Chapter III.).

17. Magnetic Leakage

This absence of magnetic insulators leads to *magnetic leakage, i.e.* the straying of magnetic lines to places where they are useless, or even harmful.

Thus in Fig. 4.07 suppose a flux is to be produced in the air-gap G, by a current in the coil A wound on the iron core BC. At the same time a number of magnetic lines *l, l, l* are produced, which are useless for the above purpose. These lines are called the *leakage flux*. This leakage takes place continuously in passing from the centre of the coil, where the flux is evidently a maximum, to either side of the air-gap, so that the value of Φ continually varies. To reduce the labour of taking this into account it is usual to assume that Φ remains constant at its maximum value for the iron part of the circuit: the error thus introduced is on the safe side, as it causes the M.M.F. calculated to be slightly greater than what is actually necessary. This leakage is taken into account by means of a Hopkinson leakage coefficient, this being defined as the ratio of the maximum flux to the useful flux, or in symbols

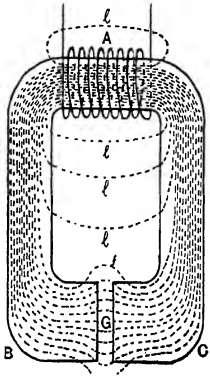


Fig. 4.07.—MAGNETIC LEAKAGE AND FRINGING.
l l, Leakage lines.
f f, Fringing.

$$u = \frac{\Phi_m}{\Phi}; \text{ and so } \Phi_m = u \,.\, \Phi.$$

At the same time the fringing which occurs at the edges of the air-gap (as *f, f,* Fig. 4.07) increases the effective cross-section of the air-gap above that of the pole-faces, and this may be taken into

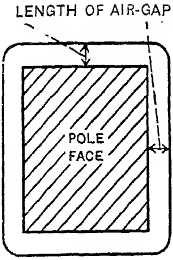


Fig. 4.08.—ALLOWANCE FOR FRINGING.

account approximately by adding a strip whose width is half the length of the gap all round the area (as shown in Fig. 4.08) when the two pole faces are of equal size, or a strip of width equal to the gap length when one is much larger than the other. In the latter case the addition is made to the area of the smaller pole face (see further Chapter IX., Art. 22).

18. Magnetic Circuit Calculations

The chief difficulty introduced by the before-mentioned variation of μ is that it is impossible to calculate directly the flux produced by a given M.M.F., owing to μ being unknown till the flux-density is found. The reverse problem is soluble, viz. the calculation of the ampere-turns required to produce a given flux. When the former problem arises it has to be attacked indirectly (see Example 3).

The latter problem is solved by calculating the ampere-turns for each uniform part of the magnetic circuit along a mean line which is drawn to scale and measured: the separate ampere-turns are then added together (see Example 2), *i.e.* the formula used is—

$$AT = \frac{10}{4\pi}\left\{\frac{\mathbf{B}_1 l_1}{\mu_1} + \frac{\mathbf{B}_2 l_2}{\mu_2} + \text{etc.}\right\} \qquad \text{N.B. } \frac{10}{4\pi} = 0{\cdot}80,$$

where l_1, l_2, . . . = lengths of mean magnetic paths in centimetres.

For working in inches and lines per square inch this becomes

$$AT = \frac{10}{4\pi}\left\{\frac{\mathbf{B}'l_1'}{\mu_1} + \frac{\mathbf{B}_2'l_2'}{\mu_2} + \ldots\right\} \times \frac{2{\cdot}54}{(2{\cdot}54)^2}$$

$$= 0{\cdot}313\left\{\frac{\mathbf{B}'l'_1}{\mu_1} + \frac{\mathbf{B}_2'l_2'}{\mu_2} + \ldots\right\}.$$

If, however, the tables or curves give ampere-turns per cm. $\left(= \frac{10}{4\pi}\frac{\mathbf{B}}{\mu}\right)$ or ampere-turns per inch $\left(= 0{\cdot}313\frac{\mathbf{B}'}{\mu}\right)$ it is simpler to use these directly, instead of obtaining the value of μ and then calculating as above.

Example 2. *A ring is made up partly of wrought-iron of* 6 *sq. cm. cross-section and* 20 *cm. mean length, and partly of cast-iron of* 15 *sq. cm. cross-section and* 60 *cm. mean length, with an air-gap* 5 *mm. wide in the cast-iron. Calculate the ampere-turns needed to produce fluxes of* (*a*) 60 *kilolines and* (*b*) 90 *kilolines respectively.* [*Take a leakage factor* 1·15 *and neglect fringing.*]

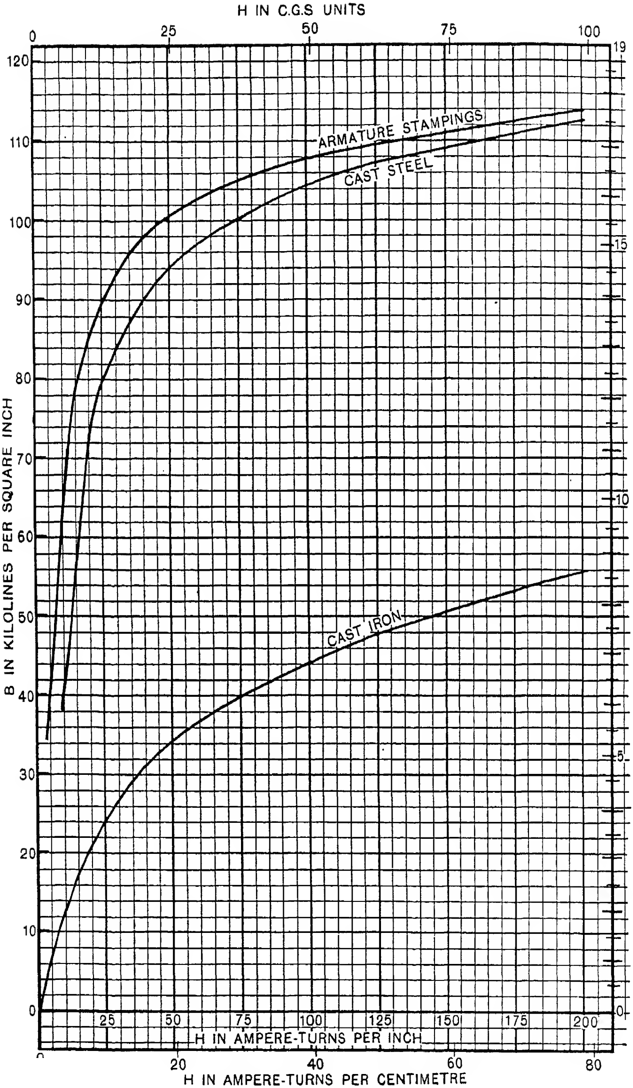


Fig. 4.09.—Magnetic Curves for Iron and Steel.

(*a*) Flux in iron = $1{\cdot}15 \times 60 = 69$ kilolines.

Tabulate as follows:—

Part of Circuit	Flux in Kilo-lines	Sectional Area of Circuit Sq. Cm.	B (Kilo-lines / Sq. Cm.)	Mean Length of Circuit Cm.	Ampere-turns per Cm.	Ampere-turns
Air-gap ..	60	15	4·00	0·5	3200	1600
Cast-iron ..	69	15	4·60	60	14·5	870
Wrought-iron	69	6	11·50	20	6·5	130
					Total	2600

Ans. 2600 ampere-turns.

B is obtained by dividing the various fluxes by the corresponding areas. The lengths are obtained from the scale diagram of the circuit.

The ampere-turns per centimetre for the air-gap $= \frac{10}{4\pi} \times \mathbf{B} \times 10^3$. For the iron they are obtained from the curves plotted from Table D in Fig. 4.09.

(*b*) Iron flux = $90 \times 1{\cdot}15 = 103{\cdot}5$ kilolines.

	Kilolines	Area Sq. Cm.	B Kilolines/Sq. Cm.	Length Cm.	$\mathcal{A}$ per Cm.	
Air-gap	90	15	6·00	0·5	4800	2400
C.I. ..	103·5	15	6·90	60	39·8	2390
W.I. ..	103·5	6	17·25	20	61·6	1230
					Total	6020

N.B.—The leakage factor would actually be greater in the second case than in the first.

Example 3. *Find the flux produced by* 4000 *ampere-turns in the ring considered in Example* 2.

This cannot be done directly.

(i) If the reluctance of the iron were negligible, then

$$\Phi = \frac{\frac{4\pi}{10}\mathcal{A}}{l_a} = \frac{\frac{4\pi}{10} \times 4000 \times 15}{0{\cdot}5} = 150\,000 \text{ lines approx.};$$

$\therefore$ if total iron reluctance = air-gap reluctance

$$\Phi = 75 \text{ kilolines};$$

$\therefore$ flux in iron = $75 \times 1{\cdot}15 = 86{\cdot}25$ kilolines.

	Φ Kilolines	Area Sq. Cm.	B Kilolines/Sq. Cm.	Length Cm.	per Cm.	
Air-gap	75	15	5·0		4000	2000
C.I. ..	86·25	15	5·75	60	23·4	1404
W.I. ..	86·25	6	14·375	20	11·3	226

Total 3630.

(ii) Therefore 4000 ampere-turns will produce a larger flux than the above; if the reluctance remained constant the air-gap flux produced would be

$$\frac{4000}{3630} \times 75 \text{ kilolines} = 82{\cdot}6 \text{ kilolines.}$$

Owing to increase of the reluctance the flux actually produced will be less than this [compare Example 2 (*b*)].

Try 80 kilolines; ∴ iron flux = 80 × 1·15 = 92 kilolines.

	Φ Kilolines	Area Sq. Cm.	B Kilolines/Sq. Cm.	Length Cm.	AT per Cm.	AT
Air-gap	80	15	5·33	0·5	4260	2130
C.I. ..	92	15	6·13	60	28·3	1698
W.I. ..	92	6	15·33	20	1·16	322

Total 4150

If a straight line law is assumed to hold for flux and ampere-turns between the above two values, then—

$$\text{Air-gap flux for 4000 ampere turns} = 75 + \frac{4000 - 3630}{4150 - 3630} \times (80 - 75)$$
$$= 78{\cdot}6 \text{ kilolines.}$$

If greater accuracy is required calculate the ampere-turns for this flux, and then re-calculate the value as above: or plot flux against AT from the four cases calculated and interpolate by means of the graph.

19. Remanence and Coercive Force

When a piece of iron or steel is magnetised electrically and the current is reduced again to zero the iron retains some of its magnetism, which is named the **residual magnetism.** The flux-density in this state is called the remanence, and is much greater in some materials than in others. It is thus evident that the relation between **B** and **H** must be different during the decrease of the current to what it is with increasing currents.

If the test piece is unmagnetised at the start, and the magnetic force is gradually increased, the relation between this and the resulting flux-density will follow some curve such as OA (Fig. 4.10). On reducing **H** gradually to zero, **B** will diminish as shown by AB,

and OB represents the remanence. The specimen can be demagnetised by applying a sufficient negative (or reversed) magnetic force, named the **coercive force**. The relation between **B** and **H** during this process is represented by BC, and OC is the coercive force.

20. Hysteresis

On further increasing the negative, or reversed, magnetic force the specimen becomes magnetised in the opposite direction: this is shown by CD. If the maximum negative value of **H** equals the

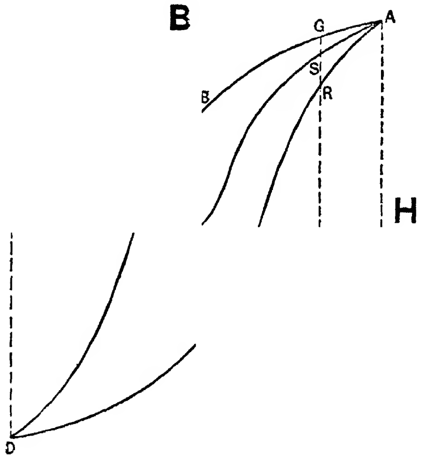


Fig. 4.10—MAGNETISATION CURVE SHOWING HYSTERESIS, ETC.

maximum positive value (*i.e.* OM = OL), then the flux-density produced will be equal in the two cases, but, of course, in opposite directions (*i.e.* DM = AL).

On bringing **H** gradually back to its original positive value the curve followed will be DEFA; which curve can be obtained by giving ABCD half a turn about O as centre.

If **H** is again reduced to zero the curve AB is again followed.

This difference in the values of **B** for a given value of **H** according to the previous treatment is known as **hysteresis**.

The curve ABCDEFA is known as the hysteresis loop.

By approaching a given value of **H** in a suitable manner the flux-density can be given any value within certain limits. *E.g.* in Fig. 4.10, for the value of **H** represented by OQ, **B** can have *any* value between QR and QG. The permeability will consequently have a similar range, but the value usually stated is that corresponding to **B** = QS, viz. the value obtained by starting with an unmagnetised specimen and bringing up the value of **H** gradually. If a higher value of **H** is available the range of **B** can be extended, since the new hysteresis loop will enclose the original one.

A large amount of residual magnetism does not necessarily mean a large coercive force, and the latter is of equal importance for permanent magnets. For these iron is useless, in spite of its considerable residual magnetism, for the coercive force is very low, *i.e.* the magnetism is easily destroyed. Hard steel is always employed owing to its high coercive force: small quantities of tungsten (about 5 %) improve its quality in this respect. Still higher values are obtained by adding to steel a large amount of cobalt (20% to 60%), together with a little tungsten or chromium, or both. This is, however, much more costly than tungsten steel.

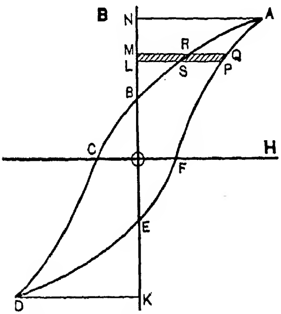


Fig. 4.11.—Loss of Energy through Hysteresis.

21. Loss of Energy Due to Hysteresis

In order to magnetise a piece of iron energy must be supplied to it, and in losing its magnetism it gives out energy. Owing to hysteresis there is, on the whole, a loss of energy in passing through a cycle of magnetisation. For it can be proved that in passing from the magnetisation represented by the point P on the **B-H** curve to that represented by Q (see Fig. 4.11) the energy absorbed by the specimen

$$= \left(\frac{1}{4\pi} \times \text{area of PQML}\right) \text{ ergs per c.cm.};$$

similarly in passing from R to S the energy given out

$$= \left(\frac{1}{4\pi} \times \text{area SRML}\right) \text{ ergs per c.cm.}$$

Thus in passing round the hysteresis loop the net energy absorbed

$$= \left(\frac{1}{4\pi} \times \text{area ABCDEF}\right) \text{ergs per c.cm.}$$

The energy is supplied by the magnetising current which has to overcome the back E.M.F. generated whenever **B** is increased, and similarly energy is restored by the generation of a forward E.M.F. whenever **B** is diminished.

It is by a consideration of the energy thus absorbed or produced in the magnetising coil that the above relations are obtained.

Thus in going from P to Q the average current is

$$\frac{10}{4\pi} \times \frac{(\text{mean value of LP and MQ})}{\text{no. of turns per cm.}} \text{ amperes.}$$

Since $$\mathbf{H} = \frac{4\pi}{10} \times \text{ampere-turns per cm.},$$

and the E.M.F. produced $= \dfrac{\text{lines cut per second}}{10^8}$ volts.

If a cubic centimetre is considered—

$$\text{E.M.F.} = \frac{\text{change of } \mathbf{B} \times \text{turns per cm.}}{10^8 \times \text{time of change}} \text{ volts} = \frac{\text{LM} \times \text{turns cm.}}{10^8 \times t};$$

$$\therefore \text{ joules} = \text{EI}t = \frac{10}{4\pi} \times \frac{(\text{mean of LP and MQ}) \times \text{LM}}{10^8}$$

$$= \frac{1}{4\pi} \times \frac{\text{area of PQML}}{10^7} \text{ joules} = \frac{1}{4\pi} \times (\text{area of PQML}) \text{ ergs.}$$

The scales used for **B** and **H** must be taken into account: thus if the scales are 1 cm. = 2,000 lines per sq. cm., and 1 cm. = 5 C.G.S. units of magnetic force, and the area of the loop is 19·4 sq. cm., then the energy absorbed per complete cycle

$$= \frac{1}{4\pi} \times 19{\cdot}4 \times 2000 \times 5 \text{ ergs per c.cm.} = 1{\cdot}53 \times 10^5 \text{ ergs/c.cm.}$$

$$= 15{\cdot}3 \times 10{,}000 \div 10^7 \text{ joules (or watt-seconds) per c.cm.}$$

$$= {\cdot}0153 \text{ joule per c.cm.}$$

It is worth noting that if **H** is plotted in ampere-turns per centimetre instead of in C.G.S. units the energy lost per cycle is

$$\left(\frac{1}{10} \times \text{area ABCDEF}\right) \text{ergs per c.cm.}$$

or $$\left(\frac{1}{10^8} \times \text{area ABCDEF}\right) \text{joules per c.cm.}$$

The true test of the value of a material for permanent magnets is the maximum value of the product of flux-density and negative magnetic force during removal of the residual magnetism, *i.e.* along BC in Fig. 4.10. On this basis 35 % cobalt steel is about 80 % better than tungsten-steel.* A fairly accurate comparison can be obtained by multiplying coercive force by remanence for different materials.

22. Steinmetz Law

If the range of the cycle of magnetisation is increased the hysteresis loop, and therefore the loss per cycle, becomes larger (see Fig. 4.12). Steinmetz has shown experimentally that the energy lost varies as $\mathbf{B}^{1.6}$ for the values usual in engineering, where $\mathbf{B}$ = maximum flux-density in each direction. This law does not hold for high values of $\mathbf{B}$, the inaccuracy beginning to be important at about 10,000 lines per sq. cm.†

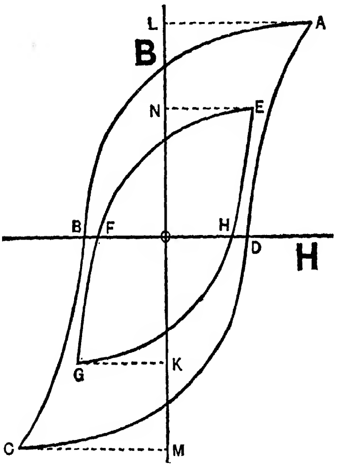


Fig. 4.12.—VARIATION OF HYSTERESIS LOSS WITH FLUX-DENSITY.

Thus approximately—

Energy lost per cycle = $h.\mathbf{B}^{1.6}$ ergs per c.cm.,

where h is a *hysteretic coefficient,* depending on the material but constant for any particular material.

Another way of stating this quality of the material is in watts per lb., at 50 (or 100) cycles per second, and at some definite flux-density (see Example below).

For good quality sheet iron h has values between ·0010 and ·0015; for poor qualities up to ·0025; for annealed steel ·008; and for hardened steel ·025.

* See Evershed, *J.I.E.E.*, vol. 58, pp. 797 *seq.*

† The index value of 1·6 is only an average, at very low flux-densities it is more nearly 2.

Example 4. *The hysteresis loss for a particular quality of iron is* 1 *watt per lb. at* 50 *cycles per second, and* 50,000 *lines per sq. in.*

(*a*) *Calculate the loss for* 30 *lb. of iron at* 42 *cycles per second, and* 90,000 *lines per sq. in., assuming Steinmetz's Law to hold.*

(*b*) *If the specific gravity of the iron is* 7·8, *calculate the hysteretic constant.*

(*a*) $\text{Loss} = 30 \times 1 \times \frac{42}{50} \times \left(\frac{90000}{50000}\right)^{1\cdot 6}$ watts

$= 30 \times 0\cdot 84 \times (1\cdot 8)^{1\cdot 6}$ watts

$= 25\cdot 2 \times 2\cdot 56$ watts

$= 64\cdot 4$ watts.

$\log 1\cdot 8 = 0\cdot 2553$

$0\cdot 6 \times \cdot 2553 = \cdot 1532$

$\therefore 1\cdot 6 \log 1\cdot 8 \quad 0\cdot 4085$

$= \log (2\cdot 56)$.

(*b*) 50,000 lines per sq. in. $= \frac{50000}{6\cdot 45}$ lines/sq. cm. $= 7750$ lines/sq. cm.

1 lb. of iron $= \frac{453\cdot 6}{7\cdot 8}$ c.cm. $= 58\cdot 2$ c.cm. 1 watt $= 10^7$ ergs per sec.

$\therefore$ Loss per cycle per c.cm. $= \frac{1 \times 10^7}{50 \times 58\cdot 2} = 3440$ ergs at 7750 lines/sq. cm.;

$\therefore 3440 = h\,(7750)^{1\cdot 6}$;

$\therefore h = \frac{3440}{(7750)^{1\cdot 6}} =$

$\log 3440 = 3\cdot 5366$

$\log 7750 = 3\cdot 8893$

$0\cdot 6 \log 7750 = 2\cdot 3336$

$\therefore 1\cdot 6 \log 7750 = 6\cdot 2229$

TESTING SECTION

23. The Ballistic Galvanometer

The usual method of measuring permeability is by the use of a *ballistic galvanometer*. This may be of either the moving coil or moving magnet type. It measures the quantity of electricity discharged through it, and should possess

(*a*) a long "period" or time of swing,

(*b*) little "damping," *i.e.* when set swinging each successive swing or "throw" should be only slightly less than the previous one.

A long period is obtained by the use of moving parts of comparatively high moment of inertia, and a long fine suspension.

It is necessary in order that the discharge may be completed before the moving parts have moved far from their zero position.

Under the above conditions the quantity (Q) discharged through the galvanometer is given by

$$Q = \left\{ \frac{t}{2\pi} \times a \times \left(1 + \frac{\lambda}{2}\right) \right\} d,$$

where $d =$ "throw" of galvanometer (first swing),

$a =$ current to produce unit steady deflexion,

$t =$ time in seconds of an oscillation,

$\left(1 + \frac{\lambda}{2}\right) =$ correcting factor for damping,

$\lambda =$ "logarithmic decrement," *i.e.* the difference between the Napierian (or hyperbolic) logarithms of successive swings. (See further Art. 26.)

[N.B.—Instead of $\left(1 + \frac{\lambda}{2}\right)$ the factor $\sqrt{\frac{d_1}{d_2}} = \sqrt{\frac{d_2}{d^3}}$, etc., or $\sqrt[4]{\frac{d_1}{d^3}}$ may be used, where d_1, d_2, d_3, etc., are successive swings in alternate directions.]

The above may be written $Q = k.d$, where k is constant, provided the damping is constant. Thus the galvanometer is a quantity (or coulomb) meter, and is so used for measuring the discharge from condensers.

24. Flux-Measurement by Ballistic Galvanometer

If a coil, connected to a ballistic galvanometer, has a change produced in the number of magnetic lines linked with the coil, *e.g.* by placing a magnet in it, a momentary E.M.F. will be produced (see Chapter VIII.) causing a discharge through the galvanometer.

The mean value of the E.M.F. $= \frac{\Phi T}{t \times 10^8}$ volts (see Chapter II.),

where $\Phi =$ *change* of flux,

$T =$ number of turns in coil,

$t =$ time (in seconds) to cause the change of flux.

If $R =$ resistance (ohms) of coil and galvanometer circuit, the mean current in galvanometer $= \frac{E}{R} = \frac{1}{R} \cdot \frac{\Phi T}{t \times 10^8}$ amperes;

$\therefore$ the quantity sent through the galvanometer

$$= \text{current} \times \text{time} = \frac{\Phi T}{Rt \times 10^8} \times t = \frac{\Phi T}{R \times 10^8} \text{ coulombs.}$$

But the throw of the galvanometer measures the quantity (see preceding Art.), hence Φ can be determined if T and R are known; *i.e.* any change of flux can be measured.

25. Calibration of Galvanometer

When numerical, and not merely comparative, values of quantity or flux are required the galvanometer must be calibrated. This may be done—

(*a*) By charging a standard condenser to a known E.M.F. (*e.g.* that of a standard cell), and discharging it through the galvanometer (see Art. 27).

(*b*) By producing a known change of flux in a coil having a known number of turns.

Corrections must be applied for damping, unless this remains constant during both calibration and the subsequent use of the galvanometer. This is the case only if the resistance of the galvanometer circuit is kept constant, which is impossible with method (*a*), since the resistance of a condenser is enormous; with method (*b*) this condition can be satisfied.

Method (*b*) may be subdivided into: (1) the "earth inductor" method, (2) the standard solenoid method.

The earth inductor consists of a large coil with many turns, which is rotated so as to cut the whole, or the vertical or horizontal component of the earth's field. Its use requires an accurate determination of the earth's field at the place of use.

The standard solenoid consists of a long solenoid uniformly wound on a non-magnetic core, with a secondary coil of many turns placed at its centre and connected to the galvanometer. This secondary is either wound closely over the solenoid, or placed inside it (see Fig. 4.17, p. 91).

The following is an example of such a solenoid:—

Length = 100 cm. Mean diam. = 7·33 cm.

No. of turns = 2720; $\therefore$ turns per cm. = 27·2.

Secondary coil:—

No. of turns = 128 (single layer).

Mean diam. = 5·40 cm.; $\therefore$ mean area = 22·9 sq. cm.

With this coil, $\mathbf{H} = (4\pi/10) \times$ ampere-turns per cm.

$$= (4\pi/10) \times 27{\cdot}2 \times \mathrm{I},$$

where I = current in coil in amperes.

This must be corrected for the solenoid not being of infinite length. This reduces the magnetic force in the ratio $l/\sqrt{(l^2 + d^2)}$,

i.e. in the ratio of axial length of solenoid to the length of its diagonal. With the above proportion of length to diameter this correction amounts to 0·27 per cent.;

$$\therefore\ \mathbf{H} = \frac{4\pi}{10} \times 27{\cdot}2\,\mathrm{I} \times (1 - {\cdot}0027) = 34{\cdot}1\,\mathrm{I} = \mathbf{B}\ (\text{since } \mu = 1);$$

$$\therefore\ \text{no. of lines through secondary} = 34{\cdot}1\,\mathrm{I} \times 22{\cdot}9 = 781\,\mathrm{I};$$

$\therefore$ no. of "cuts" in secondary *per ampere change* in primary

$$= 781 \times 128 = 1{\cdot}000 \times 10^5.$$

Thus if the *reversal* of 3 amp. in the primary causes a throw of 121 divisions of the galvanometer scale, the "constant" k' in the equation $\Phi T = k' \times d$'s $\frac{}{121} = 4{\cdot}96 \times 10^3$ for the particular resistance of galvanometer circuit used in the calibration.

26. Use of Series Resistance for Magnetic Testing

It follows from the formulae of Art. 24 that an increase of resistance in series with the galvanometer diminishes the quantity sent through it, since $Q \propto \frac{1}{R}$, where R = *total* resistance of galvanometer circuit. It must not, however, be assumed that the throw varies inversely as the resistance, for the damping is diminished (especially with a moving coil galvanometer) by an increase of resistance. If the damping is determined for each case and the correction applied (see Art. 23), then the relation (corrected throw) $\propto 1/R$ may be used. On the other hand the use of a shunt of the ordinary pattern will cause little change in the quantity passing through the galvanometer: *e.g.* if a $\frac{1}{9}$ shunt is used only $\frac{1}{10}$ of the whole quantity Q passes through the galvanometer; but Q will be nearly 10 times as great as before, since the resistance of the shunted galvanometer is $\frac{1}{10}$ of that of the galvanometer alone.

Example 5. If galvanometer resistance = 450 ohms,
resistance of rest of circuit = 10 „
„ „ shunt = 50 „
Then resistance of shunted galvanometer = 45 „

$\therefore$ the quantity sent round the circuit is increased by the use of the shunt in the ratio $\frac{45 + 10}{45 + 10}$ 8·36;

$\therefore$ the quantity sent through the galvanometer of 8·36 = 0·836 of the quantity with no shunt.

A further disadvantage of a shunt is that the damping will be very largely increased by its use, necessitating a separate calibration of the shunted galvanometer.

A shunt of the " Universal " pattern would be better, but still unsatisfactory, since a varying portion of it will be shunted by the secondary coil, and so the damping will vary considerably.

27. Calibration by Condenser

An alternative method of calibrating a ballistic galvanometer is by the use of a standard condenser and a standard cell.

These are connected as shown in Fig. 4.13. The well-insulated charge-and-discharge key enables the condenser to be both charged from the cell and discharged through the galvanometer. This is done, and the first throw (d_1) noted. The quantity (Q) discharged is known from the E.M.F. (E) of the cell and the capacitance (C) of the condenser, since Q = CE. *E.g.* if a Weston cell of 1·018 volts E.M.F. and a condenser of $\frac{1}{2}$ microfarad capacitance are used, the quantity discharged is $\frac{1}{2} \times 1{\cdot}018 = 0{\cdot}509$ micro-coulomb.

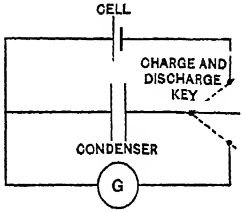


Fig. 4.13.—CALIBRATION BY CONDENSER.

For a complete test this should be repeated with other capacitances or other voltages so as to test the galvanometer at all parts of its scale. The relation of quantity to throw is usually a straight-line one, but not always.

Since the galvanometer when used for magnetic testing forms part of a closed circuit, whereas in the above test it is on open circuit, the damping will be different in the two cases and must be determined for each.

This is done by a test similar to the above, except that the cell E.M.F. and condenser capacitance need not be known. They are chosen so as to give a large first throw, and successive swings (d_1, d_2, d_3, etc.) are observed as they gradually decrease owing to damping.

The logarithms of the successive throws are then plotted as shown in Fig. 4.14. The points should lie on a straight line, *i.e.*

$$\log d_1 - \log d_2 = \log d_2 - \log d_3 = \text{etc.} = \lambda.$$

If hyperbolic logarithms are used this constant difference between the logarithms of successive swings is the " logarithmic decrement "

(λ) (see Art. 23). If ordinary logarithms are employed the constant difference must be multiplied by 2·303 to obtain the true logarithmic decrement.

The experiment is repeated with the galvanometer circuit permanently completed through the resistance with which it is to be afterwards used, and the increased logarithmic decrement (λ') obtained.

Then, since $Q = c.d_1\left(1 + \frac{\lambda}{2}\right)$, where c is a constant (see Art. 23), the value of c can be determined from the first two experiments, since Q, d_1, and λ are all known.

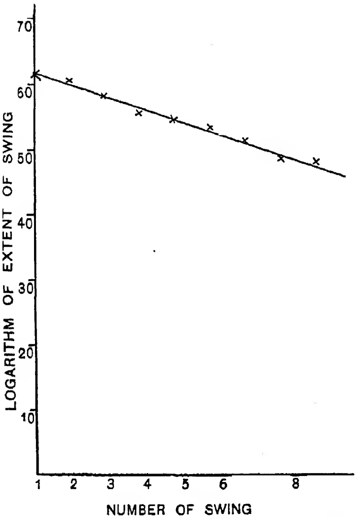


Fig. 4.14.—DETERMINATION OF LOGARITHMIC DECREMENT.

Under the changed conditions

$$= c.d_1\left(1 + \frac{}{}\right).$$

Therefore the quantity corresponding to any observed throw can be determined, since c and λ' are known.

This method is much less suitable for magnetic testing than that of Art. 25, which should always be used if the necessary apparatus is available. This is partly because the condenser method takes longer, and partly because the formula for the effect of damping is only approximate, and the inaccuracy becomes appreciable when there is considerable damping. *E.g.* the error is about $\frac{1}{2}$ per cent. when λ is 0·2 and about 2 per cent. when λ is ·4 (corresponding to $\frac{\text{second throw}}{\text{first throw}} \doteq \frac{9}{11}$ and $\frac{2}{3}$ respectively).

28. Form of Specimen

This is usually either a long round rod or a circular ring. The latter has the advantage of absence of any demagnetising effect,

but the disadvantage that each specimen has to be separately wound with its magnetising and search coils. If used it should be of a section similar to that shown in Fig. 4.15, *i.e.* with its (radial) breadth small compared with its diameter. The actual ratio of breadth to diameter is preferably only half that shown in the figure. This is to minimize the crowding of the lines towards the inner face of the ring, and the consequent error in assuming the flux-density to be uniform. It should not be forged, as the welded joint will introduce an error of unknown amount. It may be turned out of plate, or built up of a number of thin annealed stampings.

A rod can be placed in an already wound coil, the same coil being used for any number of successive specimens. The ratio $\frac{\text{length of rod}}{\text{diameter of rod}}$ should be kept large, so that the self-demagnetising effect (Art. 9) is small: an error in estimating this effect will then cause no serious inaccuracy in the calculated value of **H**.

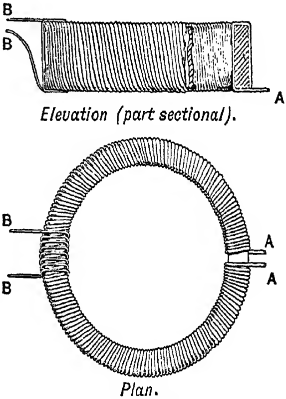


Fig. 4.15.—RING FOR MAGNETIC TESTING.

A A, Ends of magnetising coil.
B B, Ends of testing coil.

29. The Bar and Yoke Method

This method combines some of the advantages of both ring and rod. The specimen is a rod, so that ready wound coils may be used. It has its magnetic circuit completed by a massive yoke of iron, so that the demagnetising effect is eliminated almost entirely.

Various forms of yoke may be used, three of which are shown in Fig. 4.16. (A) is the double yoke used by Hopkinson, the originator of the method. In this the specimen must be a good fit in the holes in the yoke through which it passes. (B) shows Ewing's method with two test-bars magnetised in opposite directions, each by its own coil. At each end they make butt joints with massive iron yokes. The ends of the specimens require accurate preparation and their lengths must be exactly equal, and there are 4 joints in the magnetic circuit. (C) shows a simple U-shaped yoke, with the test-bar clamped to it at each end by two nuts. The coil is shown on the yoke. This is slightly more convenient than to have it

surrounding the bar, but the latter method is more accurate as it avoids errors due to magnetic leakage.

Whatever form of yoke is employed an allowance must be made for the ampere-turns required to send the flux through the yoke. The joints introduce an inaccuracy which may be important when absolute, and not merely comparative, results are required.

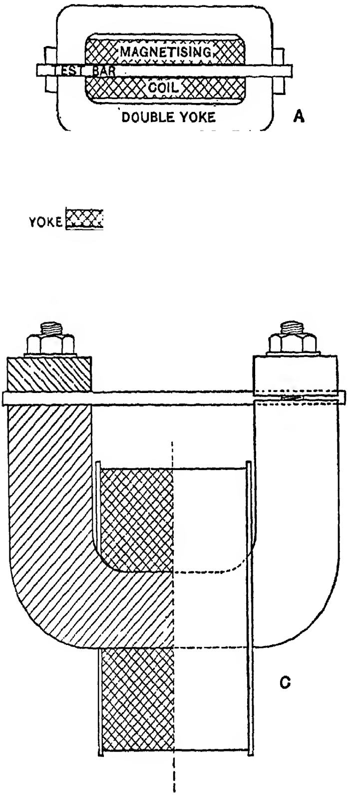


Fig. 4.16.—BAR AND YOKE METHOD.

30. Ballistic Test for Permeability

The coil for magnetising the specimen is connected to a suitable battery, through a reversing key, an ammeter, and a variable rheostat. The magnetising force applied can then be adjusted to the various values required, and completely reversed at will. Fig. 4.17 represents the connexions for a test on a long rod, and for the calibration of the galvanometer by a standard solenoid (Art. 25). The two-way switch is moved to the right for the calibration. On changing it over the current can be sent through the coil which surrounds the specimen, thus magnetising it. The current, and so the magnetism, can be reversed by the reversing key.

Thus the whole flux cuts the search coil (wound over the specimen) *twice*, and causes a throw of the galvanometer, whence the flux can be determined. If the resistance of the galvanometer circuit is kept unaltered during both calibration and test, the

damping correction will be constant and so need not be taken into account.

Example 6. Rod (cast-iron) 61·0 cm. long; 0·631 cm. diam.;
$\therefore$ area = 0·312 sq. cm.
Magnetising coil 100 cm. long, 3800 turns.
Magnetising current 0·31 amp.
Search coil 50 turns.
Galvanometer throw on reversing current 27·2.
Galvanometer constant (Art. 25) $4{\cdot}96 \times 10^3$;

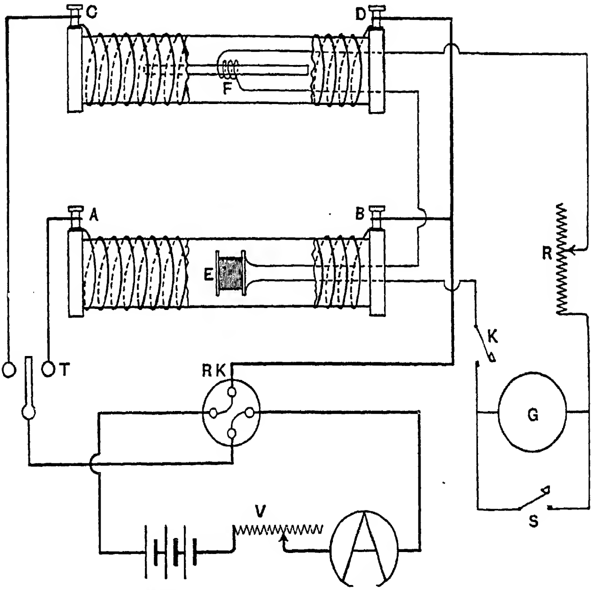


Fig. 4.17.—MAGNETIC TESTING BY BALLISTIC GALVANOMETER.

E, Fine wire coil at centre of coil AB for calibrating galvanometer. F, Fine wire coil wound on the iron test bar. K, Galvanometer circuit key. R K, Reversing key. R, Galvanometer series resistance. S, Galvanometer short-circuiting key. T, Two-way switch. V, Variable resistance.

$$\therefore \text{ No. of ``cuts'' of lines of force} = 27{\cdot}2 \times 4{\cdot}96 \times 10^3 = 1{\cdot}35 \times 10^5;$$

$$\therefore \text{ flux in rod} = \frac{1{\cdot}35 \times 10^5}{2 \times 50} = 1{\cdot}35 \times 10^3 \text{ lines};$$

$$\therefore \text{ flux-density } (\mathbf{B}) = \frac{1{\cdot}35 \times 10^3}{0{\cdot}312} = 4330 \text{ lines per sq. cm.}$$

Apparent magnetising force $\quad \overline{10\ l} \quad \frac{4\pi}{10} \times \frac{0{\cdot}31 \times 3800}{100}$,

$= 14{\cdot}8.$

$\frac{\text{Length of rod}}{\text{Diam. of rod}} \quad 96{\cdot}7; \qquad \frac{N}{4\pi} = {\cdot}00038$ (from Table E);

$\therefore$ demagnetising force $= \frac{N}{4\pi}B = {\cdot}00038 \times 4330 = 1{\cdot}65.$

(see below for a graphical method of making the corrections for the demagnetising force).

$\therefore$ true value of $H = 14{\cdot}8 - 1{\cdot}65 = 13{\cdot}15$ (C.G.S.);

$\therefore$ permeability $(\mu) = \frac{B}{H} = \qquad = 329.$

Typical results of tests by the above and other methods are given in Table D, and plotted in Fig. 4·09. Scales of AT per inch and AT per cm. are given as well as of H in C.G.S. units. The former are more convenient for magnetic calculations, *vide* Examples 2 and 3, pp. 75 and 77.

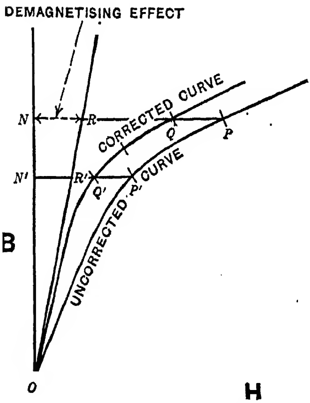


Fig. 4.18.—GRAPHICAL CORRECTION FOR SELF-DEMAGNETISATION.

When a series of readings have been taken in a test as in the above example, the corrections for the effects of the demagnetising force may be done graphically, as indicated in Fig. 4.18 and explained below:—

First the values of B are plotted against the corresponding values of the apparent magnetising force, as shown by the "uncorrected curve," OP'P. Then for one value of B (ON in Fig. 4.18), the demagnetising force is calculated as in the above example. PQ is made equal to this value of the demagnetising force, so that Q is a point on the "corrected curve"; *i.e.*, it gives the true value (NQ) of H corresponding with this particular value of B.

To obtain other points on the "corrected curve" NR is made equal to QP. A straight line, OR'R, is drawn from the origin to R, and produced if required. Then for any value of B, such as ON' in Fig. 4.18, N'R'Q'P' is drawn to the "uncorrected curve" and parallel to the H axis, cutting OR in R'. A length P'Q' is marked off equal to R'N'. Then Q' is another point on the "corrected curve."

This construction may be repeated for as many points on the "uncorrected curve" as desired, and the "corrected curve" drawn through the new points.

TABLE D.

MAGNETIC FORCE			CAST-IRON			CAST-STEEL.			ARMATURE STAMPINGS, WROUGHT IRON OR STEEL FORGINGS		
H C.G.S.	AT PER CM.	AT PER IN.	B	B′	μ	B	B′	μ	B	B′	μ
2·5	2·0	5·0	1·2	7·7	480	2·5	16	1000	5·5	35·4	2200
5	4·0	10·1	2·0	12·9	400	6·0	39	1200	10·1	65·0	2020
7·5	6·0	15·1	2·7	17·4	360	8·5	55	1130	12·5	80·5	1670
10	8·0	20·2	3·3	21·3	330	11·2	72	1120	13·1	84·5	1310
15	11·9	30·3	4·2	27·1	280	13·2	85	880	14·6	94·0	970
20	15·9	40·4	4·85	31·3	243	14·1	91	705	15·3	98·5	765
25	19·9	50·5	5·35	34·5	214	14·7	95	588	15·8	102	632
30	23·9	60·6	5·8	37·4	193	15·1	97	503	16·1	104	537
40	31·8	80·8	6·4	41·2	160	15·7	101	393	16·45	106	411
50	39·8	101	6·9	44·5	138	16·2	104·5	324	16·75	108	335
60	47·7	121	7·3	47	122	16·6	107	277	17·0	109·5	283
80	63·6	162	8·0	51·5	100	17·1	110	214	17·4	112	218
100	79·6	202	8·6	55·5	86	17·5	113	175	17·7	114	177

[Note Fig. 4.09, p. 76, is plotted from the above figures.]

B = flux-density in kilolines (thousands of lines) per sq. cm.

B′ = ,, ,, ,, ,, ,, sq. in.

N.B.—AT per cm. = 0·8 **H** approx. (½ per cent. inaccuracy).

AT per inch = 2·0 **H** ,, (1 ,, ,, ,,).

Or in words: *magnetic force* (C.G.S. units) = *ampere-turns per half-inch.*

TABLE E.—DEMAGNETISING FACTORS.

(Partly from *The Magnetic Circuit*, by H. du Bois.)

	Cylinder			Ellipsoid of Revolution		
$\frac{\text{Length}}{\text{Diam.}} = f$	$f^2\bar{N}$	$\bar{N}$	$\frac{\bar{N}}{4\pi}$	N	f^2N	$\frac{N}{4\pi}$
10	19·8	0·198*	·0158	0·2549	25·5	·0203
15	24·3	0·108*	·0086	0·1356	30·5	·0108
20	27·6	0·069*	·0055	0·0848	34·0	·00675
25	30·6	0·049*	·0039	0·0579	36·2	·00461
30	32·4	0·036*	·00286	0·0432	38·8	·00344
40	35·7	0·0223*	·00177	0·0266	42·5	·00212
50	40·5	0·0162	·00129	0·0182	45·4	·00145
60	42·4	0·0118	·00094	0·0132	47·5	·00105
70	43·7	0·0089	·00071	0·0101	49·5	·00080
80	44·4	0·0069	·00055	0·0080	51·2	·00064
90	44·8	0·0055	·00044	0·0065	52·5	·00052
100	45·0	0·0045	·00036	0·0054	54·0	·00043
150	,,	0·0020	·00016	0·0026	58·3	·00021
200	,,	0·0011	·000090	0·0016	62·0	·000125
300	,,	0·00050	·000040	0·00075	67·5	·000060
400	,,	0·00028	·000022	0·00045	72·0	·000037
500	,,	0·00018	·000014	0·00030	75·0	·000024
1000	,,	0·00005	·000004	0·00008	80·0	·000006

Demagnetising force $= \frac{N}{4\pi}$ B.

For a long ellipsoid $\frac{N}{4\pi} = \frac{1}{f^2}(\log_e 2f - 1)$.

Note that the value of $f^2\bar{N}$ becomes constant for values of f above 100, *i.e.* the demagnetising factor is then inversely proportional to f^2.

N.B.—The columns $\frac{\bar{N}}{4\pi}$ and $\frac{N}{4\pi}$ have been added to those given by Du Bois, and a few errors in the others have been corrected. The figures marked * are due to S. P. Thompson and E. W. Moss (*Proc. Phys. Soc.*, Vol. XXI., p. 630).

31. Hysteresis Test

For the purpose of obtaining the shape of the *hysteresis loop* (Art. 20) the ballistic test is modified. In place of the key which simply reverses the magnetising current a key (K) with 6 terminals is employed, together with a second adjustable resistance, R_2, and a short-circuiting switch, S. These are connected as shown in Fig. 4.19, the remaining connexions being as in the previous test (Fig. 4.17).

When S is closed, K acts simply as a reversing key, as can be seen by tracing the path of the current, first with the switch in its right-hand position as shown by the full lines, secondly with it in its left-hand position as shown by the dotted lines.

With S open, the change of K from left to right reverses the current, and at the same time diminishes its value as it must now pass through R_2 as well as through R_1.

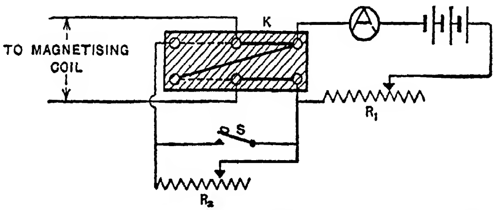


Fig. 4.19.—CONNEXIONS FOR OBTAINING HYSTERESIS LOOP.*

The procedure is as follows:—

(Remember that **H** depends on the magnetising current, so that an adjustment of **H** means a corresponding adjustment of the current.)

Close S and adjust **H** to the full value required, *i.e.* OL in Fig. 4.20.

Reverse **H** several times so as to eliminate irregularities due to previous magnetisations. This should be done before every reading. Close galvanometer circuit. Quickly increase R_1 so as to reduce **H** to a smaller value, such as OQ in Fig. 4.20. Note the resulting " throw," whence the *change* of flux-density (NG) can be calculated. Thus the position of G is determined.

* By connecting the ammeter to the left-hand bottom corner of K, instead of as shown, the resistances in its two positions (with S closed) are made more nearly equal.

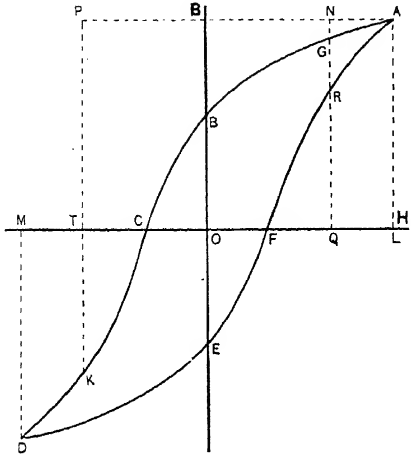


Fig. 4.20.—DETERMINATION OF HYSTERESIS LOOP.

By restoring R_1 to its previous value, and repeating the above process with different changes of **H**, any number of points on the portion AB of the hysteresis loop may be determined.

To obtain points on BCD the start is as before: but instead of increasing R_1, S is opened and K changed over. Thus **H** is reversed and diminished, *e.g.* changed from OL to OT; while the galvanometer "throw" determines PK, the change of flux-density. Thus the position of K is fixed. By repeating this second process with various values of R_2 any number of points on BCD may be determined.

The return half of the loop, DEFA, can then be obtained by reversing ABCD, which has been determined (see Art. 20); or it may be found by a similar experiment with the original current reversed.

32. Alternative Methods of Hysteresis Testing

Instead of restoring **H** to its original value after obtaining the point G on the loop, it may be further reduced, *e.g.* to OV (Fig. 4.21). The throw of the galvanometer will then give WJ, and thus settle the position of J. Continuing in this way the curve AB is determined.

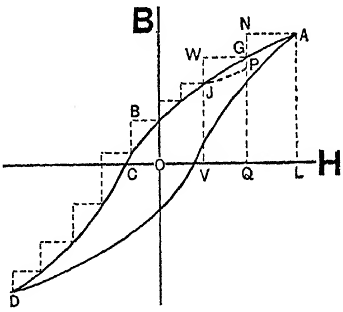


Fig. 4.21.—STEP-BY-STEP METHOD FOR HYSTERESIS LOOP.

H is then reversed, and increased in the reverse direction by small amounts, thus obtaining BCD.

This is known as the *step-by-step method*. Its main disadvantage is that an error in determining any step affects the position of all the points subsequently determined; further, the errors may accumulate and cause some of the points to be displaced from their correct position considerably.

If it is desired to repeat any particular step (*e.g.* QV), **H** must first be restored to its maximum value (OL), then reduced to its value (OQ) before this step. If it were merely restored to the value before the step, **B** would not rise to the value which it had before (QG) but to some lower value (such as QP). On reducing **H** (to OV) the throw would not depend on the same reduction of **B** as in the first instance (viz. WJ), but would be smaller (viz. proportional to the difference between QP and VJ). By previously raising **H** to its full value (and preferably reversing it several times as well) this is avoided, and each step may be repeated accurately.

33. Ewing's Hysteresis Tester

A number of instruments have been devised for commercial tests when only the energy lost by hysteresis (Art. 21) is required. A typical one is *Ewing's Hysteresis Tester*, which is shown in Fig. 4.22.

The specimen consists of a bundle of strips about 3 in. long and $\frac{5}{8}$ in. wide, a sufficient number being taken to give a total thickness of about $\frac{1}{4}$ in. This bundle is held in the clamps, A (Fig. 4.22), and rotated between the poles of a C-shaped permanent magnet, B, by means of the rubber-covered wheel, C, and the hand-wheel, D. The magnet is supported on a knife-edge, E, and carries a pointer by means of which the amount of its movement can be read on the scale, F. A vane attached to the magnet moves in an oil bath, G, so as to steady the deflection. H is an arrangement for lifting the magnet off its knife-edge and clamping it when not in use.

PRINCIPLE OF THE INSTRUMENT.—When the specimen is rotated it is magnetised in alternate directions, and thus hysteresis loss occurs. This causes a drag tending to stop the rotation of the specimen, and an equal but opposite drag tending to pull the magnet round in the direction of rotation. Now the loss of energy per cycle due to hysteresis is constant for a fixed maximum flux-density; therefore the *power* lost by hysteresis is equal to $n \times$ loss per cycle $\times$ constant, where n is the number of cycles per second.

The power required to rotate the specimen is proportional to the driving torque × revolutions per second; and consequently the extra power required owing to hysteresis is proportional to the torque due to hysteresis × n. Equating these two expressions for the power lost by hysteresis the result obtained is—

$$\text{Hysteresis torque} \propto \text{loss per cycle,}$$

and this torque is independent of the speed.

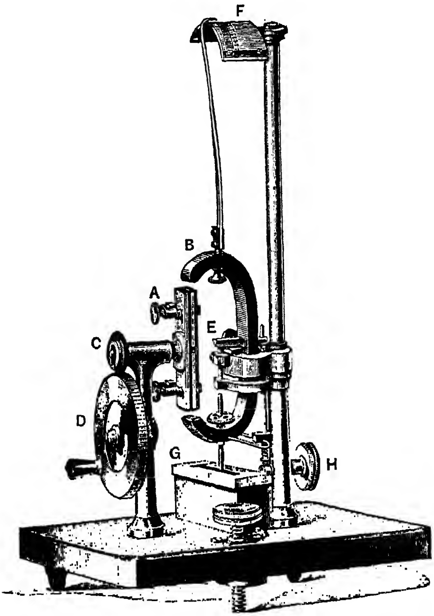


Fig. 4.22.—EWING'S HYSTERESIS TESTER.

The deflexion of the magnet depends on the torque exerted on it, and the average torque on the magnet is merely that due to hysteresis. Hence the deflexion of the magnet is a measure of the hysteresis loss per cycle. It is, however, not directly proportional to the loss, but the relation is a straight line one.

METHOD OF USE.—To interpret the results a number of standard specimens with known hysteretic constants are supplied. Three of

these are tested in the instrument, and the observed deflexions are plotted against their hysteretic constants. The points thus obtained are joined by a straight line (see Fig. 4.23) which is then used to determine the hysteretic constants of the specimens under test. Thus if ON (Fig. 4.23) is the deflexion with one of the test pieces, PN is the hysteretic constant for the iron of which it consists. This constant can be stated in whatever units are most convenient.

The flux-density produced is about 4000 lines per sq. cm. A change in the strength of the magnet does not affect the accuracy of the results, since all the deflexions will be altered in the same ratio. Moreover the exact cross-section of the specimen is unimportant, provided it is fairly near that of the standards, for any variation in this will be balanced almost exactly by the opposite change in the flux-density.

To avoid errors due to changes in the standards with lapse of time these must be checked occasionally by comparison with specimens which have been tested ballistically. This possible variation is one of the reasons why three standards should be used in every test, instead of only two, which are sufficient if they are accurate. The third standard reading also serves as a check on the accuracy with which the readings are taken.

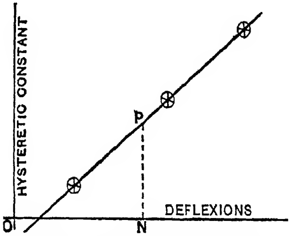


Fig. 4.23.—HYSTERETIC CONSTANT BY EWING'S HYSTERESIS TESTER.

34. The Fluxmeter

The Fluxmeter, due to Grassot, is an instrument by which the change of flux in a search coil can be measured as in the ballistic galvanometer. It has the advantages over the latter of giving a result which is independent of the rate of change of flux (cf. Art. 23), and of giving instead of a throw a deflexion which remains practically constant for ten seconds or thereabouts.

It consists of a moving-coil ballistic galvanometer in which the controlling force of the suspension has been reduced as much as possible. When a change of flux takes place in the search coil the galvanometer coil is brought to rest chiefly by electromagnetic damping, *i.e.* by the retarding force due to currents in the coil,

induced by the coil's motion. The effects of this difference of construction are as stated above. In addition the deflection is independent of the resistance in the galvanometer circuit within wide limits, *i.e.* the resistance must not be increased so much that the electromagnetic damping is decreased till air-damping becomes appreciable in comparison with it.

The general appearance of the instrument is shown in Fig. 4.24. The scale has 100 divisions on each side of a central zero, each corresponding to a change of 10^4 linkages. By using search coils

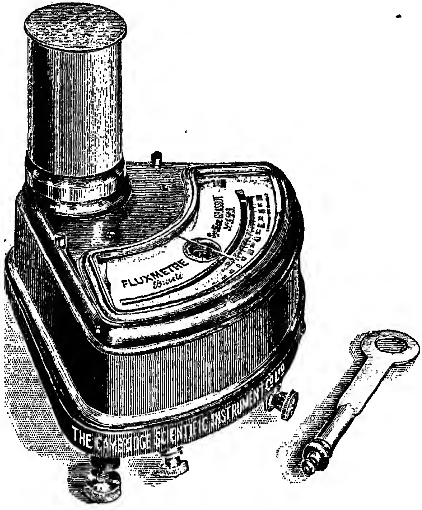


Fig. 4.24.—GRASSOT FLUXMETER.

with various numbers of turns and suitable sizes a wide range of results can be obtained.

For more accurate work a mirror is fitted and a lamp and scale used in the ordinary way.

35. Bismuth Spiral

The metal bismuth possesses the peculiar property of increasing in electrical resistance when placed in a magnetic field. This property is utilised for measuring the strength of such fields.

The bismuth is used in the form of a flat wire spiral (see Fig. 4.25) about 0·8 cm. diameter contained between two sheets of mica.

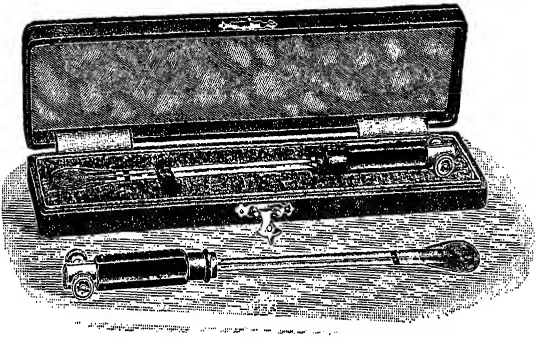


Fig. 4.25.—BISMUTH SPIRALS FOR MAGNETIC TESTS.

The ends of the spiral are connected by stiff varnished copper strips to terminals in an ebonite handle. The resistance (R_0) is measured by the P.O. box or other suitable method when the spiral is in zero

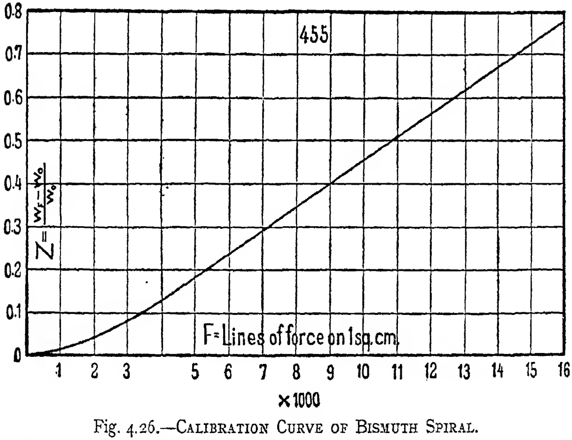


Fig. 4.26.—CALIBRATION CURVE OF BISMUTH SPIRAL.

W_0 = resistance of spiral in zero field; W_F = resistance of spiral in field of strength F.

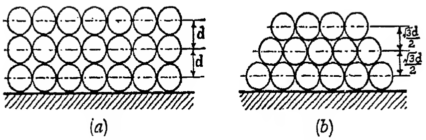


Fig. 4.27.—ARRANGEMENTS OF WIRES IN MAGNET WINDING.

resistance $\frac{R_1 - R_0}{R_0}$ is calculated, and the flux-density obtained thence. Since the relation between flux-density and fractional increase of resistance is not a simple one and varies somewhat with the particular piece of bismuth used, a curve is used to obtain the flux-density. This is supplied by the makers, an example being shown in Fig. 4.26.

MAGNET-COIL WINDING SECTION

36. Arrangement of Layers : Bedding

The space occupied by a coil of a given number of turns of any size of round wire will depend on the way in which the successive layers lie above each other, whether with the wires directly above each other, or with those of the second layer in the hollows of the first. See (*a*) and (*b*) in Fig. 4.27.

With arrangement (*b*) the distance between successive layers is ·866 $\left(=\frac{\sqrt{3}}{2}\right)$ of that in (*a*) for the same size of wire, and the total space occupied is reduced in a rather less degree, or the number of turns which can be wound in a given space is increased nearly in the ratio $\frac{2}{\sqrt{3}}$, *i.e.* by 15 per cent. Since, however, each turn of the second layer must cross a turn of the first owing to their being wound in opposite directions along the length of the coil, it is safer (and usual) to assume, in making calculations, that arrangement (*a*) is the actual one. The actual diminution of space occupied due to "bedding" is usually from nothing to 5 per cent. in practice, instead of the 13·4 per cent. (100 — 86·6) which might occur if all the wires could be arranged as in (*b*).

37. Relations of Winding and Spool

Fig. 4.28 represents the cross-section of a magnet spool or bobbin which is to be filled with round wire of diameter *d*, *over its*

insulation. The unit of length employed is immaterial provided all the quantities are expressed in the same unit.

The following relations follow from what has been stated above.

$$\text{No. of wires per layer} = \frac{b}{d} = \frac{\text{net length of spool}}{\text{diameter of wire}}.$$

$$\text{No. of layers} = \frac{h}{d} = \frac{\text{depth of winding}}{\text{diameter of wire}}.$$

$$\therefore \text{ No. of turns (or ♉)} = \frac{bh}{d^2} = \frac{\text{winding space}}{\text{square of diam. of wire}}.$$

Further—
$$h = \frac{D_2 - D_1}{2}.$$

$$\text{Mean diameter of coil (D)} = \frac{D_1 + D_2}{2}.$$

$$\text{Mean length of a turn } (l_m) = \pi D = \pi \frac{D_1 + D_2}{2}.$$

$$\begin{aligned}\text{Total length of wire (L)} &= l_m \times \text{♉} \\ &= \pi \frac{D_1 + D_2}{2} \cdot \frac{bh}{d^2}. \\ &= \frac{\pi b}{2d^2}(D_1 + D_2)\frac{D_2 - D_1}{2} \\ &= \frac{\pi b\,(D_2^2 - D_1^2)}{4d^2}.\end{aligned}$$

It is usually preferable to determine l_m and ♉ separately and then multiply them, rather than use the above formula. (See Example 7.)

Total cross-section of winding $=$ area of wire $\times$ no. of turns $= \frac{\pi d^2}{4} \times \frac{bh}{d^2} = \frac{\pi}{4} \times bh = 0{\cdot}785 \times$ (winding space).

This includes the insulation and shows that the waste spaces (if no bedding occurs) always amount to 21·5 per cent. of the total space, independently of what size of wire is used.

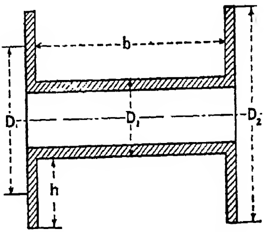


FIG. 4.28.—CROSS-SECTION OF MAGNET SPOOL.

Similarly:—total volume of winding (including insulation)

$$= \text{area of wire} \times \text{total length}$$

$$= \frac{\pi d^2}{4} \times l_m \times \text{ʊ} = \frac{\pi d^2}{4} \times \frac{bh}{d^2} \times l_m$$

$$= \frac{\pi}{4} bhl_m = 0{\cdot}785 \times \text{winding space} \times \text{mean length of a turn},$$

which again is independent of the diameter of the wire.

Example 7. *A magnet bobbin is* $\frac{5}{8}$ *in. diam.,* 1 *in. diam. over the flanges and* 2 *in. long between flanges. Find the length of wire,* 18 *mils diam. over the insulation to fully wind it.*

The winding space is $\frac{1 \text{ in.} - \frac{5}{8} \text{ in.}}{2} = \frac{3}{16}$ in. deep;

$\therefore$ area of winding space $= 2$ in. $\times \frac{3}{16}$ in. $= \frac{3}{8}$ sq. in.;

$\therefore$ no. of turns $= \frac{3}{8} \div (\cdot 018)^2 = 1157$.

Mean diameter $= \frac{1 + \frac{5}{8}}{2} = \frac{13}{16}$ in.;

$\therefore$ mean length of a turn $= \frac{13}{16}\pi = 2{\cdot}55$ in.;

$\therefore$ total length of wire $= 1157 \times 2{\cdot}55$ in. $= 2950$ in.

$= 246$ ft.

$= 82$ yd.

38. Weight and Resistance Relations

It follows from the last relation of Art. 37 that if the ratio of insulation to metal is constant and the same materials are used (double-cotton-covered copper is usual), the weight of the winding depends only on the size of the spool and not on the diameter of wire used. The above assumption is approximately correct for large and medium sized wires, but ceases to be so for fine wires, for which the insulation occupies a larger proportion of the available space and so reduces the total weight.

If d_1 = diameter of wire under the insulation, the resistance

$$= \frac{\rho L}{A} = \frac{\rho l_m \times \text{ʊ}}{0{\cdot}785 d_1^2} = \frac{\rho l_m \times bh}{0{\cdot}785 \times d_1^2 \times d^2}.$$

Hence, assuming as before that $\frac{d_1}{d}$ is constant, it follows that the *resistance of the winding of a given spool varies inversely as the fourth power of the diameter of the wire;* or *varies directly as the square of the number of turns* $\left(\text{since this varies as } \frac{1}{d^2}\right)$.

When the above assumption cannot be made the relations become less simple, but can be worked out by the above principles.

Example 8. *The resistance of the wire on a bobbin fully wound with silk-covered wire 7 mils. diam. is found to be 120 ohms. What will be the resistance if the same bobbin be equally wound with 10 mils diam. wire? The thickness of covering to be taken as 1 mil in each case.* [C. & G., II.

Covered diam. $= 7 + 2 \times 1 = 9$ mils in first case
and $= 10 + 2 = 12$,, second case;

$\therefore$ no. of turns is reduced in the ratio $\frac{(9)^2}{(12)^2} = \frac{9}{16}$,
and the total length is reduced in the same ratio.

But the resistance per unit length is reduced inversely as the square of the diameter of the uncovered wires, *i.e.* as $\frac{7^2}{10^2} = \frac{49}{100}$;

$$\therefore \text{ new resistance} = \frac{9}{16} \times \frac{49}{100} \times 120 \text{ ohms}$$
$$= 33{\cdot}4 \text{ ohms.}$$

Note that in this case the resistance is not inversely proportional to $(\text{diam.})^4$ nor directly proportional to $\eth^2$.

39. Ampere-Turns and Voltage Relationship

A problem which frequently occurs in connexion with dynamos is the production of a given number of ampere-turns when a given voltage is applied to the winding. If the inner and outer diameters of the spool are known the size of wire to use can be determined by the relation

$$R = \frac{E}{I}, \text{ and therefore } \frac{R}{\eth} = \frac{E}{I\eth}$$

or
$$\text{resistance per turn} = \frac{\text{volts}}{\text{ampere-turns}}.$$

Since the mean length of a turn is known the size of wire can then be found. (See Examples 9 and 11.)

The important point to notice is that the ampere-turns produced by a given voltage depend on the size of wire and not on the number of turns used, or, in other words, the ampere-turns are independent of the length of the winding, if mean length of turn is fixed.

The number of turns can then be settled by taking a suitable current-density, say 800 amperes per sq. in., and thus obtaining the current which, divided into the ampere-turns, gives the turns. Or, alternatively, the length of the winding can be determined from the current-density *independently of the size of wire*, for

$$\frac{\rho l}{A} = R = \frac{E}{I}; \quad \therefore l = \frac{E}{\rho(I/A)} = \frac{\text{volts}}{\rho \times \text{current-density}}.$$

Thus for any given current-density there must be a certain number of *yards per volt*, about 45 yards per volt for 800 amperes per sq. in. (See Examples 9 and 10.)

When the length and cross-section of the wire are known the resistance can be calculated, either from a wire table or from the resistivity of copper. In carrying out such calculations the rise of temperature due to the passage of the current must be taken into account (cf. Example 10). The amount of this rise will depend mainly on the size and proportions of the coil, and on the current-density used. (See further Chapter XII.)

The weight of the wire can be calculated similarly by the use of a wire table or from the Sp. Gr. of copper (8·89).

Example 9. *Work out a rule for finding the diameter of a wire to be wound on a given field magnet bobbin, of which the inner and outer diameters of the winding space and its available length are known, in order that when a given voltage is applied it will yield a prescribed number of ampere-turns of excitation. If a bobbin has a winding length of* 8 *in., and the inner and outer diameters of the available winding space are* 9 *in. and* 15 *in respectively, and if* 50 *volts is to produce* 10,000 *ampere-turns of excitation, of what diameter must the copper wire be?*

[*N.B.—Take the resistance of copper for this purpose as such that a bar* 1 *ft. long and* 1 *sq. in. in cross-section will have a resistance of* 9 *microhms.*]

[C. & G., II.

For the answer to the first portion of the question see Arts. 37 and 39. The size of wire can be determined quite apart from the winding length.

$$\text{For resistance per turn} = \frac{\text{volts}}{\text{ampere-turns}} = \frac{50}{10000} = 0{\cdot}005 \text{ ohm,}$$

$$\text{and} \qquad \text{length of a turn} = \pi \times \frac{9+15}{2} = 37{\cdot}7 \text{ in.}$$

$$R = \frac{\rho l}{A}; \quad \therefore A = \frac{\rho l}{R} = \frac{9 \times 37{\cdot}7}{12 \times 10^6 \times {\cdot}005} = {\cdot}00566 \text{ sq. in.;}$$

$$\therefore \text{diameter} = \sqrt{\frac{4}{\pi}A} = \sqrt{\frac{{\cdot}00566}{{\cdot}785}} = {\cdot}085 \text{ in.}$$

The nearest standard sizes are 14 S.W.G. (·080 in. diam.) and 13 S.W.G. (·092 in. diam.), so the winding might be made up partly of each size, or entirely of the larger wire so as to be on the high side with the ampere-turns.

No more is asked for, but the following further calculations are made to illustrate Art. 37.

Taking 13 S.W.G. for the winding and adding 12 mils for double cotton covering, overall diameter = ·104 in.

$$\text{Depth of winding} = \frac{15-9}{2} = 3 \text{ in.;}$$

$$\therefore \text{winding space} = 3 \times 8 = 24 \text{ sq. in.}$$

$$\therefore \text{no. of turns} = \frac{24}{({\cdot}104)^2} = 2220;$$

$$\therefore \text{total length of winding} = 2220 \times \frac{37{\cdot}7}{36} \text{ yd.}$$

$$= 2325 \text{ yd.;}$$

$\therefore$ weight of copper (see Table B) $\frac{}{1000} \times 76{\cdot}88$ lb.

$= 179$

Resistance of winding $\frac{9 \times 2325 \times 3}{10^6 \times {\cdot}006648} = 9{\cdot}44$ ohms.

${\cdot}006648$ sq. in. being the area of 13 S.W.G. (see Table B, p. 61);

$\therefore$ current in winding $= \frac{50}{9{\cdot}44} = 5{\cdot}30$ amperes;

$\therefore$ ampere-turns $= 5{\cdot}30 \times 2220 = 11770.$

The excess over the 10,000 required is due to the use of 13 S.W.G. instead of ${\cdot}085$ in. diam. wire.

Current density $= \frac{5{\cdot}30}{{\cdot}006648} = 797$ amperes per sq. in.

Yards per volt $= \frac{2325}{50} = 46{\cdot}5.$

Example 10. *Calculate the length of winding required per volt for a current-density of* 900 *amperes per sq. in. if the working temperature of the wire is* 60° *C.* (= 140° *F.*).

Resistivity of copper at 15° C. $= 0{\cdot}666 \times 10^{-6}$ ohm-inch

$\therefore$ at 60° C. it $= 0{\cdot}666 \times 10^{-6}\{1 + (60 - 15) \times {\cdot}00401\}$,, ,,

$= 0{\cdot}666 \times 10^{-6} \times 1{\cdot}180 = 0{\cdot}786 \times 10^{-6}$,, ,,

(see Chapter III., Art. 3, and Table A, p. 60);

. per volt $= \frac{10^6}{0{\cdot}786 \times 900}$ in. (see Art.

$\frac{10^4}{0{\cdot}786 \times 9 \times}$ $39{\cdot}3$ yd.

Example 11. *Calculate size and total length of wire to give* 13,000 *ampere-turns when supplied at* 78 *volts, with current-density and temperature as in Example* 10, *the mean length of turn being* 66 *in.*

From Example 10, total length $= 39{\cdot}3 \times 78 = 3065$ yd.

Resistance per turn $= \frac{78}{13000}$ $0{\cdot}0060$ ohm;

$\therefore$ resistance per 1000 yd. $\frac{0{\cdot}0060 \times 1000 \times 36}{66}$

$3{\cdot}27$ ohms at 60° C.;

$\therefore$ resistance per 1000 yd. at 15° C. $\frac{{\cdot}27}{8} = 2{\cdot}77$ ohms.

12 S.W.G. is very near this (see Table B), but is about 2 per cent. too small in cross-section.

Check Calculations. Resistance of 12 S.W.G. $= 2{\cdot}83$ ohms per 1000 yd. at 60° F. $= (2{\cdot}83 \times 1{\cdot}18 =)\ 3{\cdot}34$ ohms per 1000 yd. at 140° F.;

total resistance $= 3{\cdot}34 \times \frac{3065}{1000}$

current $\frac{78}{10{\cdot}24}$ $7{\cdot}62$ amperes;

$$\therefore \text{current-density} = \frac{\quad}{\cdot 008495} \quad \text{amperes per sq. in.}$$

$$\therefore \text{ampere-turns} = 7{\cdot}62 \times 1670 = 12{,}700,$$

which is slightly below value owing to the smallness of the wire used.

40. Force Between Magnetised Surfaces

This force is applied in practice in electromagnets for lifting iron and steel plates, etc., and in clutches for connecting and disconnecting shafting, etc. The stress (force per unit area) can be determined by the following considerations. Imagine the surfaces separated by a narrow air-gap in which the flux-density is $\mathbf{B}$ lines per square centimetre: by definition the force on unit magnetic pole in this gap is $\mathbf{B}$ dynes, and $\frac{\mathbf{B}}{2}$ dynes may be supposed to be due to the action of each surface.

If the pole be placed in one of the surfaces this will exert no normal force on the pole, *e.g.* if the surface have North polarity the unit N. pole would be repelled by it at a point either just above or just below the surface, hence at a point on the surface the force is neither outwards nor inwards. Hence the force $\frac{\mathbf{B}}{2}$ dynes is entirely due to the other surface. But there are $\mathbf{B}$ lines proceeding from every square centimetre of the surface, and 4π lines come from unit pole, so the pole strength per sq. cm. is $\frac{\mathbf{B}}{4\pi}$. Therefore the stress is $\frac{\mathbf{B}}{2} \times \frac{\mathbf{B}}{4\pi} = \frac{\mathbf{B}^2}{8\pi}$ dynes per sq. cm. Or the total force is $\frac{\mathbf{B}^2 A}{8\pi}$ dynes (A being the area in sq. cm.) $= \mathbf{B}^2 A/2{\cdot}47 \times 10^7$ kg. $= 4{\cdot}06\ \mathbf{B}^2 A \times 10^{-8}$ kg.

Alternatively, since $\mathbf{B} = \mathbf{H}$ in air, it follows from the relation proved in Art. 21 that the energy contained in an air gap is $\mathbf{B}^2/8\pi$ ergs per c.cm. For the curve FA in Fig. 4.11 becomes a straight line through the origin, say OA. And the area of the triangle OAN $= \frac{1}{2}\text{ON}.\text{NA} = \frac{1}{2}\mathbf{B}.\mathbf{H} = \frac{1}{2}\mathbf{B}^2$.

$\therefore$ energy absorbed in magnetising the gap is $\frac{1}{4\pi} \times$ (area of the triangle) $= \mathbf{B}^2/8\pi$ ergs per c.cm.

Hence if the gap length is increased by a small amount δl without changing $\mathbf{B}$ the increase of energy is $A.\delta l.\mathbf{B}^2/8\pi$ ergs. This must be equal to the work done in moving the surfaces apart, which is total force $\times\ \delta l$. Hence the total force is $A.\mathbf{B}^2/8\pi$ dynes, as proved above.

MODIFICATION FOR ENGLISH UNITS.—If $\mathbf{B}_1$ = flux-density in lines per sq. in. and A_1 = area in sq. in.

$$\text{Force} = \frac{\mathbf{B}_1{}^2A_1}{8\pi} \times \frac{6{\cdot}45}{(6{\cdot}45)^2} \text{ dynes}$$

$$= \frac{\mathbf{B}_1{}^2A_1}{8\pi} \times \frac{1}{6{\cdot}45 \times 981 \times 453{\cdot}6 \text{ lb.}}.$$

$$= \frac{\mathbf{B}_1{}^2A_1}{7{\cdot}21 \times 10^7}\text{lb.} = \frac{1{\cdot}386}{10^8}\,\mathbf{B}_1{}^2A_1 \text{ lb.}$$

The formula may also be written in the form $\dfrac{\Phi^2}{8\pi.A}$ dynes, which shows that for a given total flux a smaller area gives a bigger pull; thus coned pole pieces are advantageous provided the coning does not diminish the flux seriously by the increase of reluctance thus caused.

The maximum stress commercially possible is about 200 lb. per sq. in.

The same formula applies when the surfaces are some distance apart, the main difficulty in applying it being doubtfulness as to the amount of leakage and fringing.

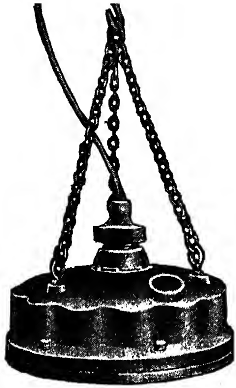
41. Attraction of Coil on Iron Core

This action is employed in measuring instruments, and in circuit breakers. Exact calculation is usually impossible, but certain general relations can be obtained and approximate values of the force calculated.

When the centres of a symmetrical coil and bar coincide there is no force between them. This is evident from symmetry.

With a bar more than twice as long as the coil the force will increase from a very small value when one end of the bar is just entering, to a maximum when it has got nearly through the coil, after which it will diminish again, reaching zero when the centre of the bar reaches the centre of the coil, as stated above.

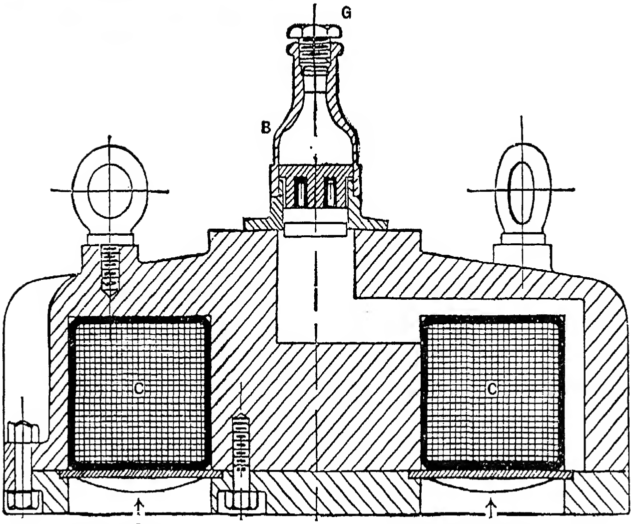
With a very short core the pull is a maximum when its centre is opposite the mouth of the coil.



(a) General view.

With intermediate lengths of core the position of maximum pull lies between these two, *e.g.* with a 10 in. coil and a 12 in. core the maximum pull is when the core has more than 6 in. and less than 10 in. of its length within the coil.

The maximum pull is approximately $\frac{1}{4}\cdot\frac{B^2A}{8\pi}$ dynes,* B being calculated for a magnetic circuit consisting of the iron and the air space inside the coil only. With short cores the demagnetising effect comes in as explained in Art. 9.



(b) Cross-section.

Fig. 4.29.—LIFTING MAGNET.

B, Terminal box. C C, Coil. G, Gland. P, Centre pole. Q Q, Outer pole. S S, Coil shield.

---

* "On the Predetermination of Plunger Electromagnets." S. P. Thompson. *The Electrician*, Vol. 53, p. 917.

42. Lifting Magnets

Lifting magnets are employed for lifting iron or steel plates, pig iron, scrap iron, etc.

An example made by the Witton-Kramer Co. is shown in Fig. 4.29.

This magnet is of the "pot" type, *i.e.* with one pole in the centre and the other in the form of a ring concentric with the central pole. The shell is of cast magnet steel with strengthening ribs and suspension lugs in one piece. The coil of copper wire or strip is held in place and protected by a "coil shield" of non-magnetic manganese steel or phosphor bronze. The coil shield is held by the pole shoes, which are bolted to the main casting. The coil, and afterwards the whole magnet, is impregnated in vacuo with an asphalt compound. The terminal box of cast-iron is attached to the magnet shell and is filled with a waxen compound. The leads enter through glands so that the box is water-tight. The supply must be direct current of a p.d. not exceeding 500 volts.

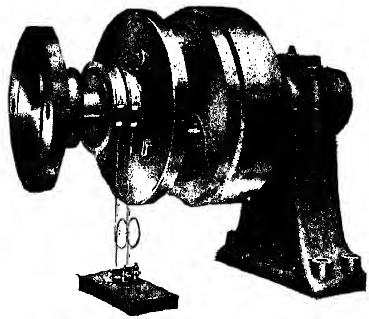


Fig. 4.30.—MAGNETIC CLUTCH, WITH COVER REMOVED TO SHOW COILS.

43. Magnetic Clutch

Magnetic clutches are another application of electromagnetism. They are employed for connecting at will two lengths of shafting in line, or for other similar purposes.

An example made by the Witton-Kramer Co. is shown in Fig. 4.30.

The clutch consists of a thick disc of high permeability steel, keyed to the main shaft, carrying pole pieces to which the magnet coils are fitted. The ends of the coils are connected to slip-rings, to which current is conveyed by means of brushes. A sheet iron cover is provided to protect the coils from external injury. To the pulley or driven shaft a soft wrought-iron disc is fitted, and when the magnet coils are excited this disc is attracted to the pole shoes, and thus caused to rotate with the main portion of the clutch. The coils are wound for a standard voltage of 50 volts, a suitable series resistance and switch being supplied to reduce the supply voltage to this figure.

QUESTIONS ON CHAPTER IV

1. Sketch the form and distribution of the lines of magnetic force in the field surrounding a long straight conductor which carries a current. Give a second sketch showing similarly the lines of force in the field of two parallel straight conductors carrying currents flowing in the same direction. [C. & G., I.

2. Define the terms "intensity of field," "permeability," and "ampere-turns." Calculate the intensity of the field at the central point of a solenoid 100 cm. long, of small diameter, wound with 6000 turns of wire, traversed by a current of 5 amperes. [C. & G., I.

3. State the law of the magnetic circuit for the case in which the area and material are different in the various parts of the circuit. If the magnetising current is doubled, to what extent is the total flux altered; similarly, how is the flux affected by halving the current?

4. A ring-shaped electromagnet has an air-gap 6 mm. long and 20 sq. cm. in area, the mean length of the core being 50 cm., and its cross-section 10 sq. cm. Calculate, approximately, the ampere-turns required to produce a field of strength $\mathbf{H} = 5000$ in the air-gap. (Assume permeability of iron as 1800.) [C. & G., II.

5. What is an electromagnet? In what way does its magnetism depend on its core? What are the reasons which determine, in any case, whether the coil should consist of few turns of thick wire or of many turns of thin wire? [C. & G., I.

6. What rules can you give about winding electromagnets? What are the circumstances that determine the selection of any particular size of wire for the coil? [C. & G., I.

7. The air-gap area of each pole of a smooth-core four-pole dynamo is 300 sq. cm., and the distance from pole-face to core is 5 mm. How many ampere-turns must be used on each pair of poles for air-gap excitation alone if the magnetic density in the air-gap is 10,000 lines per square centimetre? [C. & G., II.

8. Calculate the ampere-turns required to produce the following flux-densities ($\mathbf{B}$) in an iron ring of 60 cm. mean circumference and 2·4 sq. cm. cross-section. $\mathbf{B} = 5000$; 10,000; 12,000; 14,000. Corresponding permeabilities 2500; 2000; 1500; 1000. Plot *total flux* against ampere-turns and find the number of ampere-turns required for a total flux of 30,000 lines.

9. If a saw-cut 1 mm. wide is made in the above ring, calculate the extra ampere-turns needed in each case (including that with 30,000 lines), and the total required.

10. Calculate the ampere-turns needed to produce a flux of 40,000 lines in a ring 80 cm. mean circumference and 4·2 sq. cm. cross-section. Given that for the iron of which it consists,

when $\mathbf{B}$	5000;	10,000;	14,000
	2500;	2000;	1000.

11. If a saw-cut 1 mm. wide is made in the above ring, how many extra turns are needed?

12. An iron ring consists of 60 cm. of 10 sq. cm. section, 35 cm. of 20 sq. cm. section, and 25 cm. of 30 sq. cm. section. If it is wound with 10 turns per cm., what current will produce a flux of 180,000 lines?

What current will produce half this flux, and what is the ratio of these currents?

13. If a gap 6 mm. wide is made in the above ring in its thickest part, what current is now required for the full flux and the half flux, and what is the ratio of these currents?

14. What is the magnetisation curve of a material? Sketch approximately to scale such curves for air, hard steel, wrought-iron, and cast-iron. [C. & G., II.

15. A mild steel circular ring of mean diameter 14 cm. and rectangular section 5 mm. by 10 mm. has 704 turns wound uniformly on it. A secondary coil of 20 turns is connected to a ballistic galvanometer which gives a throw of 1 division for 2000 cuts. The following observations were obtained by reversing currents in the primary coil:—

Current:	0·05;	0·1;	0·2;	0·3;	0·4;	0·5;	0·75;	1·0;	1·5;	2·0;	3·0;	4·0;	5·0 A.
Throw:	10;	61;	98;	115;	126;	133;	147;	154;	159;	162;	167;	171;	173.

Plot the **B** — **H** and permeability — **B** graphs for this steel.

16. The following results were obtained for a steel casting:—

H =	3;	5;	10;	15;	20;	20 C.G.S. units.
B =	6400;	9600;	13,050;	14,600;	15,300;	16,000 lines per sq. cm.

Plot a curve with ampere-turns per inch as abscissae and lines per square inch as ordinates, and one with μ as ordinates.

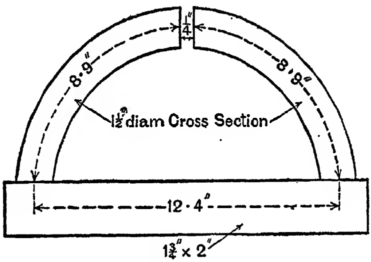


Fig. 4.31.

17. An electromagnet is made of steel of the above quality to the given dimensions. Find the ampere-turns necessary to produce a flux of $1 \cdot 7 \times 10^5$ lines.

18. Find the number of yards per volt for a current density of 1000 A. per sq. in. and a working temperature of 75° C.

Use the result to find the length of winding, size of wire, and winding space for a coil to produce 10,000 ampere-turns when supplied at 100 volts, if length of mean turn is 4 ft.

19. Calculate the size of wire for the cylindrical shunt field coil of a 6-pole 500-volt dynamo, the ampere-turns per pole being 5000 and the mean length of a turn 80 cm. Determine the minimum weight of copper required so that the temperature rise may not exceed 35° C., assuming that all the heat is dissipated from the cylindrical surface of the coil at the rate of 3·3 watts per sq. dm. per degree Centigrade rise. Specific resistance of copper 1·7 microhms at 15° C., density 8·9. [Lond. Univ., El. Mach.

20. Find the force of attraction in pounds weight between the square-faced ends of two rods of iron placed in a solenoid. The area of the iron rods is 6

sq. cm., the permeability of the iron is 1000, and the magnetic force produced in the iron by the solenoid is 14 C.G.S. units. [Lond. Univ., El. Tech.

21. How does the pull which an open solenoid exerts on its core vary with the position of the core in the following cases: (*a*) when the core itself is much longer than the solenoid; (*b*) when the core is from one-quarter to one-half as long as the solenoid; (*c*) when a very short cylinder or a sphere of iron is used instead of the usual core? In the case (*a*) state also what difference, if any, will result if the solenoid is surrounded by a cylindrical jacket of iron, closed at the bottom by an iron disc. State also how the pull, in any given case, will be affected by a reduction of the excitation to one-half its normal value. [C. & G., II.

22. A solenoid, 12 in. long, wound on a brass tube $1\frac{1}{4}$ in. external diameter, is coiled with a No. 13 S.W.G. wire, having 10 layers with 115 turns in each layer, and can carry 5 amperes without undue rise of temperature. Calculate the strength of the magnetic field produced (*a*) at the centre of the coil, (*b*) at the open mouth of the coil. What is the maximum pull you would expect it to exert on a cylindrical rod of soft iron half an inch in diameter and 15 in. long? In what position of this rod will the pull be a maximum? [C. & G., II.

23. A magnetic track brake is energised by the car motors, and each pole face has an area of 2·25 sq. in. Calculate the pull per pole in tons when the magnetic induction is 18,000. [Lond. Univ., El. Mach.

24. A smooth-core armature, working in a four-pole field magnet, has a gap (from iron to iron) of 0·5 in. The area of surface of each pole is one square foot. The flux from each pole is 7 megalines. Find (*a*) the mechanical force with which the pole attracts the armature; (*b*) the amount of energy expressed in joules that is stored in the four gaps. (N.B.—746 joules = 550 ft.-lb. at London; 1 ft. = 30·48 cm.; 1 lb. = 453·6 grm.) [C. & G., II.

25. Explain why the holding-on force of a magnet is proportional to the square of the flux-density at the surfaces in contact. Which will exert the greater hold-on force—a magnet having a flux of 400,000 magnetic lines and a contact surface of 8 sq. in., or one having 500,000 lines and a contact surface of $12\frac{1}{2}$ sq. in.? [C. & G., II.

26. One field coil of a dynamo has to give 10,000 ampere-turns with a potential difference of 110 volts between its terminals. The mean length of one turn of wire on the coil is 0·4 m. Find the diameter in millimetres of the copper wire required; also find the number of turns of wire in the coil for a permissible loss of 90 watts. Take the resistance of a copper wire 1 m. long and 1 sq. mm. in cross-section at the temperature of the coil as 0·02 ohm. [C. & G., II.

CHAPTER V

ALTERNATING CURRENTS

1. An Alternating Current or Voltage

An alternating current or voltage is one which flows or acts first in one direction and then in the opposite alternately, hence the name. In most cases the time-intervals for the two directions are equal, and the values of the alternating quantity are the same in magnitude though opposite in direction after the expiration of equal fractions of the intervals. Stated graphically this implies that if the current or voltage be plotted on a time base, as ABCDEFG in Fig. 5.01 (the opposite directions being indicated by drawing the graph above and below the time-base), then the time-interval, AD, for one direction is equal to DG, the time-interval for the other

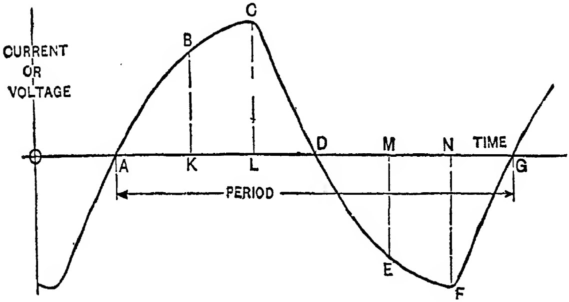


Fig. 5.01.—Alternating Current or Voltage.

direction. Further, if any fraction of AD, such as AK, be taken and DM is made equal to AK, then the values BK and EM, at these instants are equal in magnitude but opposite in direction. And in particular the maximum positive (CL) and negative (FN) values are equal in magnitude, and occur at equal times (AL, DN) after a zero value.

Thus the complete graph consists of a series of *half-waves* of the same shape (such as ABC, CDE, EF, FG, etc., in Fig. 5.02), alternately above and below the time-axis to represent the opposite directions of flow of the current or of action of the pressure. A positive half-wave and a negative half-wave together form a

complete wave (ABCDE in Fig. 5.02, ACDFG in Fig. 5.01) or cycle of changes. The time occupied by a complete cycle (AG in Fig. 5.01; AE or CF or EG or etc., in Fig. 5.02) is called the periodic time or period: this is usually constant or changing relatively slowly.

The number of periods or cycles in a second is called the periodicity or, more often, the frequency of the alternating current or voltage.

$$\text{Evidently: (period in seconds)} = \frac{1}{\text{ency}}$$

The maximum value (BM positive and DN negative in Fig. 5.02) is called the peak value, or the crest value, or sometimes the amplitude.

2. Sine Waves

If a coil of any fixed shape is rotated with uniform angular

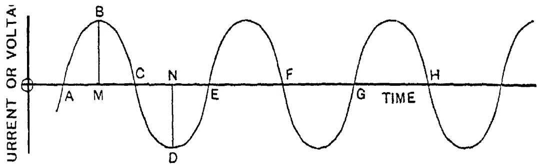


Fig. 5.02. A SERIES OF WAVES.

velocity about an axis, fixed relatively to the coil, in a stationary uniform magnetic field, the graph of the E.M.F. produced in the coil plotted on a time base is a *sinusoidal* or *sinoidal wave* (usually called a *sine-wave* for brevity). That is, it can be expressed by an equation of the form: $e = a \sin (bt + c)$,

where e = the E.M.F. produced at any instant;

t = time elapsed at that instant since the fixed zero of time;

and a, b, and c are constants.

Since the value of the sine of an angle oscillates between $+1$ and -1, the value of a is equal to E_m, the crest value of e. Moreover if time is measured in seconds, the value of b is equal to ω, the angular velocity of the coil in radians per second (see Fig. 5.04).

The truth of the above can be proved most easily in the case of a single rectangular turn, rotating about an axis midway between two of its sides, in a field perpendicular to this axis.

Let l = length in cm. of sides parallel to axis;

d = length in cm. of sides perpendicular to axis;

$\mathbf{B}$ = flux-density (uniform) in lines per sq. cm.;

α = angle between plane of coil at any instant and its position when perpendicular to field (see Fig. 5.03);

and ω = angular velocity of coil in radians per second.

Then the velocity of the sides $(l) = \omega . \frac{d}{2}$ cm. per sec.;

their velocity component (v) perpendicular to the field $= \omega . \frac{d}{2} \sin \alpha$;

$\therefore$ the E.M.F. of the turn $= 2\mathbf{B}lv = 2\mathbf{B}l . \omega \frac{d}{2} \sin \alpha$,

$$= \mathbf{B} . ld . \omega \sin \alpha,$$

$$= \mathbf{B} . A . \omega \sin \alpha,$$

$$= \Phi_m . \omega \sin \alpha;$$

where A = area of coil in sq. cm., and Φ_m = maximum flux through the coil.

These last two expressions can be proved to be true for any shape of coil and for any axis.

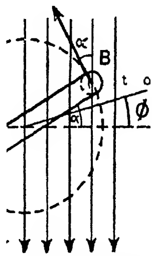


Fig. 5.03.—COIL ROTATING IN UNIFORM FIELD.

If ϕ ("phi") is the value of α at the start (*i.e.* when $t = 0$), then $(\alpha - \phi)$ is the angle turned through in t seconds, and so is equal to ωt,

hence $$\alpha = \omega t + \phi;$$

$$\therefore \text{ the E.M.F. of the coil} = \omega \Phi_m . \sin (\omega t + \phi),$$

which is of the above form: $e = a \sin (bt + c)$.

The angle ϕ is called the **phase** of the E.M.F. It is chiefly the **phase differences** of two or more alternating quantities that are of importance.

If the E.M.F. is plotted on a time-base as in Fig. 5.04, the angle turned through can also be measured along the base, since it is proportional to the time. Thus for any point P on the graph, the ordinate PN represents the E.M.F. after a time t seconds has elapsed,

and $\quad$ ON $= t$, on the scale of time,

and $\quad$ ON $= \omega t$, on the angle scale.

Further, if A is the point at which the E.M.F. is zero, OA on the angle scale is equal to ϕ, and PN is proportional to the sine of the angle AN.

This E.M.F. goes through a complete cycle once in every revolution ($= 2\pi$ radians), *i.e.* AE = period in seconds = 2π radians;

$$\therefore \text{ the frequency } (f) = \omega/2\pi;$$

$$\therefore \omega = 2\pi f;$$

$$\therefore e = 2\pi f \Phi_m \sin(2\pi ft + \phi).$$

If the coil contains several turns wound together, the value of the E.M.F. is increased correspondingly, and the formula becomes—

$$e = \qquad \sin$$

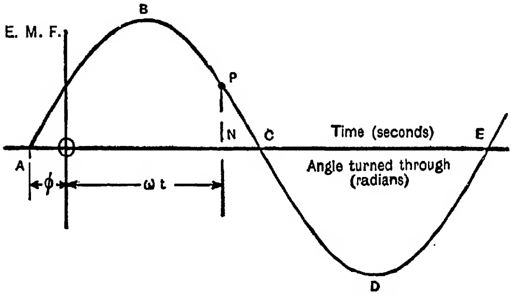


Fig. 5.04.—Sinusoidal E.M.F. Wave.

Though actual alternating voltages and currents are not always sine waves, the aim of designers of alternators is to obtain wave-forms as close to this shape as possible, since otherwise various undesirable effects may be produced (see Volume II.). This is, therefore, taken as the standard wave-form, which has the further advantage of greatly simplifying both theory and calculations.

In this book sine waves will be assumed as a rule, with occasional notes on the effects of other wave-forms, leaving a fuller treatment of these to Volume II.

3. Relation of E.M.F. and Flux

It may be noted that the flux through the coil in Fig. 5.03 at any moment is

$$Bld \cos a = \Phi_m \cos a = \Phi_m \cos(\omega t + \phi).$$

Now a cosine wave is of the same shape as a sine wave, but is a quarter of a period ($=\frac{\pi}{2}$ radians $= 90°$) ahead of it in phase, since

$$\cos(\omega t + \phi) = \sin\left(\omega t + \phi + \frac{\pi}{2}\right);$$

$$\therefore \text{ Phase of the flux} = \left(\phi + \frac{\pi}{2}\right).$$

Thus the flux through the coil and the E.M.F. produced are related as shown in Fig. 5.05; *i.e.* they are both sinusoidal waves, but differ in phase by quarter of a period, the flux leading. Note in particular that the E.M.F. is maximum when the flux is zero, and vice versa.

A negative flux means that it passes through the coil in the

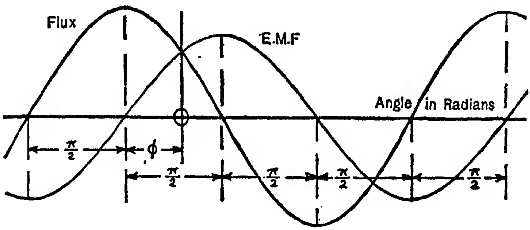


Fig. 5.05.—FLUX AND E.M.F. WAVES.

opposite direction, which is the case when a (Fig. 5.03) lies between one and three right angles in value (90° to 270°). The directions to be reckoned positive are so chosen that a current driven by a positive E.M.F. tends to produce a positive flux. Thus, if in Fig. 5.03 an E.M.F. acting down through the paper in A and upwards in B is taken as positive, then, by the Right-handed Screw Rule, the positive direction of the flux is vertically down. By applying the Right-hand Rule it will be found that the E.M.F. is then positive while B is higher than A, and negative when B is lower, corresponding to the graphs in Figs. 5.04, 5.05. The Right-hand Rule is as follows:—Place the thumb and the fore and middle fingers of the right hand at right angles to one another. Point the thumb in the direction of motion of the conductor, and the forefinger in the direction of the flux; then the middle finger gives the direction of the E.M.F.

This relation of the E.M.F. and flux waves can be proved readily by the use of the calculus as follows:—

Let the flux $(\Phi) = \Phi_m \cos\ (\omega t + \phi) = \Phi_m \sin\left(\omega t + \phi + \frac{\pi}{2}\right)$.

Then for a single turn of any shape the E.M.F. (e) is given by:—

$$e = -\frac{d\Phi}{dt} = \omega\Phi_m \sin\ (\omega t + \phi),$$

as already obtained.

4. Mean and Effective Values

The mean value of an alternating voltage or current is zero, since the negative half-wave is the exact reverse of the positive half-wave. But for certain purposes (*e.g.* electro-chemical work with rectifiers) it is necessary to know the mean value of each half-wave.

This can be found for any shape of wave as follows: Considering the rotation of a single turn as in Art. 2, average E.M.F. = mean rate of cutting lines of force during a half-cycle (or half-revolution) = 2* × (maximum flux)/(time of half-revolution)

$$= \frac{4 \times \text{maximum flux}}{\text{periodic time}}$$

$$= 4 \times \text{maximum flux} \times \text{frequency} = 4f\Phi_m.$$

The ratio to the crest valve is ·637 for any sinusoidal wave. For other wave forms it can be found by plotting the half-wave on a time (or angle) base and finding the mean height $\left(\text{or } \frac{\text{area}}{\text{base}}\right)$ for the figure formed by the half-wave and its base (see Example 1).

For most purposes the **effective** (or **virtual**) value is more important. This may be defined as follows:—

The effective value of an alternating $\left\{\frac{\text{current}}{\text{voltage}}\right\}$ is equal to the steady $\left\{\frac{\text{current}}{\text{voltage}}\right\}$ which $\left\{\frac{\text{flowing in}}{\text{applied to}}\right\}$ the same resistance causes heat to be produced at the same rate as the mean rate with the alternating $\left\{\frac{\text{current}}{\text{voltage}}\right\}$, all the conditions being the same.†

* Because flux through coil is reversed in half a revolution.

† Allowance must be made for effects due to the A.C. itself, cf. Art. 7.

Since the rate of heat production in a given resistance varies as (current)^2, the average rate for an A.C. depends on the average value of its square;

$$\therefore\ (\text{effective value})^2 = \text{mean value of (alternating current)}^2;$$

$$\therefore\ \textit{effective value} = \sqrt{(\text{mean value of A.C.})^2}$$

$$= \textit{square root of mean square.}$$

The effective value is consequently often called the R.M.S. value.

The same applies to voltage, since rate of heat production varies as (voltage)^2.

The ratio $\left(\dfrac{\text{effective value}}{\text{maximum value}}\right)$ is called the *amplitude factor* of a wave, and for all sinusoidal waves it has the value $\dfrac{1}{\sqrt{2}} = \cdot 707$.

The reciprocal of the amplitude factor, *i.e.* the ratio

$$\left(\frac{\text{maximum}\ \cdot\ \ \text{e}}{\text{effective value}}\right),$$

is called the peak factor, or crest factor, and for all sinusoidal waves it has the value $\sqrt{2} = 1{\cdot}414$.

These values for sinusoidal waves are obtained as follows:—

The maximum E.M.F. (see Art. 2) is $2\pi f\Phi_m$;

$$\therefore\ \frac{\text{average E.M.F.}}{\text{maximum E.M.F.}} = \frac{4}{2\pi} = \frac{2}{\pi} = \cdot 637.$$

Again let $\quad i = \mathrm{I}_m \sin \omega t,$

where $\quad \mathrm{I}_m = \text{maximum value of current},$

and $\quad \omega = 2\pi \times \text{frequency},$

and so i is an alternating current whose value is zero when $t = 0$ (see Fig. 5.c6).

Then the rate of heat production in a resistance of *one* ohm is

$$i^2 = \mathrm{I}_m{}^2 \sin^2 \omega t.$$

But $\quad \sin^2 \omega t = \tfrac{1}{2}(1 - \cos 2\omega t),$

and the mean value of $\cos 2\omega t$ over the period (or half-period) is zero;

$\therefore$ mean value of $\sin^2 \omega t$ is $\tfrac{1}{2}$;

$\therefore$ mean rate of heat production is $\tfrac{1}{2}\mathrm{I}_m{}^2$.

(Note that this is half the maximum rate of heat production.)

$$\therefore\ \text{effective value of current} = \sqrt{\tfrac{1}{2}\mathrm{I}_m{}^2} = \mathrm{I}_m/\sqrt{2} = \cdot$$

$$\therefore\ \text{amplitude factor} = \cdot 707, \text{ as stated above.}$$

This will apply equally to any sinusoidal wave.

Alternatively:—Mean value of $i^2 = \int_0^{2\pi} I_m^2 \sin^2 \omega t \, d.\omega t \div$:

$$= \frac{I_m^2}{2\pi}\left[\frac{1}{2}\left(\omega t - \frac{\sin 2\omega t}{2}\right)\right]_0^{2\pi} = \frac{I_m^2}{2\pi}\left[\pi\right] = \frac{I_m^2}{2}; \text{ etc.}$$

The ratio $\left(\frac{\text{R.M.S. value}}{\text{Mean value}}\right)$ is called the **form factor** of the wave. It cannot be less than unity. For sinusoidal waves its value is

$$\left(\frac{1}{\sqrt{2}}\right) \div \left(\frac{2}{\pi}\right) = \frac{\pi}{2\sqrt{2}} = 1 \cdot 111.$$

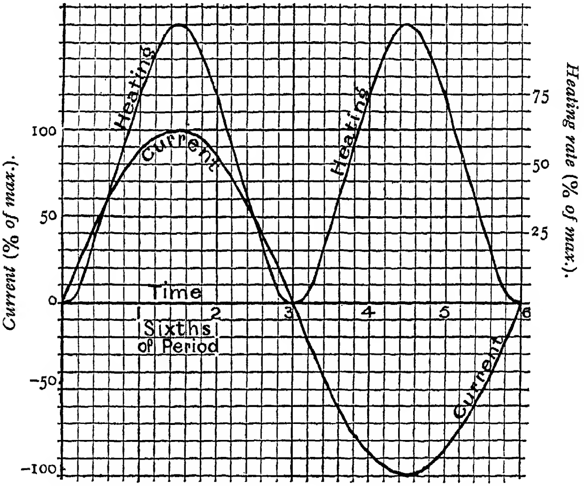


Fig. 5.06.—EFFECTIVE VALUE OF A SINUSOIDAL CURRENT.

Example 1. *Plot the following alternating voltage, the values being given at intervals of $\frac{1}{24}$th of period:—*

0, 26, 51, 89, 131, 173, 225, 298, 231, 164, 88, 30, 0, — 26, *etc.*

Determine its mean and effective values, and its peak and form factors.

The wave is plotted in Fig. 5·07.

The mean value is the mean height of the positive half-wave. Using the end-ordinate method:—

Mean value

$$= \frac{1}{12}\{0 + 26 + 51 + 89 + 131 + 173 + 225 + 298 + 231 + 164 + 88 + 30\}$$
$$= \frac{1}{12} \times 1506 = 125{\cdot}5 \text{ volts.}$$

(Note that $\dfrac{\text{mean value}}{\text{peak value}} = \dfrac{125{\cdot}5}{298} = {\cdot}421$ as against ·637 for sine wave.)

The mean square may be found by squaring all the ordinates.

$$\text{Mean square} = \tfrac{1}{12}\{0 + 676 + 2\,601 + 7\,921 + 17\,161 + 29\,929 + 50{,}625 + 88\,804 + 53\,361 + 26\,896 + 7\,744 + 900\}$$

$$= \tfrac{1}{12} \times 286\,618 = 23\,885;$$

$$\therefore \text{ effective value} = \sqrt{23885} = 154{\cdot}5 \text{ volts;}$$

$$\therefore \text{ peak factor} = \frac{298}{154{\cdot}5} = 1{\cdot}90 \text{ (as against } 1{\cdot}41 \text{ for sine wave)}$$

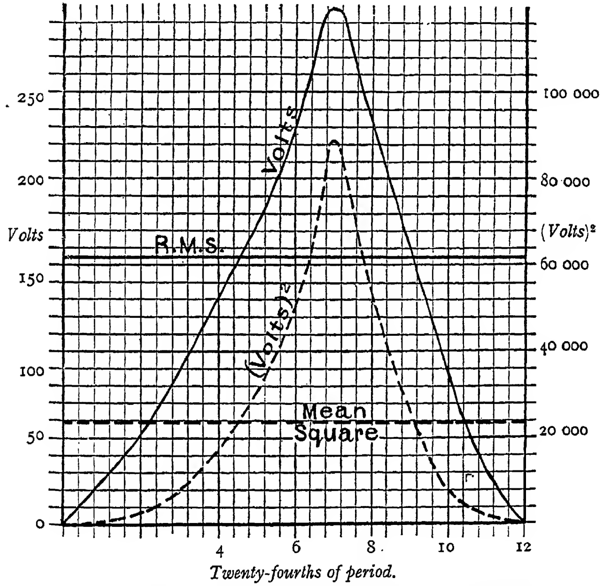


Fig. 5.07.—EFFECTIVE VALUE OF NON-SINUSOIDAL VOLTAGE.

and form factor = $\dfrac{}{125{\cdot}5}$ 1·23 (as against 1·11 for sine wave).

N.B.—A peaked wave has high values for its peak and form factors, while a flat-topped wave has low values for these.

5. "Clock," "Crank," or Vector Diagrams

If a line CB of fixed length is rotated about one end C, the length of its projection (CN) on a fixed line CD is equal to CB sin α, where $\alpha = \angle ACB$, AC being perpendicular to CD, and, therefore, parallel

to BN. Thus, if QM is drawn proportional to α from a fixed point Q, and in line with CA produced, and MP is made equal and parallel to CN, then

$$MP = CB \sin \alpha = CB \sin QM.$$

Therefore if this is repeated for various positions of BC, the point P will trace out a sine-wave, and so can represent an alternating voltage or current. It is convenient when drawing the curve to take the positions of BC at equal angles apart (say 30°). The corresponding positions of M are then at equal distances apart. If ϕ is the phase of this alternating quantity, the origin of time (O) must be taken to the right of Q, so that QO $= \phi$ on the angle scale. Evidently CB = the crest value of the alternating quantity; and time is proportional to the angle turned through by CB.

Now if another alternating quantity is to be represented this can be done by taking another line (CE in Fig. 5.08) equal to the crest

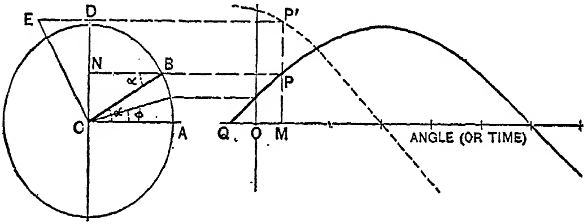


Fig. 5.08.—RELATION OF CRANK DIAGRAM AND TIME GRAPH.

value of the second quantity and projecting from CE as it rotates about C, in the same way as for CB.

If the two quantities have the same frequency the two "cranks" or vectors CB, CE will rotate at the same rate. But if the phase of the second quantity (ϕ') differs from that of the first then, so as to make the origin of time the same, when $\angle$ BCA $(\alpha) = \phi$, $\angle$ ECA $= \phi'$, and so $\angle$ ECB $= (\phi' - \phi) =$ phase difference. Since the two lines CE, CB rotate at the same rate, this angle (ECB) *always* has this value. For instance, if the first quantity is the E.M.F. in Arts. 2, 3, and the second is the flux, then CB $= \omega\Phi_m$; CE $= \Phi_m$; and $\angle$ BCE = a right angle.

Consequently a simple diagram, consisting of the lines CB, CE alone, gives most of the information obtainable from the two complete sinusoidal waves, and the former takes very much less trouble to draw. Such a diagram is called a "clock" (or "crank") or vector diagram.

6. Vector Addition

When the sum is required of two quantities alternating sinusoidally with the same frequency (for examples see Arts. 8, 13), it can be obtained by vector addition; *i.e.* if the "cranks" or vectors representing the two quantities are CA, CB (in Fig. 5.09), the parallelogram CADB is completed, and CD is then the vector representing the sum of the two quantities.

For proof draw AN, BM, and DL, perpendicular to the vertical through C. Then CN is equal to the value of the first quantity at the instant chosen, and CM is equal to the value of the second quantity at the same instant. Therefore the value of the sum of the two quantities at this instant = CN + CM = ML + CM = CL, for CN = ML because CA and BD are equal and parallel.

But CL is the projection of CD on the vertical, and therefore, a sinusoidal wave drawn by means of CD will have its ordinate equal to the sum of the ordinates of the waves of the two given quantities at each point. At some points one or both of the latter are negative, but the above relation still holds good provided the signs are taken into account; *e.g.* if one is negative the vector sum will be equal to the arithmetical difference, and will be positive or negative according as the larger ordinate is the positive or the negative one.

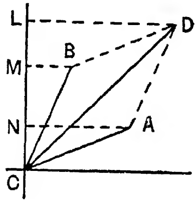


FIG. 5.09.—VECTOR ADDITION.

Since the ratio $\left(\dfrac{\text{effective value}}{\text{maximum value}}\right)$ is constant for sinusoidal waves (see Art. 4), vector addition may be applied equally well to the effective values. The only difference is that all the lines are reduced to $\cdot 707 \left(= \dfrac{1}{\sqrt{2}}\right)$ times their length when maximum values were used. Alternatively, the same diagram may be used to represent both maximum and effective values, the scale for the latter then being $1{\cdot}414$ ($= \sqrt{2}$) times that for the former.

7. Inductance

When an alternating P.D. is applied to a circuit the effective current produced differs as a rule from that set up by a steady P.D., equal to the R.M.S. value of the alternating P.D., applied to the same circuit. It is of great importance for the study of A.C.

phenomena to understand clearly the reasons for the difference, and the factors affecting it.

The cause of this difference is not that the relation: Current $= \dfrac{\text{P.D.}}{\text{Resistance}}$ is inapplicable to an alternating current, but that two further considerations must be taken into account, viz.:

(*a*) $i = \dfrac{v}{R}$ applies to instantaneous values but not necessarily to R.M.S. values;

(*b*) the action of the alternating current may produce an E.M.F. which changes the instantaneous value of the current from $\dfrac{v}{R}$ to $\dfrac{e}{R}$, where e = instantaneous value of the E.M.F. set up by the current.

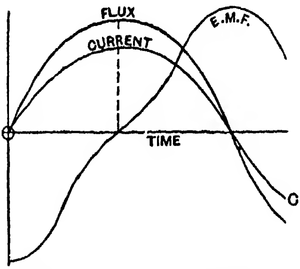


Fig. 5.10.—CURRENT AND SELF-INDUCED F.N.F.

The usual cause of this E.M.F. is that the magnetic flux due to the A.C. links with the circuit, and when the flux changes with the current an E.M.F. is induced in the circuit called the self-induced E.M.F. The value of this depends on the rate of change of the flux, and acts in the direction which opposes this change (by Lenz's Law, see Appendix A); *i.e.* when the current is increasing the self-induced E.M.F. opposes it, and when the current is decreasing this E.M.F. assists it.

The relation between the three quantities is therefore usually somewhat as shown in Fig. 5.10, the flux and current changing together and the E.M.F. being small when they are large but changing slowly, and large when they are small but changing rapidly. The E.M.F. lags behind the flux and the current by quarter of a period (or 90°) if the half-waves of the latter are symmetrical, which is usually the case but not always.

If the flux wave is sinusoidal the E.M.F. wave is sinusoidal too. For let the equation of the flux wave be $\Phi = \Phi_m \sin \omega t$,

then $$e = -\frac{d\Phi}{dt} \times \mho = -\,\omega\Phi_m \mho \cos \omega t,$$

where $\mho$ = number of turns in the coil,

i.e. the E.M.F. wave is a cosine wave reversed, and so lags by $\frac{1}{4}$ period as stated above.

The value of $(\Phi_m \mho)$ depends partly on that of the maximum current (I_m) and partly on the coil and its magnetic circuit, and so may be equated to LI_m, where L depends only on the coil and its surroundings.

Since
$$LI_m = \Phi_m \mho,$$
$$L = \Phi_m \mho / I_m,$$

i.e. L in C.G.S. units is equal to the number of magnetic linkages per C.G.S. unit current; and so L is the inductance (see Chap. II., Art. 1).

When the magnetic circuit is of constant permeability the flux is proportional to the current; $\therefore\ L = \frac{\Phi\mho}{i} = \text{constant};$

$$\therefore\ e = -\frac{d(Li)}{dt} = -L\frac{di}{dt},$$

and for a sine wave $e = -L\frac{d\,(I_m \sin \omega t)}{dt} = -\omega LI_m \cos \omega t;$

$$\therefore\ \text{Max. value of } e = \omega LI_m,$$

and
$$\text{R.M.S. value of } e = \omega LI,$$

where
$$I = \text{R.M.S. value of current.}$$

From the relation $e = -L.di/dt$ an alternative definition of inductance is obtained as follows:—*The inductance of a coil (in C.G.S. units) is equal to the (C.G.S.) E.M.F. self-induced by a current changing at the rate of* 1 *C.G.S. unit per second.*

The practical unit of inductance is called the Henry and may be defined as follows:—*The inductance of a coil in henries is equal to the number of volts self-induced by a current changing at the rate of* 1 *ampere per second.*

Hence 1 Henry = 10^9 C.G.S. units of inductance;

since 1 ampere = $\frac{1}{10}$ C.G.S. unit of current, and to induce 1 volt requires a change of 10^8 magnetic linkages per sec.

The connexion with the magnetic circuit of the coil can be obtained as follows:—

$$\text{Flux} = (\text{M.M.F.})/\text{Reluctance.}$$

Hence
$$\Phi_m = (4\pi/10)\, I_m \mho / R.$$

But, as was shown above, the inductance is given by

$$L = \Phi_m \mho / I_m.$$

Substituting for Φ_m, and cancelling out I_m, results in

$$L = (4\pi/10)\,\mho^2/R.$$

This shows that to obtain a large inductance the electric circuit should have many turns, and that secondly the reluctance of the magnetic circuit should be kept small.

Though the current has been cancelled out it nevertheless affects the value of the inductance if the magnetic circuit consists partly or wholly of magnetic material. For the value of the reluctance changes inversely as the permeability and this depends on the flux-density. This in turn depends on the current, and so the inductance has different values with different currents.

In the majority of cases this effect is negligible and so the inductance is treated as a constant of the circuit. It should be borne in mind that this is not exact when the main part of the magnetic circuit is of iron.

8. Reactance and Impedance

The ratio of the effective value of the self-induced E.M.F. to the effective value of the current is called the **Reactance** of the circuit. If volts and amperes are the units employed for the first two then the reactance is in ohms: or in symbols

$$X = \frac{E_s}{I}$$

where X = reactance in ohms, E_s = self-induced E.M.F. in volts (R.M.S.), and I = current in amperes (R.M.S.).

Since E_s has been shown to be equal to ωLI (Art. 7), it follows that $X = \omega L$. Thus the reactance varies directly as the frequency and is not, like the inductance, dependent only on the circuit. In particular for steady currents (D.C.) the reactance becomes zero.

When there is a self-induced E.M.F. the ratio $\left(\frac{\text{Applied P.D.}}{\text{Current}}\right)$ is no longer equal to the resistance as it is for steady currents, and this ratio is then called the **Impedance**;

i.e. $Z = \frac{V}{I}$,

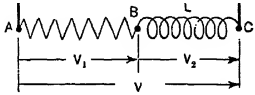
where Z = impedance in ohms,

V = applied P.D. in volts (R.M.S.),

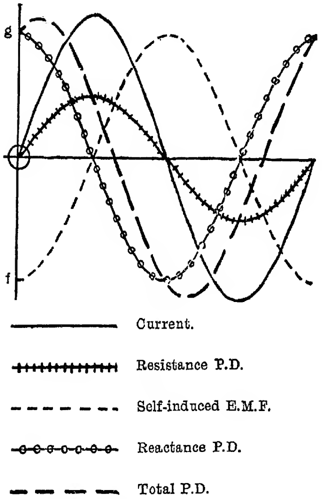
I = current in amperes (R.M.S.).

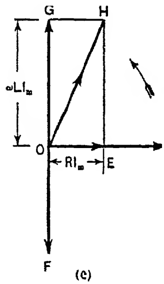
Consider a circuit consisting of an inductionless resistance (R), and an inductance (L) of negligible resistance in series [Fig. 5.11 (*a*)].

If i amperes is the instantaneous value of the current flowing at any instant through the two from A to C (*i.e.* when flowing from C to A, i is a negative quantity) the P.D. across R of A above B is



(a)
CIRCUIT DIAGRAM.





(*b*) TIME GRAPHS. (*c*) VECTOR DIAGRAM.

Fig. 5.11.—RESISTANCE AND REACTANCE IN SERIES.

Ri volts. If the current wave is a sine wave, $i = I_m \sin \omega t$, the resistance P.D. is likewise a sine wave being given by $R.I_m \sin \omega t$.

At the same instant the self-induced E.M.F. is $-L\frac{di}{dt}$, and the P.D. across L of B above C must balance this and so is equal to $L\frac{di}{dt}$.

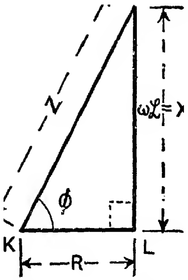


Fig. 5.12.—IMPEDANCE TRIANGLE.

For a sine wave of current this P.D. is $\omega LI_m \cos \omega t$ (see Art. 7) as shown in Fig. 5.11(*b*). The total P.D. of A above C at any instant is the sum of the P.D.s of A above B and of B above C at the same instant, just as for a steady P.D. (*i.e.* with direct current);

total P.D. $= Ri$ [illegible] for all wave forms

$= RI_m \sin \omega t + \omega LI_m \cos \omega t$ for sine wave.

As shown in Art. 6, this latter sum can be obtained by vector addition. In Fig. 5.11(*c*) let OD be the current vector; then OE drawn in same direction and equal to RI_m is the resistance P.D. The self-induced E.M.F. is represented by OF $= \omega LI_m$, a right angle behind OD, and the P.D. of B above C is given by OG equal in magnitude to OF but drawn in the opposite direction. Complete the rectangle OEHG. Then OH is the vector for the resultant P.D.

Now $$OH^2 = OE^2 + OG^2;$$

$$\therefore \text{ resultant P.D. (max.)} = \sqrt{\{(RI_m)^2 + (\omega LI_m)^2\}}$$
$$= I_m\sqrt{\{R^2 + (\omega L)^2\}};$$
$$\therefore \text{ resultant P.D. (R.M.S.)} = I\sqrt{\{R^2 + (\omega L)^2\}};$$
$$\therefore \text{ impedance of AC} = \frac{\text{P.D.}}{\text{Current}} = \sqrt{\{R^2 + (\omega L)^2\}}.$$

From this last result it follows that if a right-angled triangle KLM is drawn with the sides (KL, LM) containing the right angle representing the resistance and reactance respectively to the same scale, the hypotenuse, KM, represents the impedance on the same scale.

The phase difference between the total P.D. and the current is given by the angle HOE in Fig. 5.11(*c*). But the triangle MKL is similar to the triangle HOE since each side is equal to the corresponding side of the latter, divided by I_m. Therefore the angle MKL is equal to the angle HOE;

$$\therefore \tan \phi = \frac{\omega L}{R} = \frac{X}{R}; \text{ and } \cos \phi = \frac{R}{Z}$$

where Z = impedance of whole circuit

and ϕ = phase difference between current and total P.D.;

and the current lags behind the voltage.

If the resistance and reactance are combined instead of being in separate parts of the circuit all the above still holds good except that the resistance and reactance P.D.s (V_1 and V_2) cannot be measured directly.

Example 2. *An inductionless resistance of* 10 *ohms is in series with an inductive coil of* 15 *ohms reactance at* 50 ~ *and of negligible resistance.* (*a*) *Calculate the current flowing when a P.D. of* 200 *volts at this frequency is applied to the terminals, and the P.D. across each part of the circuit.*

(*b*) *Find the total P.D. required to produce the same current as before at* 25 ~.

(*a*) Total impedance $= \sqrt{(10)^2 + (15)^2}$ (see Fig. 5.13).

$$= \sqrt{325}$$

$$= 18{\cdot}0 \text{ ohms;}$$

$$\therefore \text{ current} = \frac{200}{18{\cdot}0} = 11{\cdot}1 \text{ amperes.}$$

P.D. across resistance $= 11{\cdot}1\text{A} \times 10 = 111$ volts.

" " reactance $= 11{\cdot}1 \times 15$ 167 volts.

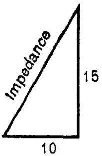


Fig. 5.13.— At 50~

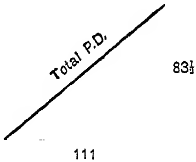


Fig. 5.14.—At 25~.

(*b*) The P.D. across the resistance is unchanged.

The P.D. across the reactance is proportional to the frequency, and therefore its new value is $\frac{25}{50}$ of 167 volts $= 83\frac{1}{2}$ volts;

$$\therefore \text{ total P.D.} = \sqrt{\{(111)^2 + (83\tfrac{1}{2})^2\}} \text{ (see Fig. 5.14)}$$

$$= 139 \text{ volts.}$$

9. Power in A.C. Circuits

In a D.C. circuit the power in watts is equal to the product of the volts and the amperes. In A.C. circuits this remains true for *instantaneous values*. But the average power is usually not equal to the product of the R.M.S. values of the volts and amperes (nor of their mean values). For sinusoidal waves it can be shown that:—

$$\text{average power} = VI \cos \phi$$

where $V, I =$ R.M.S. values of volts and amperes,

$\phi =$ their angular phase difference;

and $\cos \phi$ is called the power factor.

For let $v = V_m \sin \omega t$ be the equation of voltage wave
and $i = I_m \sin (\omega t - \phi)$ the equation of current wave.

Then the equation of the power wave is:—

$$w = v \times i = V_m I_m \sin \omega t \sin (\omega t - \phi), \text{ (see Fig. 5.15)}$$
$$= \tfrac{1}{2} V_m I_m \{\cos \phi - \cos (2\,\omega t - \phi)\}.$$

The mean value of the second term in the bracket taken over a period (or half a period) is zero;

$$\therefore \text{ mean value of } w = \tfrac{1}{2} V_m I_m \cos \phi$$
$$= VI \cos \phi$$

since $V_m = \sqrt{2}\cdot V$, and $I_m = \sqrt{2}\cdot I$ (see Art. 4).

Note that the power wave is of double the frequency of the voltage and current waves and so goes through a complete cycle of

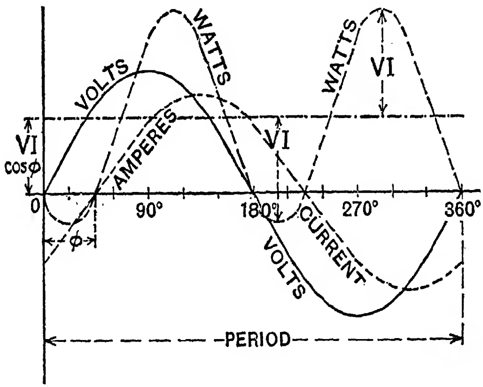


Fig. 5.15.—POWER WITH SINUSOIDAL WAVES.

changes in *half* a period. Further, that the instantaneous value of the watts varies between VI $(1 + \cos \phi)$ and $-$ VI $(1 - \cos \phi)$, and that the average power is half the (algebraic) sum of these. Negative values mean that at the corresponding instants the circuit is returning power to the supply instead of absorbing it.

With non-sinusoidal waves the power may still be written VI $\cos \phi$ so that the power factor is $\cos \phi$, but as a rule ϕ is in such cases not exactly equal to the angle of phase difference between current and voltage, which may then be denoted by ψ ("psi").

Example 3. *The P.D. applied to a circuit and the current flowing in it have the following values at corresponding instants, $\frac{1}{24}$th period apart.*

P.D. 0, 26, 51, 89, 131, 173, 225, 298, 231, 164, 88, 30, 0, etc., volts.
Current −39, −17, 0, 16, 31, 50, 61, 74, 85, 98, 81, 65, 39, etc., amp.

Find the R.M.S. values of the P.D. and of the current, the average power, and the power factor.

The R.M.S. P.D. has been found in Ex. 1 to be 154·5 volts.

Proceeding similarly with the current wave:—

$$\text{mean square of current} = \tfrac{1}{12}\left\{\begin{aligned}&1521 + 289 + 0 + 256 + 961 + 2500 + 3721\\ &+ 5476 + 7225 + 9604 + 6561 + 4225\end{aligned}\right\}$$
$$= \tfrac{1}{12} \times 42\,339 = 3528;$$
$$\therefore \text{ R.M.S. current} = \sqrt{\{3528\}} = 59{\cdot}4 \text{ amperes.}$$

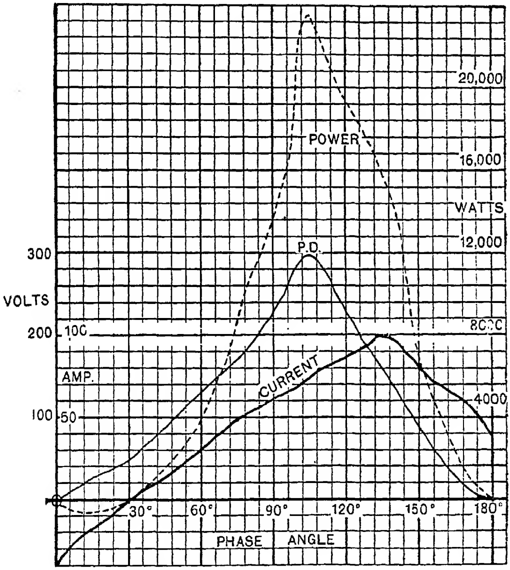


Fig. 5.16.—POWER WITH NON-SINUSOIDAL WAVES.

By multiplying corresponding values of volts and amperes the power wave ordinates for half a period are found to be
0, −442, 0, 1424, 4061, 8650, 13725, 22052, 19635, 16072, 7128, 1950, watts,
those for the next half period being a repetition of these.

Sum of positive ordinates = 94697 watts
Sum of negative ordinates = −442 watts

Complete sum (12 ordinates) = 94255 watts

$\therefore$ Average power $= \frac{1}{12}$ of 94255
$= 7855$ watts.

$$\text{Power factor} = \frac{\text{Watts}}{\text{Volts} \times \text{Amperes}} \cdot \frac{7855}{154{\cdot}5 \times 59{\cdot}4} \quad {\cdot}856.$$

Equating this to cos ϕ makes $\phi = 59°$ whereas (ψ) the angle of lag of current at zero value $= 60°$.

10. Components of Voltage and Current

It follows from Art. 8 that in a circuit containing resistance and reactance the total R.M.S. P.D. is the vector sum of IR and IX (see Fig. 5.17), and that these are respectively equal to V cos ϕ and V sin ϕ. These are called the *components* of the voltage.

Since average power $= \text{VI} \cos \phi$

$$= (\text{current}) \times \text{V} \cos \phi,$$

the former (V cos ϕ) is called the power component, or the active voltage.

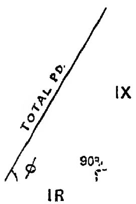


Fig. 5.17.
COMPONENTS OF VOLTAGE.

The other component (V sin ϕ) is often called the wattless component because it does not imply the expenditure of any power on the average, that which is supplied during one quarter period being returned during the next quarter period. An alternative name is the reactive voltage; and the quantity VI sin ϕ is called the reactive volt-amperes.

In the same way the current may be divided into two components I cos ϕ and I sin ϕ, similarly named.

To obtain a physical image of this a non-inductive resistance (R) and a reactance (X) of negligible resistance may be supposed connected in parallel [see Fig. 5.18 (*a*)].

Then if $\quad \text{I} \cos \phi = \text{current in R},$

and $\quad \text{I} \sin \phi = \text{current in X}$

these currents are respectively in phase with, and lagging 90° behind, the common terminal P.D.(V). Therefore:—

$$\begin{aligned} \text{Total current} &= \textit{vector} \text{ sum of separate currents} \\ &= \sqrt{\{(\text{I} \cos \phi)^2 + (\text{I} \sin \phi)^2\}} \qquad [\text{see Fig. } 5.18\ (b)] \\ &= \text{I}\sqrt{\{\cos^2 \phi + \sin^2 \phi\}} = \text{I}, \end{aligned}$$

and ϕ is the angle of lag of total current behind the voltage.

Note that $\quad \text{I} \cos \phi = \dfrac{\text{V}}{\text{R}}, \text{ and I} \sin \phi = \dfrac{\text{V}}{\text{X}}$

and therefore $I = V\sqrt{\left\{\frac{1}{R^2} + \frac{1}{X^2}\right\}} = V.\frac{\sqrt{R^2 + X^2}}{RX}$;

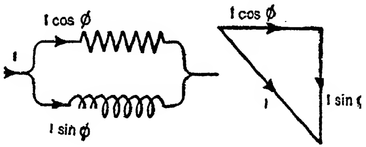
$$\therefore \text{ combined impedance } = \frac{V}{I} = \frac{RX}{\sqrt{R^2 + X^2}} \quad \text{(cf. Art. 8).}$$

It is often convenient to split the voltage or the current into these two components in cases where the resistance and reactance are not separate (for example see Art. 13).

11. Choking Coils

When for any purpose electric power is required at a pressure lower than that of the supply, the most straightforward method with D.C. is to absorb the excess pressure in a series resistance. (For other methods used with D.C. see Chap. XVI., Arts. 8-10.) This can be done with A.C. too, but usually it is better to use a *choking coil, i.e.* a coil whose reactance is large compared with its resistance.

The advantage of this is that the total power absorbed is much less than with a resistance. If the choking coil has only reactance (X) and the apparatus in series has only resistance (R) the power absorbed $= VI \cos \phi = V.I.\frac{R}{Z} = I^2R =$ power absorbed by resistance alone (since $\frac{V}{Z} = I$). Whereas with a series resistance (R_s) the power absorbed $= I^2(R + R_s) = VI$; *i.e.* it is greater than the power taken by the useful resistance alone in the ratio $\frac{R + R_s}{R}$ $= \frac{V}{V_r}$, where $V_r =$ P.D. across useful resistance $=$ IR.



(*a*) CIRCUIT DIAGRAM. (*b*) VECTOR DIAGRAM.

Fig. 5.18.—COMPONENTS OF CURRENT.

With an actual choking coil of effective resistance R_c, the power taken by the whole circuit is increased similarly in the ratio $\frac{R + R_c}{R}$; but since R_c can be made much less than R_s the saving is still considerable (see Example 4). The lower the power factor of the coil the higher is its efficiency.

The choking coil acts by taking in energy during part of the period and restoring some* of it during the next part in a manner analogous to that of a flywheel. The amount of energy thus stored and returned can be proved equal to $\frac{1}{2}LI^2_m$, and therefore to LI^2 for sinusoidal waves.

For if the current I_m is reduced to zero in t sec. at a uniform rate the induced E.M.F. has the value $-L.di/dt = L.I_m/t$ during this time, and the average current is $\frac{1}{2}I_m$. Hence the work done $= V.I.t = L(I_m/t).\frac{1}{2}I_m.t = \frac{1}{2}LI_m^2$. And this value is unchanged if the current is reduced to zero in any other manner. If L is in henries and I_m is in amperes the energy is in joules (watt-seconds); but if L and I are in C.G.S. units the energy is in ergs.

In the former case if the iron reluctance is negligible and air-gap is of length l cm. and cross-section A sq. cm., with a maximum flux-density of B lines per sq. cm., $I_m = (10/4\pi)\,Bl/\mho$, and $L = BA\mho/10^8$; $\therefore\ \frac{1}{2}LI_m^2 = (B^2/8\pi).Al/10^8 =$ ergs in gap$/10^8 =$ joules in gap (see Chap. IV., Art. 42).

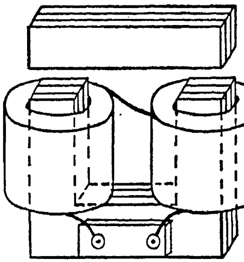


Fig. 5.19.—CHOKING COIL.

A choking coil (see Fig. 5.19) consists of many turns of wire wound on a laminated iron core, preferably with a short air-gap (or gaps) in the magnetic circuit to reduce the distortion in the wave form which variable permeability and hysteresis produce. Its *effective resistance* (R_c) is greater than the *ohmic resistance* (*i.e.* that due to the coils alone) by an amount which takes into account the losses due to hysteresis and eddy currents in its core, and is given in ohms by

$$R_c = \frac{\text{watts absorbed by coil}}{(\text{current})^2}.$$

This effective resistance increases with the frequency (but not in direct proportion to it) because the iron losses increase with frequency (see Chap. XII., Art. 3) while the ohmic resistance is independent of the frequency.

Example 4. *A choking coil is required to enable a number of incandescent (non-inductive) lamps to take* 3·2 *A. at* 110 *volts from an A.C. supply at* 210

* All, if its resistance is negligible.

volts. If the effective resistance is 4 ohms, find its reactance, and compare its efficiency with that of a series resistance for the same purpose.

$$\text{The resistance of the lamps} = \frac{110}{3\cdot 2} \quad 34\cdot 4 \text{ ohms;}$$

$$\text{total resistance in circuit} = 34\cdot 4 + 4 = 38\cdot 4 \text{ ohms.}$$

$$\text{The total impedance} = \frac{210}{3\cdot 2} = 65\cdot 6 \text{ ohms;}$$

$$\begin{aligned}\therefore \text{ reactance of coil} &= \sqrt{\{(65\cdot 6)^2 - (38\cdot 4)^2\}} \\ &= \sqrt{\{104 \times 27\cdot 2\}} \\ &= 53\cdot 2 \text{ ohms.}\end{aligned}$$

$$\begin{aligned}\text{Efficiency} &= \frac{34\cdot 4}{38\cdot 4} = 1 - \frac{4}{8\cdot 4} = 1 - \cdot 104 \\ &= \cdot 896 = \quad \cdot 6 \text{ per cent.}\end{aligned}$$

With a series resistance:—

$$\text{Total resistance in circuit} = \frac{210}{3\cdot 2} \quad 65\cdot 6 \text{ ohms;}$$

$\therefore$ series resistance $= 65\cdot 6 - 34\cdot 4 = 31\cdot 2$ ohms;

$\therefore$ efficiency $= \dfrac{34\cdot 4}{65\cdot 6} : : \cdot 524 = 52\cdot 4$ per cent.

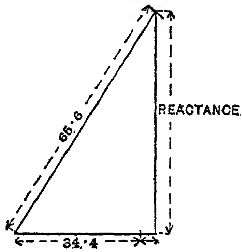


Fig. 5.20.—VECTOR DIAGRAM OF CHOKING COIL CIRCUIT.

Alternatively:—

Power used by lamps $= 110^{\text{v}} \times 3\cdot 2^{\text{A}} = 352$ watts

Power absorbed by coil $= (3\cdot 2)^2 \times 4 = 41$ watts;

$$\text{efficiency of coil} = \frac{352}{352 + 41} = 1$$

$$\frac{41}{393} \quad 89\cdot 6 \text{ per cent.}$$

Power absorbed by series resistance $(210 - 110)^{\text{v}} \times 3\cdot 2 = 320$ watts;

$$\text{efficiency of series resistance} = \frac{352}{352 + 320} \quad 52\cdot 4 \text{ per cent.}$$

12. Capacitance Current

When a steady P.D. is applied to a condenser, current flows until in a brief time the P.D. between the plates becomes equal to that applied. Thereafter no current flows, unless the condenser is a leaky one, in which case a smaller current continues to flow through the insulation between the plates (*e.g.* a cable, see Chap. III., Art. 15).

If the P.D. applied is an alternating one of steady R.M.S. value an alternating current will flow whose R.M.S. is constant (except sometimes during the first few oscillations). With sinusoidal waves the value of this current is given by:—

$$I =$$

where $\quad I =$ R.M.S. value of current in amperes,

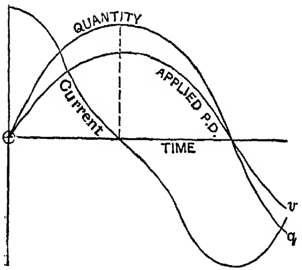


Fig. 5.21.—CAPACITANCE CURRENT WITH NON-SINUSOIDAL WAVES.

$\omega = 2\pi \times$ frequency,

C = capacitance of condenser in farads,

V = R.M.S. value of applied P.D. in volts,

and this current *leads* the P.D. by $\frac{1}{4}$-period (see Ex. 5).

Proof.—Let the voltage wave be given by $v = V_m \sin \omega t$.

Then the quantity in the condenser at any instant is given by:—

$$q = Cv$$
$$= CV_m \sin \omega t.$$

Current flowing rate of change of quantity,

$$= \frac{dq}{dt}$$
$$= CV_m \frac{d \sin \omega t}{dt}$$
$$= \omega CV_m \cos \omega t,$$

i.e. it leads the P.D. by $90° = \frac{1}{4}$-period.

Max. value of current $= \omega\, CV_m$;

$\therefore$ R.M.S. „ „ „ $= \frac{\omega\, CV_m}{\sqrt{2}}$

$$= \omega CV$$

With non-sinusoidal waves the current $\left(= C\frac{dv}{dt}\right)$ has a wave-form different from the P.D.s, but the angle of lead remains $\frac{1}{4}$-period if each half of either wave is symmetrical, and then the other wave is also symmetrical (see Fig. 5.21).

Example 5. (*a*) *Calculate the current taken by a cable* 11 *miles long having a capacitance of* 0·31 *microfarad per mile when connected to a supply at* 6 600 *volts* 50 ~, *with no load on the cable.*

(*b*) *If a non-inductive load of* 100 *kilowatts comes onto the cable, what is the new value of the cable current?*

(*a*) Capacitance of cable $= 11 \times 0{\cdot}31\mu F = 3{\cdot}41\ \mu F$;

$\therefore$ Capacitance current $= \omega CV = 2\pi \times 50 \times \frac{3{\cdot}41}{10^6} \times 6600$

$$= 7{\cdot}06 \text{ amperes.}$$

(*b*) Load current $= \frac{100 \times 1000}{6600} = 15{\cdot}15$ amperes.

This is in phase with the P.D. but the capacitance current leads the P.D. by 90°. The total current is the *vector* sum of the two currents as the two circuits are in parallel;

$\therefore$ total current $= \sqrt{\{(15{\cdot}15)^2 + (7{\cdot}06)^2\}} = \sqrt{\{274{\cdot}5\}}$

$$= 16{\cdot}6 \text{ amperes.}$$

13. Capacitance and Impedance in Series

If a condenser of capacitance C farads [AB in Fig. 5.22(*a*)], is connected in series with an impedance of Z ohms [BC in Fig. 5.22 (*a*)] made up of R ohms resistance and L henries inductance, the respective P.D.s are given by:—

$$V_1 = \frac{I}{\omega C}; \text{ and } V_2 = IZ = I\sqrt{\{R^2 + (\omega L)^2\}}.$$

The total P.D. (V) is the vector sum of V_1 and V_2. The value of this can be found by splitting V_2 into its components RI and ωLI (see Art. 10). Then, if in Fig. 5.22 (*b*), OP represents V_1, lagging 90° behind the current, and OQ represents V_2, OR its power component, and OS its reactive component, the total P.D. is given by OW, the diagonal of the parallelogram OPWQ. But OW is likewise the vector sum of OP, OR, and OS; and OP and OS are in exactly opposite directions, and so their vector sum is equal to their arithmetical difference = OT (where ST = OP);

$$\therefore\ V = OW = \sqrt{\{OR^2 + OT^2\}} = \sqrt{\left\{(RI)^2 + \left(\omega LI - \frac{I}{\omega C}\right)^2\right\}}$$

$$= I\sqrt{\left\{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right\}}.$$

The current will lag behind the total P.D. [as shown in Fig. 5.22 (*b*)] or will lead it, according as ωL is greater or less than $\frac{1}{\omega C}$.

14. Resonance

In the particular case when ωL is equal to $\frac{1}{\omega C}$ (or $\omega^2 LC = 1$), the current is in phase with the total P.D., and the latter is equal to IR; *i.e.* the capacitance and reactive voltages cancel out, and the current is the same as if the resistance alone were in the circuit.

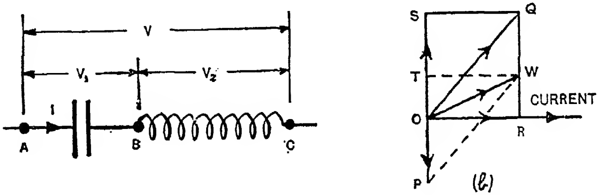


Fig. 5.22.—CAPACITANCE AND IMPEDANCE IN SERIES.

The P.D. across the impedance is then greater than the total P.D., and the capacitance P.D. may also be greater [see Ex. 6 (*a*)].

With a given capacitance and inductance the frequency at which these effects occur is given by:—

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{1}{LC}}.$$

This state is known as *resonance,* from analogy with sound, because the frequency is the same as that at which electricity would oscillate in a circuit consisting of the capacitance and inductance alone when short-circuited.

When the applied frequency is close to that of resonance, the P.D.s across the condenser and the impedance respectively may exceed the total P.D.; and this is known as *partial* (or *imperfect*) *resonance* [see Ex. 6 (*b*)].

If the condenser (Fig. 5.23) is initially charged to a potential V_m, on closing the switch the condenser will discharge. By the time that the condenser is discharged a considerable current is flowing in the inductance, and as it diminishes the condenser is recharged to its original P.D. (if there is no resistance in circuit) but in the reverse direction. It then discharges again and thus there is an oscillation of electricity in the circuit, *i.e.* an alternating current of constant effective value.

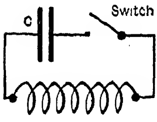


Fig. 5.23.—OSCILLATORY CIRCUIT.

Originally the energy contained in the condenser amounts to $\frac{1}{2}CV_m^2$. For if the condenser is discharged at a uniform rate in t seconds the current during discharge is $Q_m/t = CV_m/t$. And the mean voltage during discharge is $\frac{1}{2}V_m$. Hence the energy given out is $V.It = \frac{1}{2}V_m.(CV_m/t).t = \frac{1}{2}CV_m^2$. If C is in farads and V in volts the energy is in joules (watt-seconds). At the moment of complete discharge all this energy is in the inductance;

$$\therefore \tfrac{1}{2}CV_m^2 = \tfrac{1}{2}LI_m^2 \qquad \text{(cf. Art. 11).}$$

$$= \frac{1}{2}\frac{V_m^2}{\omega^2 L} \qquad \left(\text{since } I_m = \frac{V_m}{\omega L}\right)$$

$$\therefore \omega^2 CL = 1, \text{ as stated above.}$$

Example 6. *A coil of* 0·64 *henry inductance and* 40 *ohms resistance is connected in series with a condenser of* 12 *microfarads capacitance.*

(*a*) *Find the frequency for resonance, and the P.D.s on the coil and on the condenser and across the two, when a current of* 1·5A *at this frequency is flowing;*

(*b*) *find the three P.D.s when the same current at* 50 ∼ *flows;*
(*c*) *repeat this for* 75 *cycles per sec.*

(*a*) Frequency for resonance $= \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}}$

$$= \frac{1}{2\pi}\sqrt{\left\{\frac{10^6}{0{\cdot}64 \times 12}\right\}}$$

$$= 57{\cdot}5 \text{ cycles per sec.}$$

Reactive P.D. $= 2\pi \times 57{\cdot}5 \times 0{\cdot}64 \times 1{\cdot}5 = 347$ volts.

Resistance P.D. $= 40 \times 1{\cdot}5 = 60$ volts.

Coil P.D. $= \sqrt{\{60^2 + 347^2\}} = 352$ volts.

Condenser P.D. $= \dfrac{1{\cdot}5 \times 10^6}{2\pi \times 57{\cdot}5 \times 12} = 347$ volts.

Total P.D. = Resistance P.D. = 60 volts.

(*b*) Reactive P.D. $= 2\pi \times 50 \times 0{\cdot}64 \times 1{\cdot}5 = 302$ volts;

$\therefore$ Coil P.D. $= \sqrt{\{60^2 + 302^2\}} = 308$ volts.

Condenser P.D. $= \dfrac{1{\cdot}5 \times 10^6}{2\pi \times 50 \times 12} = 398$ volts.

Total P.D. $= \sqrt{\{60^2 + (302 - 398)^2\}}$ (cf. Fig. 5·22.)

$= \sqrt{\{60^2 + 96^2\}} = 113$ volts.

Current leads the total voltage.

(*c*) Reactive P.D. $= 302 \times 75/50 = 453$ volts;

$\therefore$ Coil P.D. $= \sqrt{\{60^2 + 453^2\}} = 457$ volts.

Condenser P.D. $= 398 \times 50/75 = 265$ volts.

Total P.D. $= \sqrt{\{60^2 + (453 - 265)^2\}}$

$= \sqrt{\{60^2 + 188^2\}} = 197$ volts.

Current lags behind the total voltage.

15. Mechanical Analogies

The behaviour of A.C. circuits may be understood more clearly by comparing them with certain mechanical arrangements which act in similar fashion. For instance, if a twisting moment is applied to a heavy flywheel it gathers speed gradually. If this torque is reversed after a time, the flywheel continues to move in the opposite direction to the force at a decreasing velocity, then comes to rest, and only after this does it begin to move in the direction of the force.

This is analogous to the application of an alternating P.D. to an inductance (see Art. 7), the velocity of the flywheel corresponding with the magnitude of the electric current. If the torque varies sinusoidally with time the velocity does the same, but the velocity is zero when the force is at its maximum, and the velocity reaches its

maximum when the force has fallen again to zero, *i.e.* the changes in velocity lag behind those of the force by $\frac{1}{4}$ period, just in the same way as the current lags behind the applied P.D.

Moreover, the greater the moment of inertia of the flywheel, the greater the force required to produce a given velocity. And again, the greater the frequency (*i.e.* the shorter the period) the greater the force required. And, thirdly, the greater the velocity to be produced the greater the force. These correspond with the proportionality of the P.D. to (*a*) the inductance, (*b*) the frequency, and (*c*) the current.

Electrical resistance can be represented by bearing or air friction, or by the immersion of the flywheel in a more or less viscous liquid. These produce a force which always opposes the motion, and which increases with the velocity of the flywheel. If the viscosity is great and the inertia small, the maximum velocity will occur almost at the same instant as the maximum force. As the inertia increases or the viscosity decreases, the time lag of the maximum velocity increases, tending towards the $\frac{1}{4}$ period lag of the case of no viscosity.

A spring is analogous to an electrical condenser. If a force is applied to its free end the deflexion (electrical charge or quantity) is proportional to the force (P.D.) unless the elastic limit (dielectric strength) is exceeded. If an alternating force is applied the deflexion at each instant is proportional to the force, but the velocity (current) is not. The force is maximum, positive or negative, at each end of the movement, but the velocity is zero at these instants. On the other hand, the maximum velocity occurs at the centre of the movement and the force is then zero. Moreover, the maximum velocity in either direction occurs $\frac{1}{4}$ period before the maximum force in the same direction, corresponding with the leading current taken by a condenser (Art. 12).

The effect of frequency is the same in the two cases. For if this is altered and the maximum value of the force is unchanged, the same maximum displacement is produced in a time which is shorter the greater the frequency. Thus the maximum velocity increases directly as the frequency, in the same way as does capacitance current.

By combining inertia with an elastic body, *e.g.* a weight supported by a spring, a model of an electrical oscillating circuit (Art. 14) is obtained. If this is displaced from its equilibrium position and then released it will oscillate at a frequency depending on the elasticity of the spring and the inertia of the weight. The amplitude,

and therefore the velocity, of the oscillations gradually diminishes owing to air or other resistance, but the frequency is constant.

If an alternating force of this frequency is applied, the maximum displacement (and therefore the maximum velocity and stress) is much larger than if the same force is applied to the spring alone or to the weight alone. The maxima are limited only by the resistances, and so a small force of the resonating frequency may cause a breakdown just as in the corresponding electrical case.

The analogy between the mechanical and electrical systems is so close in all these cases that anyone familiar with the former will find little difficulty in extending the comparison to many other cases.

16. Skin Effect

When a conductor of large cross-section carries an alternating current the centre portions have a larger E.M.F. induced in them than the outer portions, since the former have a larger flux linked with them; *e.g.* with a straight conductor of circular cross-section the lines of force are concentric circles the whole of which link with the axis of the conductor: but a filament of the conductor at a distance from the axis does not link with those lines of force which lie between it and the axis.

The result of this difference in the induced E.M.F.s is to make the current density vary over the section, increasing towards the surface. In addition a phase difference is produced between the currents in different parts. As the centre is approached the current lags more behind the current at the surface owing to the greater inductance of the central parts.

The total current is the *vector* sum of the currents in the elements of the sectional area, and this is necessarily less than the arithmetical sum. But the heating produced (I^2R loss) depends on the latter; and is further increased by the departure from uniform current density.

Therefore, defining resistance as:—

$$(\text{watts lost in heating the conductor})/(\text{current})^2,$$

it follows that the resistance of a conductor to alternating currents is greater than its resistance to steady currents. The impedance of the conductor is higher still owing to its reactance with A.C.

Obviously, this effect will increase with the size of conductor since for the same current density there will be more internal flux. Similarly, higher frequencies produce a greater effect since these produce greater phase differences: whereas higher resistivity reduces the effect because it reduces the phase differences.

For conductors of circular section and of non-magnetic material the amount of the increase depends on a quantity $(m) = \pi d\sqrt{2f/\rho'}$ $\doteqdot 0{\cdot}1d\sqrt{2f/\rho}$, where d = diameter in cm.; f = frequency in cycles per sec.; ρ' = resistivity in C.G.S. units; ρ = resistivity in microhm-cm. $= \rho' \div 10^3$.

When m is less than 1 the increase is under $\frac{1}{2}$ per cent., but becomes greater rapidly for higher values of m, reaching 100 per cent. when m is 4·9. For higher values, (A.C. resistance)/(D.C. resistance) $= 0{\cdot}353m + 0{\cdot}27$ approx., and for very high values, this ratio $= m/2\sqrt{2} = 0{\cdot}3536m$ approx.

At a frequency of 50 ∼ the increase of resistance is negligible for conductors, other than iron, of less than 0·6 in. diameter. For a round copper conductor 0·8 in. diameter, the increase is about 3 per cent. rising to 7 per cent. for 1 in. diameter, and to 30 per cent. for $1\frac{1}{2}$ in. diameter. For other non-magnetic metals the same percentage increases occur at diameters which are greater in proportion to $\sqrt{}$(resistivity), *i.e.* which give the same conductance per unit length, as the expression for m shows.

With iron and other magnetic materials the effect is much greater since, with a given current, a greater magnetic flux will be carried by the conductor itself. The effect is represented by increasing m in the ratio of $\sqrt{\text{permeability}}$. Thus the amount of the increase will vary with the current, being greatest at a value which causes such a flux in the conductor that the average permeability is a maximum. For both higher and lower values of the current the increase of resistance will be smaller. Moreover the hysteresis loss will be included in the heating loss, and so the properties of the material in this respect will affect the ratio.

For steel rails, used as a return conductor in tram or train traction, an approximation for working conditions may be made by assuming that at 25 ∼ only the outside layer to a depth of 3 mm. carries the current, and 4 mm. for 15 ∼.*

When m is large $\quad R_a/R = m/2\sqrt{2}$;

$$\begin{aligned}\therefore\ R_a &= (m/2\sqrt{2})\,(\rho' l/\tfrac{1}{4}\pi d^2)\\ &= (\pi d\sqrt{f}/2\sqrt{\rho'})\,(4\rho' l/\pi d^2)\\ &= \rho' l/(\tfrac{1}{2}d\sqrt{\rho' f}) = \rho' l/(\pi d\sqrt{\rho'/f}/2\pi).\end{aligned}$$

Thus the value of R_a is the same as if the current flowed only in a thin skin near the surface of thickness $\sqrt{\rho'/f}/2\pi$.

* Wilson & Lydall, *Electric Traction*, Vol. II., Chap. 8.

17. Impedances in Series

When two impedances are connected in series (Fig. 5.24) the total P.D. is the *vector* sum of the separate P.D.s. And since the current is the same in the two at each instant, the total impedance ($\bar{Z}$) is equal to

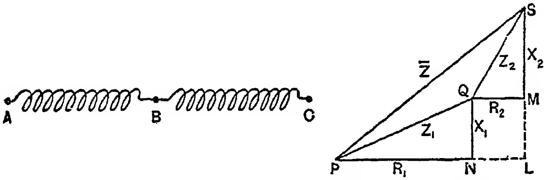
$$\left\{\frac{\text{Total P.D.}}{\text{Current}}\right\} = \frac{\text{Vector sum of separate P.D.s}}{\text{Current}}$$

$$= \text{Vector sum of impedances,}$$

i.e. total impedance = PS in Fig. 5.24, if PQ and QS represent the separate impedances in magnitude and phase.

By splitting up each impedance into resistance and reactance it follows that:—

$$Z = PS = \sqrt{\{PL^2 + LS^2\}}$$
$$= \sqrt{\{(PN + NL)^2 + (LM + MS)^2\}}$$
$$= \sqrt{\{(R_1 + R_2)^2 + (X_1 + X_2)^2\}}.$$



(*a*) CIRCUIT DIAGRAM. (*b*) VECTOR DIAGRAM.

Fig. 5.24.—IMPEDANCES IN SERIES.

In the same way it can be shown that with any number of impedances in series:—

Total impedance = Vector sum of separate impedances.

$$= \sqrt{\{(\text{Sum of resistances})^2 + (\text{Sum of reactances})^2\}}.$$

(See, however, Art. 22 for a modification of this rule under some conditions.)

Example 7. *A non-inductive coil (A) of 7 ohms resistance, is connected in series with a coil (B) of 8 ohms impedance and 6 ohms resistance, and a coil (C) of 12 ohms impedance and 3 ohms resistance, when frequency is 50 ∼.*

(*a*) *Find the total impedance at this frequency.*

(*b*) *What current flows when 200 volts 50 ∼ is applied to the terminals, and what are the P.D.s across the various coils?*

(*c*) *If coil A is short circuited, to what value does the current rise?*

(*a*) Reactance of coil B $= \sqrt{\{8^2 - 6^2\}} = \sqrt{28} = 5{\cdot}3$ ohms.

Reactance of coil C $= \sqrt{\{12^2 - 3^2\}} = \sqrt{135} = 11{\cdot}6$ ohms.

$\therefore$ Total impedance $= \sqrt{\{(7 + 6 + 3)^2 + (0 + 5{\cdot}3 + 11{\cdot}6)^2\}}$
$+ 286\} = 23{\cdot}3$ ohms.

(*b*) $$\text{Current} = \frac{200}{23{\cdot}3} = 8{\cdot}59 \text{ amperes.}$$

P.D. across A $= 8{\cdot}59 \times 7 = 60{\cdot}1$ volts
" " B $= 8{\cdot}59 \times 8 = 68{\cdot}7$ volts
" " C $= 8{\cdot}59 \times 12 = 103{\cdot}1$ volts.

Note that the sum of the P.D.s is $231{\cdot}9$V, which considerably exceeds the total P.D. of 200 volts.

(*c*) The total impedance of B and C in series

$$= \sqrt{\{(6 + 3)^2 + (5{\cdot}3 + 11{\cdot}6)^2\}} = \sqrt{\{81 + 286\}}$$
$$= 19{\cdot}3 \text{ ohms;}$$

$$\therefore \text{ increased value of current} = \frac{200}{19{\cdot}3} = 10{\cdot}4 \text{ amperes.}$$

18. Impedances and Condensers in Series

This case may be treated by firstly obtaining the total impedance of the separate impedances by the above method, secondly, calculating the single capacitance equivalent to the separate capacitances, and combining these two results as in Art. 13. As the P.D. across a capacitance is equal to $\frac{1}{\omega C}$ (Art. 12), the single capacitance ($\bar{C}$) equivalent to a number of capacitances (C_1, C_2, etc.) in series, is given by

$$\frac{1}{\bar{C}} = \frac{1}{C_1} + \frac{1}{C_2} + \text{etc.}$$

Hence the impedance of the whole circuit is equal to

$$\sqrt{\left\{(\text{Sum of resistances})^2 + \left(\text{Sum of reactances} - \frac{1}{\omega \bar{C}}\right)^2\right\}},$$

i.e. the method of Art. 17 can be used but the capacitance reactances $\left(\frac{1}{\omega C}\right)$ must be reckoned negative. The current will lag behind the P.D. if the total reactance thus obtained is positive.

Example 8. *A circuit consists of a non-inductive coil (A) of* 200 *ohms resistance, a coil (B) of* 250 *ohms impedance and* 150 *ohms resistance at* 50 ~, *and two condensers of* 30 *microfarads and* 60 *microfarads capacitance respectively, all in series.*

Find (a) the total impedance;

(b) the current with 400 *volts* 50 ~ *applied to the terminals, and the P.D. across each part of the circuit;*

(*c*) *the additional reactance or capacitance required to bring the current into phase with the total P.D.*

(*a*) Reactance of coil B $= \sqrt{\{250^2 - 150^2\}} = 200$ ohms;

$$\therefore \text{ Impedance of A and B in series } = \sqrt{\{(200 + 150)^2 + 200^2\}}$$
$$= \sqrt{\{122\,500 + 40\,000\}}$$
$$= 403 \text{ ohms.}$$

$$\text{Combined capacitance } = 1 \div (\tfrac{1}{30} + \tfrac{1}{60}) = 20\mu\text{F};$$

$$\therefore \text{ total capacitance reactance } = \frac{10^6}{2\pi \times 50 \times 20} = 159 \text{ ohms};$$

$$\therefore \text{ combined impedance } = \sqrt{\{(200 + 150)^2 + (200 - 159)^2\}}$$
$$= \sqrt{\{122\,500 + 1681\}}$$
$$= 353 \text{ ohms.}$$

(*b*) Current $= \frac{400}{353} = 1\cdot13$ amp.

P.D. across A $= 1\cdot13 \times 200 = 226$ volts.

P.D. across B $= 1\cdot13 \times 250 = 283$ volts.

[P.D. across A and B in series $= 1\cdot13 \times 403 = 455$ volts, *i.e.* this is a case of partial resonance.]

$$\text{P.D. across } 30\ \mu\text{F condenser} = \frac{1\cdot13 \times 10^6}{2\pi \times 50 \times 30} = 120 \text{ volts.}$$

$$\text{P.D. across } 60 \quad \text{condenser} = \frac{1\cdot13 \times 10^6}{2\pi \times 50 \times 60} = 60 \text{ volts.}$$

[P.D. across the two condensers in series $= \dfrac{1\cdot13 \times 10^6}{2\pi \times 50 \times 20}$ $1\cdot13 \times 159$

$= 180$ volts.

Note that this is the sum of the separate P.D.s across the two condensers, these being in phase with each other because they are both 90° behind the common current.]

(*c*) The capacitance reactance is the smaller of the two and so the current will lag behind the total P.D. To bring it into phase the capacitance reactance must be brought up to 200 ohms;

$$\text{total capacitance} = \frac{10^6}{\times 50 \times 200} = 15\cdot9 \text{ microfarads};$$

$$\therefore \text{ additional capacitance} = 1 \div \left(\frac{1}{15\cdot9} - \frac{1}{20}\right)$$
$$= 77\cdot6 \text{ microfarads}$$

connected in series with the rest of the circuit.

Or:—additional capacitance reactance $= 200 - 159 = 41$ ohms;

$$\text{additional capacitance} = \frac{10^6}{2\pi \times 50 \times 41} \quad 77\cdot6 \text{ microfarads.}$$

19. Impedances in Parallel

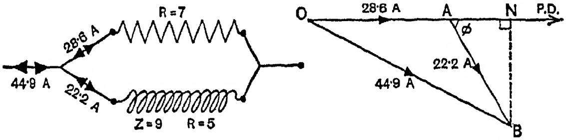
When two or more impedances are connected in parallel and an alternating P.D. applied to the common terminals, the total current flowing at any instant is the sum of the separate currents flowing at the same instant. But as a rule the effective value of the total

current is less than the sum of the effective values of the separate currents, owing to these not being in phase with each other. Therefore:—

total current = vector sum of separate currents (see Ex. 9).

This rule applies equally when one or more of the circuits contains capacitance (see Art. 12, Ex. 5 (*b*), for a simple example). As the capacitance current leads the P.D. whereas inductive resistance takes a lagging current it is possible, with a condenser and an inductive resistance in parallel, for one or both of the separate currents to *exceed* the total current (see Ex. 10). This is known as *current resonance*, to distinguish it from the (*pressure*) *resonance* of Art. 14.

Example 9. *A non-inductive coil of* 7 *ohms resistance is connected in parallel with an inductive coil of* 9 *ohms impedance and* 5 *ohms resistance at* 50 ∼. *If a P.D. of* 200 *volts* 50 ∼ *is applied to the terminals, find the current in each coil and in the mains.*



(*a*) CIRCUIT DIAGRAM. (*b*) VECTOR DIAGRAM.

Fig. 5.25.—IMPEDANCE AND RESISTANCE IN PARALLEL.

$$\text{Current in first coil} = \frac{200}{7} = 28{\cdot}6 \text{ A.}$$

$$\text{Current in second coil} = \frac{200}{9} = 22{\cdot}2 \text{ A.}$$

The mains current is the vector sum [OB, Fig. 5·25 (*b*)] of these two; which can be obtained graphically, or as follows:—

The angle of lag (ϕ) of the current in the second coil is given by $\cos \phi = \frac{R}{Z} = \frac{5}{9}$. Hence the power component (AN) of the current in this coil

$$= 22{\cdot}2 \times \tfrac{5}{9} = 12{\cdot}3 \text{ A.}$$

And the wattless component (NB) $= \sqrt{\{22{\cdot}2^2 - 12{\cdot}3^2\}}$

$= \sqrt{\{34{\cdot}5 \times 9{\cdot}9\}} = 18{\cdot}5$ A.

Hence the total current (OB) $= \sqrt{\{ON^2 + BN^2\}}$

$= \sqrt{\{(28{\cdot}6 + 12{\cdot}3)^2 + (18{\cdot}5)^2\}}$

$= 44{\cdot}9$ amperes.

Example 10. *If a condenser of* 250 μF. *capacitance is connected in parallel with the coils of the above example, calculate*

(*a*) *the total current;*

(*b*) *the total current if the non-inductive coil is removed.*

(*a*) Capacitance current $= 2\pi \times 50 \times \frac{250}{10^6} \times 200 \quad 15{\cdot}7$ A.

This leads the P.D. by 90°;

$$\therefore \text{ nett wattless current} = 18{\cdot}5 - 15{\cdot}7$$
$$= 2{\cdot}8 \text{ A. lagging;}$$

$$\therefore \text{ total current} = \sqrt{\{(28{\cdot}6 + 12{\cdot}3)^2 + (2{\cdot}8)^2\}}$$
$$= 41{\cdot}0 \text{ amperes.}$$

(*b*) Omitting the current taken by the non-inductive coil:—

$$\text{Total current} = \sqrt{\{12{\cdot}3^2 + 2{\cdot}8^2\}}$$
$$= 12{\cdot}6 \text{ amperes.}$$

This is less than either the capacitance current (15·7A) or the current in the inductive resistance (22·2A) and so is a case of partial current resonance.

If the capacitance current equals the wattless component of the current taken by the inductive resistance (or resistances) in parallel with it, the total current is in phase with the P.D. This may be termed exact current resonance (cf. Art. 17), although the frequency at which this occurs is less than that which gives minimum total current, unless the resistance is negligible. When this is the case the condition for resonance is $\omega^2LC = 1$, as for pressure resonance.

20. Admittance, Conductance, and Susceptance

The rule for circuits in parallel may be stated in a form similar to that for circuits in series by introducing the quantity called Admittance, which is defined as the ratio $\left(\frac{\text{Effective amperes}}{\text{Effective volts}}\right)$. It is therefore equal to $\frac{1}{\text{Impedance}}$, and is equal to the *amperes per volt*, just as impedance is equal to the volts per ampere.

Hence with circuits in parallel:—

$$\text{Total admittance} = \frac{\text{Total current}}{\text{P.D.}}$$

$$= \frac{\text{Vector sum of separate currents}}{\text{Common P.D.}} \quad \text{(see Art. 19)}$$

$$= \text{Vector sum of separate admittances,}$$

the admittances being drawn parallel to the respective current vectors just as impedance vectors are drawn parallel to the corresponding P.D.s.

In finding the vector sum of a number of admittances it is convenient to split them into two sets of components respectively in phase with, and 90° out of phase with the P.D., just as with the

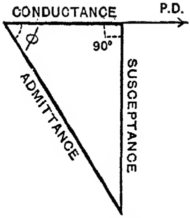


Fig. 5.26.—ADMITTANCE TRIANGLE.

currents in Ex. 9. These components are called respectively the **conductance** and the **susceptance** (see Fig. 5.26).

Thus:—Conductance (G)
= Power component of admittance
= Power component in amperes per volt

Susceptance (B)
= Reactive component of admittance
= Reactive amperes per volt.

The rule for circuits in parallel may now be written:—

Total admittance = $\sqrt{\{(\text{Sum of conductances})^2 + (\text{Sum of susceptances})^2\}}$ (see Ex. 11).

The proof of this follows the same lines as that of the similar rule for impedances in series (Art. 17).

With condensers the susceptance (= ωC for negligible leakage) must be reckoned negative, and the conductance zero.

Denoting the admittance by Y, the following relations hold good:—

$$G = Y \cos \phi = Y . \frac{R}{Z} \quad \text{(see Art. 11)}$$
$$= \frac{1}{Z}\frac{R}{Z} = \frac{R}{Z^2}$$
$$= \frac{R}{R^2 + X^2},$$

and

$$B = Y \sin \phi = Y . \frac{X}{Z}$$
$$= \frac{X}{Z^2}$$
$$= \frac{X}{R^2 + X^2},$$

and

$$Y = \sqrt{G^2 + B^2}.$$

When the reactance is zero, the *susceptance* is zero too, and the conductance is the reciprocal of the resistance just as for continuous current. But when there is reactance the conductance is less than the reciprocal of the resistance.

Similarly in the particular case of a circuit of negligible resistance the susceptance is the reciprocal of the reactance, but for all other cases its value is smaller.

Example 11. *Two coils A and B are connected in parallel and a P.D. of 300 volts 50 ∼ applied to their terminals. Given that at this frequency their impedances are 8 ohms and 11 ohms respectively, and their resistances 7 ohms and 4 ohms respectively, (a) find the currents in each, and the total current and the total power factor; (b) find the total current if a condenser of 120 microfarads capacitance is connected in parallel.*

(a) Admittance of A $= Y_A = \frac{1}{8} = 0{\cdot}125$

Power factor of A = cos $\qquad = 0{\cdot}875;$

$\therefore$ Conductance of A $= G_A = Y_A \cos\phi_A = 0{\cdot}125 \times 0{\cdot}875$
$= 0{\cdot}109,$

or $\quad G_A = \frac{R_A}{Z_A^2} = \frac{7}{8^2} = \frac{7}{64} = {\cdot}109;$

$\therefore$ Susceptance of A,

$= {\cdot}061.$

Similarly:—

$$Y_B = \frac{1}{11} = {\cdot}091$$

$$\cos \qquad \frac{4}{11} = {\cdot}364$$

$$G_B \quad \frac{4}{11^2} = {\cdot}033$$

$$B_B = \sqrt{\{({\cdot}091)^2 - ({\cdot}033)^2\}}$$
$$= {\cdot}085.$$

Total admittance

= vector sum of admittances (see Fig. 5·27)

$= \sqrt{\{({\cdot}109 + {\cdot}033)^2 + ({\cdot}061 + {\cdot}085)^2\}}$

$= {\cdot}204.$

Total current = P.D. $\times$ Total admittance $= 300 \times {\cdot}204$
$= 61{\cdot}2$ amperes.

Current in coil A $= 300 \times {\cdot}125 = 37{\cdot}5$ A.
" " B $= 300 \times {\cdot}091 = 27{\cdot}3$ A. } Sum $= 64{\cdot}8$ A.

If ϕ_T = angle of lag of total current behind P.D.

$$\cos\phi_T \quad \frac{{\cdot}1}{{\cdot}204}$$

Note that this lies between $\cos\phi_A$ and $\cos\phi_B$, as Fig. 5.27 shows must be the case.

(b) Admittance of condenser $= \frac{I}{V}$

$$2\pi \times \quad \times 120 \qquad {\cdot}038,$$

and this is all susceptance, the conductance of a condenser being zero;

$\therefore$ total susceptance of circuit $= {\cdot}061 + {\cdot}085 - {\cdot}038 = {\cdot}108;$

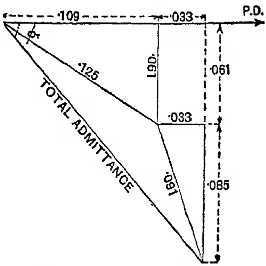


Fig. 5.27.—ADMITTANCES IN PARALLEL.

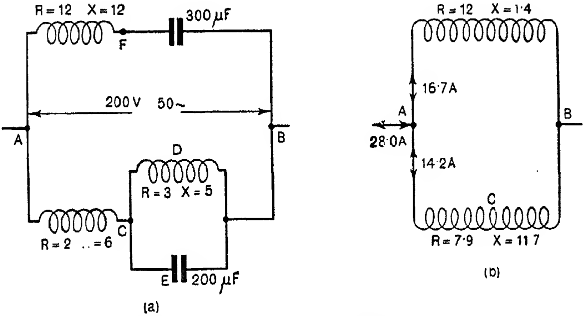
$$\therefore \text{ total admittance} = \sqrt{\{(\cdot 142)^2 + (\cdot 108)^2\}}$$
$$= \cdot 178;$$
$$\therefore \text{ total current} = 300 \times \cdot 178 = 53{\cdot}4 \text{ amperes.}$$

which is less than before by 7·8 A., due to the addition of the leading condenser current of ($300 \times \cdot 038 =$) 11·4 A.

21. General Case

Many circuits more complicated than those dealt with can be treated by reducing them to simpler equivalent circuits step by step, by the use of the methods of Arts. 17 and 18 for series circuits, and that of Art. 20 for parallel circuits. The method of Art. 19 while sufficient for plain parallel circuits is less suitable than that of Art. 20 for the more complicated cases.

The procedure is best illustrated by an example.



CIRCUIT DIAGRAM. SIMPLIFIED CIRCUIT DIAGRAM

Fig. 5.28.

Example 12. *A P.D. of* 200 *volts,* 50 ~, *is applied to the points A, B, of the circuit shown in Fig.* 5.28 (*a*). *Find the total current which flows, and the P.D.s across AC, CB, AF, FB.*

Conductance of CDB R — 3 ·088

Susceptance of CDB = X = ·147

$$\text{Susceptance of CEB} = -2\pi \times 50 \times \frac{200}{10^6} = -\cdot 063$$

$$\text{Total susceptance of CB} = \cdot 147 - \cdot 063 = \cdot 084;$$

$$\therefore \text{ Total admittance of CB} = \sqrt{\{(\cdot 088)^2 + (\cdot 084)^2\}}$$

$$\text{And } \cos\phi \text{ for complete circuit CB equals } \frac{\cdot 088}{\cdot 122};$$

$$\therefore \text{ equivalent resistance of CB} = Z \cos\phi = \frac{1}{\cdot 122} \times \frac{\cdot 088}{\cdot 122}$$
$$= 5{\cdot}9 \text{ ohms.}$$

$\left[\text{Note that this is equal to } \frac{G}{Y^2}, \text{ just as } G = \frac{R}{Z^2} \text{ (see Art. 20).}\right]$

$$\sin\phi \text{ for circuit CB} = \frac{\cdot 084}{\quad};$$

$$\therefore \text{ equivalent reactance of CB} = Z \sin\phi = \frac{1}{\cdot 122} \times \frac{\cdot 004}{\cdot 122}$$
$$= 5{\cdot}7 \text{ ohms.}$$

$\left[\text{Note that this is equal to } \frac{B}{Y^2} \text{ just as } B = \frac{X}{Z^2}.\right]$

$\therefore$ Total resistance of circuit ACB $= 2 + 5{\cdot}9 = 7{\cdot}9$ ohms.

Total reactance of circuit ACB $= 6 + 5{\cdot}7 = 11{\cdot}7$ ohms.

$$\text{Reactance of condenser FB} = -\frac{10^6}{2\pi \times 50 \times 300} = -10{\cdot}6 \text{ ohms;}$$

$\therefore$ total reactance of circuit AFB $= 12 - 10{\cdot}6 = 1{\cdot}4$ ohms.

The original circuit has now been reduced to the simpler circuit as shown in Fig 5.28 (*b*) which can be dealt with by either of the methods for parallel circuits.

Conductance of circuit ACB : $\frac{7{\cdot}9}{\quad}$.

Conductance of circuit AFB $\frac{\quad}{12^2 .}$ $\cdot 082$

$$\left.\begin{aligned}\text{Susceptance of circuit ACB} &= \frac{11{\cdot}7}{7{\cdot}9^2 + 11{\cdot}7^2} = \cdot 059\\ \text{Susceptance of circuit AFB} &= \frac{1{\cdot}4}{12^2 + 1{\cdot}4^2} = \cdot 010\end{aligned}\right\}\begin{array}{l}\text{Total}\\ \cdot 069;\end{array}$$

$\therefore$ Total admittance $= \sqrt{\{(\cdot 122)^2 +}$

$= \cdot 140;$

$\therefore$ Total current $= \cdot 140 \times 200 = 28{\cdot}0$ amperes.

Impedance of circuit ACB $= \sqrt{\{(7{\cdot}9)^2 + (11{\cdot}7)^2\}}$

$= 14{\cdot}1$ ohms;

$$\therefore \text{ Current in ACB} = \frac{200}{14{\cdot}1} = 14{\cdot}2 \text{ A.}$$

Impedance of AC (Fig. 5.28*a*)

$= \sqrt{\{2^2 + 6^2\}} = 6{\cdot}32$ ohms;

$\therefore$ P.D. across AC $= 14{\cdot}2 \times 6{\cdot}32 = 90$ volts.

$$\text{P.D. across CB} = 14{\cdot}2 \times \frac{1}{\cdot 122}$$
$$= 116 \text{ volts.}$$

Impedance of circuit AFB $= \sqrt{\{(12)^2 + (1{\cdot}4)^2\}}$

$= 12{\cdot}1$ ohms;

$$\therefore \text{ Current in AFB} = \frac{200}{12{\cdot}1} = 16{\cdot}5 \text{ amp.}$$

Impedance of AF (Fig. 5·28*a*) $= \sqrt{\{(12)^2 + (12)^2\}}$

$= 17{\cdot}0$ ohms;

$$\therefore \text{ P.D. across AF} = 16{\cdot}5 \times 17{\cdot}0 \quad 281 \text{ volts.}$$

$$\text{P.D. across FB} \quad \frac{I}{\omega C} = 16{\cdot}5 \times 10{\cdot}6 = 175 \text{ volts.}$$

22 Mutual Inductance

All the above rules hold good only on the assumption that an alternating current in one coil sets up no E.M.F. in any of the other coils of the circuit. In many cases, however, part of the flux due to the current in one coil will interlink with the turns of another coil; in such cases any change in the current will induce an E.M.F. in the second coil. It follows that a varying current in the second coil will produce a varying flux, and therefore an induced E.M.F., in the first. Hence this action is called *Mutual Induction.*

The amount of the mutual inductance (M) between two coils is measured by the volts induced in the second coil by a current in the first coil varying at the rate of 1 ampere per second (cf. Art. 7).

Alternatively, mutual inductance in henries may be defined as equal to (magnetic linkages produced in second coil per ampere in first coil) $\div 10^8$. This can be proved equal to (linkages produced in first coil per ampere in second coil) $\div 10^8$. In other words, the mutual inductance of the 2nd coil on the 1st = mutual inductance of 1st on 2nd.

If the permeability of the common magnetic circuit is not constant M will vary correspondingly, and will depend on the values of the currents in *both* coils. This, however, is only on the same lines as the fact that in such cases inductance (L) is likewise variable.

When circuits with mutual inductance are connected in series their combined impedance differs from that given by the rule of Art. 17. It will be greater or less than this according as the mutual inductances act with or against the self-inductances. A fuller treatment is given in Vol. II.

23. Power Measurement by Voltmeters and Ammeters

A wattmeter (see Chap. VII., Art. 13) is generally used to measure the power in an A.C. circuit. But if such an instrument is unavailable, or when the conditions make it inaccurate, the power can be obtained by readings of voltmeters and ammeters alone.

The " three-voltmeter " method due to Ayrton and Sumpner* is one such method, and the connexions are as shown in Fig. 5.29 (*a*).

* Proc. Roy. Soc., Vol. XLIX., 1891, p. 424.

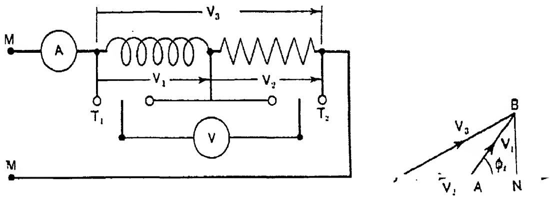
The load (L) in which the power is to be measured is connected in series with an ammeter and a non-inductive resistance (R), *e.g.* a number of glow lamps. By means of two 2-way switches (T_1, T_2) one voltmeter can be used to measure the three voltages, V_1 across the load, V_2 across the non-inductive resistance, and V_3 across the two in series.

It is more accurate to read the three voltages thus on the same instrument than to use three separate voltmeters.

Then the power absorbed by the load

$$= I.\frac{V_3^2 - V_1^2 - V_2^2}{2V_2};$$

where I = current shown by ammeter;



(a) (b)

CURRENT DIAGRAM. VECTOR DIAGRAM.

FIG. 5.29.—THREE-VOLTMETER METHOD OF POWER MEASUREMENT.

and the power supplied to the whole circuit

$$= I\frac{V_3^2 - V_1^2 + V_2^2}{}.$$

The difference between these two expressions = $V_2 . I_2$, which is the power used in R, since this is non-inductive.

Proof.—In the vector diagram [Fig. 5.29 (*b*)] let OA represent V_2, in phase with the current; and AB represent V_1, leading the current by the angle ϕ_1. Then OB represents V_3 (cf. Art. 17).

Draw BN perpendicular to the current vector.

$$\text{Then power in load} = V_1 I \cos\phi_1 = AN \,.\, I$$

But $$OB^2 = ON^2 + NB^2 = (OA + AN)^2 + NB^2$$

$$= OA^2 + 2OA \,.\, AN + AN^2 + NB^2$$

and $$AN^2 + NB^2 = AB^2;$$

$$\therefore\ 2OA \,.\, AN = OB^2 - OA^2 - AB^2;$$

$$\therefore \text{ power in load} = I \, . \, \frac{OB^2 - OA^2 - AB^2}{2OA} = I \, . \, \frac{V_3^2 - V_2^2 - V_1^2}{2V_2}$$

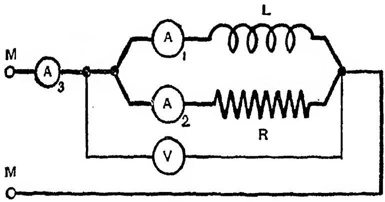
Q.E.D.

The total power $= V_3 I \cos \angle \text{BON} = I \, . \, ON$

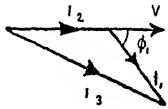
$$= I \, . \, (OA + AN) = I\left(V_2 + \frac{(V_3^2 - V_2^2 - V_1^2)}{2V_2}\right) = I\frac{V_3^2 + V_2^2 - V_1^2}{2V_2}.$$

One drawback of this method is that, since the expression for power contains the differences of squares of voltages, small errors in the voltmeter readings produce much larger errors in the value of the power. The greatest accuracy is obtained when the series resistance is chosen so that the " drop " across it is equal to that across the load, but even in this case the percentage error may easily be five or more times as great as the percentage error of the voltmeters.*

Another disadvantage is that a supply at higher than the normal voltage is required, and for accuracy (as stated in previous para-



(*a*) CIRCUIT DIAGRAM.



(*b*). VECTOR DIAGRAM.

Fig. 5.30.—THREE-AMMETER METHOD OF POWER MEASUREMENT.

graph) the excess must be considerable; 50 per cent. at least, and higher for high power factors.

This latter disadvantage is overcome in the " three ammeter method " [see Fig. 5.30 (*a*)].

In this the non-inductive resistance (R) is connected in parallel with the load, and the total current supplied is measured as well as the currents in both branches.

Then the power absorbed by the load (L) $= V \, . \, \dfrac{I_3^2 - I_1^2 - I_2^2}{2I_2}$

and total power supplied $= V \, . \, \dfrac{I_3^2 - I_1^2 + I_2^2}{2I_2}$,

where I_1 = current in load,

* See further " Small A.C. Power Measurements " (J. T. Urwin), *Electrician*, vol. 70, p. 843.

I_2 = current in non-inductive resistance,

and I_3 = total current.

The vector diagram [Fig. 5.30 (*b*)] shows that the relation between the three currents in this case is similar to that between the three P.D.s in the 3-voltmeter method. The truth of the above expressions for power can therefore be proved in the same way.

The disadvantage of the 3-ammeter method compared with the 3-voltmeter one is that it requires more elaborate switching arrangements to take all three readings on the same ammeter.

Both methods are independent of frequency; and since R.M.S. values depend on the squares of the harmonic components (see Vol. II.), they are accurate for all wave-forms.

QUESTIONS ON CHAPTER V.

1. Find the form factor of each of the following A.C. waves, the ordinates being given at intervals of $\frac{1}{24}$ of a period (*i.e.* 15°):—

(*a*)	0,	2,	6,	7,	8,	8,	8,	7,	6,	4,	2,	0, etc.
(*b*)			1,	3,	5,	7,	8,	7,	8,	7,	5,	3, etc.
(*c*)	1,	2,	4,	5,	6,	6,	5,	4,	3,	2,	1,	− 1, etc.
(*d*)	2,	5,	8,	9,	7,	5,	3,	2,	1,	1,	0,	− 2, etc.

2. Plot a sinusoidal wave of amplitude 25, and determine its virtual value by squaring ordinates.

Plot a second wave whose ordinates at intervals of $\frac{1}{24}$ of a period are:— 0, 4, 9, 15, 20, 24, 25, 24, 20, 15, etc.

Find its virtual value, its mean value, and its form-factor.

3. Plot the following alternating current wave:—

Phase angle	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°	
Amperes ..	0	1·8	4·2	7·0	9·7	11·2	12·5	12·6	11·5	7·8	4·7	2·1	0	etc.

Determine its R.M.S. value, and its form and peak factors.

What are the values of these factors for a sinusoidal wave, and what would be the maximum value of such a wave having the same R.M.S. value as the one plotted?

4. Define the terms "R.M.S. value of an alternating current" and "Phase difference between two alternating currents." Prove that with a sine wave shape the R.M.S. value is 0·707 of the crest or maximum value of the current. [C. & G., II.

5. An ammeter in a circuit attached to 100-volt mains reads 50, and the power-factor is 0·5. Assume sine curves and plot the volts and amperes to scale showing their proper phase relation. [C. & G., II.

6. Explain exactly what you mean by the inductance of a coil with an iron core (*a*) when the permeability of the iron is constant, (*b*) when the iron is saturated and the permeability varies for each new value of the flux density. [C. & G., II.

7. A non-inductive coil of 15 ohms resistance is connected in series with a coil of 10 ohms reactance and negligible resistance to a 200-volt supply.

Calculate the current, the P.D. across each coil, and the power-factor of the circuit.

The frequency is then doubled but the total P.D. is kept at 200 volts. Recalculate the above quantities.

8. A certain choking coil of negligible resistance takes a current of 8 A. at 100 volts, and 50 ~. A certain non-inductive resistance under the same conditions carries 10 A. What will the two take when put in series on 150 volts at 40 ~?

9. Find the current and power-factor for a circuit having 10 ohms resistance, and ·01 henry inductance, when supplied from a 200-volt circuit at 50 ~.

10. What must be the inductance of a circuit of 400 ohms resistance in order that the current may lag 30° in phase behind the E.M.F. when the frequency is 100 alternations per sec?

11. On a single-phase alternator the voltmeter reads 2120, the ammeter reads 285 A., and the wattmeter reads 495 kilowatts. Calculate (*a*) the power-factor, (*b*) the resistance of the circuit, (*c*) the reactance of the circuit.

12. Calculate the reactance and power-factor in a circuit of 500 ohms resistance supplied with alternating current at an E.M.F. of 240 volts, in each of the following cases:—

(*a*) current = ·48 A.
(*b*) current = ·4 A.
(*c*) current = ·36 A.

13. The air-gap under the pole of a series motor is 0·06 inch in length, and the effective area is 80 sq. in. There are 5 turns of cable round the pole, having a resistance of 0·005 ohm. Assuming that the reluctance of the iron parts of the magnetic circuit is one-fifth of the reluctance af the air-gap, calculate the voltage drop in a series coil when 150 amperes R.M.S. at 25 cycles per second are passing through it. [C. & G., II.

14. Draw on a time base one complete period of a sinusoidal alternating voltage, maximum (or peak) value 320 volts, and a sinusoidal current lagging $\frac{1}{8}$ of a period, peak value 60 amperes. From these draw the watt curve and find the mean power. What are the R.M.S. values of the above voltage and current? Calculate the apparent watts, and the ratio of the mean watts to the apparent.

15. Plot the following A.C. voltage and current:—

Phase angle	0	20	40	60	80	90	100	120	140	160	180	200	220	
v (volts)	0	92	244	388	470	502	520	464	264	112	0	−92	−244	etc.
i (centiamp)	−220	−41	120	369	652	780	875	980	843	544	220	41	−120	etc.

Draw also the watt curve. Determine the R.M.S. volts and amperes, the mean power, power-factor, and angle of lag.

16. The following are instantaneous values of a current and of the P.D. required to drive it through a certain circuit:—

Phase angle	0°	15°	30°	45°	60°	75°	90°	105°	120°	135°	150°	165°	180°
Amp. ..	−6·3	−3·9	−1·8	0	1·8	3·9	6·3	8·7	10·5	11·1	10·5	8·7	6·3
Volts ..	0	7	50	141	260	361	400	361	260	141	50	7	0

Find the R.M.S. values of current and P.D., and the peak-factor of each wave.

Find the mean power supplied to the circuit.

Compare the cosine of the angle of lag (ψ) of the current with the power-factor.

17 Explain the action of a "choking coil" in cutting down the current supplied from an alternating circuit. What advantage does it possess over a series resistance for effecting the same purpose? Why is a series resistance necessary for this purpose with direct currents?

18. A choking coil is required to cut down an A.C. supply at 230 volts to 150 volts. For what voltage should it be designed?

If the frequency is reduced to half that for which the coil was designed, what will be the voltage on the coil and the voltage on the non-inductive circuit supplied through it, if the total P.D. is unchanged?

19. Calculate the reactance and the impedance of a choking coil of 3 ohms resistance, to be used in series with a set of 115-volt lamps taking 6 amperes, to enable them to be supplied from A.C. mains at 220 volts.

Calculate the power-factor of the choking coil and of the whole circuit, and compare the power wasted in the coil with that wasted in a non-inductive resistance serving the same purpose.

20. A choking coil is required to give a supply of 14 A. to a non-inductive circuit of 10 ohms resistance, from a supply at 200 volts 50 cycles per sec. The effective resistance of the choking coil is 1·2 ohms; find its reactance, inductance, impedance, and power-factor, and the power-factor of the whole circuit.

21. A choking coil is required to enable 20 110-volt, 35-watt lamps to be supplied at their normal voltage from a 210-volt circuit. Calculate the voltage across the coil and its reactance. If six lamps are switched off, to what value does the voltage on the rest rise? Neglect resistance of choking coil, and change of resistance of lamps.

22. Calculate the reactance and impedance of a choking coil of 0·7 ohm resistance, for connexion across 110-volt supply mains in series with a non-inductive resistance which is to take 12 amperes at 42 volts.

If the frequency is halved, find the current which flows, and the P.D. across the coil and the non-inductive resistance respectively.

23. What is the current taken up by a condenser of 3 microfarads capacitance connected to the terminals of a 200-volt alternator giving 100 $\sim$?

24. A concentric cable 5 miles long is found to take 2 amperes when connected to an alternator giving a pure sine wave of 6 600 volts, frequency 50. Calculate the capacitance of the cable per mile.

25. It is desired to connect a single 50-volt 30-watt lamp to 110-volt 50-cycle mains by means of a condenser. Calculate the capacity of the condenser required and the power-factor of the load when the lamp is lighted. [C. & G., II.

26. Calculate the capacitance current of a concentric cable 8 miles long, 0·27 μF capacitance per mile, when supplied at 6 600 volts 50 $\sim$.

If a non-inductive load of 40 kilowatts comes onto the cable, what is the new value of the cable current?

27. Calculate the capacitance current taken by a concentric cable 11 miles long, 0·31 μF per mile, when supplied at 6 600 volts 50 $\sim$.

A non-inductive load of 100 kilowatts comes on to the cable. Find the total current now taken.

What difference would it make if the load were inductive?

28. A P.D. of 2 000 volts, 50 cycles per sec., is applied to test a cable of 14 μF. capacity through a resistance of 110 ohms. Find the current which will be taken from the supply, and the phase difference between the current and the impressed P.D. [C. & G., II.

29. Prove the relation between the capacity, resistance, and inductance when resonance is established in an oscillating circuit. A circuit has a capacity of 0·003 microfarad and an inductance of 0·011 millihenry. Calculate the frequency at which resonance will take place. [C. & G., II.

30. A resistance of 0·1 ohm, an inductance of 1 henry, and a capacity of 10·2 microfarads are connected in series and supplied with alternating current at 10 volts, 50 cycles. Find the voltage across the inductance and the capacity. [C. & G., II.

31. A coil of 80 millihenries inductance and 10 ohms resistance is connected in series with a condenser of 30 microfarads capacitance. A constant P.D. of 50 volts is applied to the terminals, and the frequency varied from 50 cycles/sec. to 200 cycles/sec. Plot the current against frequency.

What frequency gives resonance? Calculate for this frequency the current, and the P.D.s across the coil and condenser.

32. What is meant by resonance in an electrical circuit? A condenser of 1·5 microfarads capacity and a variable choking coil of 15 ohms resistance are connected in series to a 50-cycle 100-volt supply, the wave shape of which has a strong third harmonic. What value of the inductance will give resonance (*a*) with the third harmonic, (*b*) with the fundamental frequency?

[C. & G., II.

33. If coil A has 40 ohms resistance and 50 ohms impedance, and coil B has 50 ohms resistance, 130 ohms impedance, find the impedance of the two coils in series. What P.D. would be required to send 3 amperes through this circuit, and what P.D. will be produced across each coil? What is the power-factor of each coil, and of the two in series?

34. A circuit consists of a coil of 250 ohms impedance and 200 ohms resistance in series with a condenser of 30 μF capacitance. If a P.D. of 250 volts 50 $\sim$ is applied to the terminals, find the current, and the P.D.s across each part of the circuit.

If another condenser of 20 μF capacitance is connected in series, find the new values of the current and of the separate P.D.s.

35. A coil A has 400 ohms resistance, 400 ohms impedance.

" B " 200 " " , 250 " "

" C " 300 " " , 500 " "

Calculate the reactance in each case.

Find the impedances of A and C in series and of B and C in series.

What voltage would be required for ½ ampere through each of these circuits?

36. Define admittance, conductance, and susceptance. A coil of 10 ohms impedance, 6 ohms resistance, is in parallel with a non-inductive coil of 8 ohms resistance.

If 200 volts P.D. is applied, find the current in each and the total current.

What is the admittance of each, and of the two in parallel?

37. What is the power-factor of the above circuit? Find capacitance required in parallel with the two coils to bring the p.f. to unity if the frequency is 50 $\sim$. What is the current when this is done?

38. A current from an alternating current supply at 200 volts and 50 cycles flows through a number of lamps taking 20 amperes and through an inductive coil having a resistance of 5 ohms and an inductance of 0·05 henry. Find the total current passing to the two circuits in parallel, and calculate the power-factor of the combined circuits. [C. & G., II.

39. Two coils, one having a resistance of 2 ohms and a self-induction of 0·015 henry, and the other having a resistance of 1 ohm and a self-induction of 0·08 henry, are arranged in parallel on a 100-volt 50 frequency circuit. Find the magnitudes and phases of the currents flowing in each circuit, and of the resultant current flowing through the whole system. [C. & G., II.

40. A capacitance of 4 μF is in parallel with a resistance of 500 ohms which takes a current of power-factor = 0·866 (so that $\phi = 30°$).

Find the capacitance current, the current in the inductive resistance, and total current when 1 000 volts, 50 cycles per sec., is applied to the terminals.

41. A circuit of 200 ohms resistance and 1 henry inductance is connected across the terminals of a 10 000-volt supply, in parallel with a condenser of 6 microfarads capacitance. Find the total current taken if the frequency is 50

42. A coil of 36 ohms impedance and 11 ohms resistance is connected in parallel with a non-inductive rheostat whose conductance is varied from zero to 0·04 by four equal steps. If a P.D. of 200 volts is applied to the terminals, calculate the total current in each case; and plot the current against (*a*) the conductance of the rheostat; (*b*) the resistance of the rheostat.

43. A condenser of 8 microfarads capacitance is connected in parallel with a coil of 50 ohms resistance and 0·20 henry inductance. Calculate the separate currents and the total current when a P.D. of 200 volts at 50 cycles per sec. is applied to the terminals.

Recalculate the currents for a frequency of 100 cycles per sec.

44. Three circuits A, B, C are connected in parallel and a P.D. of 200 volts is applied to the terminals. A is non-inductive and of 12 ohms resistance; B is of 5 ohms resistance and 10 ohms reactance; C consists of a non-inductive resistance of 6 ohms in series with a condenser which causes the current to lead the P.D. by 60°. Find the current in each and the total current.

45. A resistance of 12 ohms and an inductance of 0·18 henry are in series on a 500-volt supply at 40 frequency. Find the current and power-factor. What values of resistance and inductance when placed in parallel will give the same current as the above at the same power-factor? [El. Tech.

46. An alternate current circuit includes two sections AB and BC in series. The section AB consists of two branches in parallel. The first of these is formed of a non-inductive resistance of 60 ohms in series with a condenser of 50 microfarads, while the second consists of a resistance of 60 ohms having an inductance of 250 millihenries. The section BC consists of a resistance of 100 ohms having an inductance of 300 millihenries. The frequency of the current is 50 cycles per second. The voltage across the section AB is 500. What is the voltage across the section BC? [El. Tech.

47. An air-cored choking coil is subjected to an alternating voltage of 100. The current taken is 0·1 ampere, and the power-factor is 0·2, when the frequency of the current is 50. Find the capacity of a condenser which, if placed in parallel with the coil, will cause the main current to be a minimum. What will be the impedance of this parallel combination (*a*) for currents of frequency 50, and (*b*) for currents of frequency 40? [El. Tech.

48. Give a diagram of the 3-voltmeter method of measuring A.C. power, and prove its correctness.

For what kind of tests is it most useful, and what is its main disadvantage?

If the P.D. across the non-inductive resistance is 67 volts, that across the inductive resistance 83 volts, and the total P.D. 128 volts, with 3·7 amperes flowing in the circuit; calculate the impedance of the inductive resistance and of the whole circuit, and the power used in each.

49. What is meant by the virtual or R.M.S. value of an alternating current? Explain a method by which the power expended in a circuit not of unity p.f. may be found by using three ammeters, a voltmeter and a non-inductive resistance (and show that its accuracy is not affected by the wave form of

the alternating current). What is the chief drawback to this method of measuring power? [C. & G., II.

(N.B.—The portion of the question in brackets requires Vol. II. for its elucidation.)

50. Prove the formula for the power used in a circuit when measured by the "three-ammeter method." State the advantages and disadvantages of this method.

If the readings on the ammeters are 1·35 A. (non-inductive coil), 1·92 A. (inductive coil), and 2·84 A. (total), when a P.D. of 215 volts is applied; calculate the values of:—

(*a*) the non-inductive resistance;

(*b*) the impedances of the inductive resistance and of the whole circuit;

(*c*) the power supplied to the inductive resistance.

51. Two circuits have a mutual inductance of 2 henrys. Assuming that all the lines of force induced pass through both circuits, what must be the rate of change of current in one in order that the induced E.M.F. in the other may be 10 000 volts? State the units of current and time which you employ. [C. & G., II.

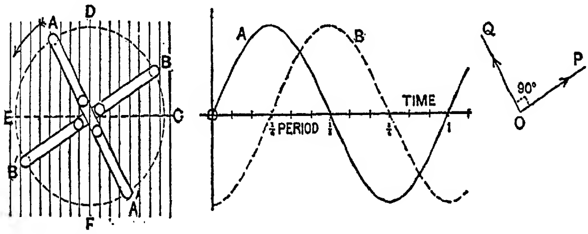
52. Point out fully the reasons that tests by such a method as the three-voltmeter measurement of alternate current power are in general unsatisfactory. Show how to deduce from the formula the percentage error in the measurement due to a given percentage error in one of the observations.

CHAPTER VI

POLYPHASE SYSTEMS

1. Polyphase Supply

In the majority of A.C. supply systems there are two or more sets of circuits, which may be either completely independent or connected at one or more points. The number of sets of circuits is nearly always 2 or 3, but in certain special cases 4, 6, or 12 sets of circuits are used (see Vol. II.). The P.D.s across each set of circuits are of the same frequency and magnitude, but they differ in phase by constant amounts. These systems are called respectively two-phase, three-phase, four-phase, etc., and are referred to collectively as polyphase systems. They have various advantages both in generation and in use, and these are outweighed by their disadvantages in the case of traction only.



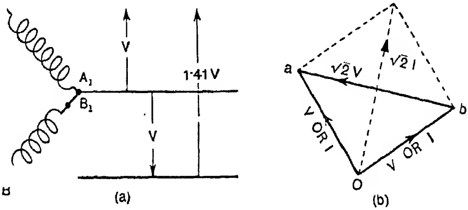
(*a*) Rotating Coils. (*b*) E.M.F. Graphs. (*c*) Vector Diagram.

Fig. 6.01.—Two-phase Supply.

2. Two-Phase Supply

If, instead of a single coil, as in Chap. V., Art. 2, two similar coils AA′, BB′, fixed at right angles to each other, are rotated with constant velocity in a uniform field [see Fig. 6.01 (*a*)], both the E.M.F.s produced in them will be sinusoidal and of the same frequency, and of equal maximum (and equal effective) values. But when A has reached D, the position of maximum E.M.F., B will be at C, and the E.M.F. in BB′ is zero. At the moment when B reaches D (maximum E.M.F.), A will have arrived at E, giving zero E.M.F. in AA′. When B comes to E (zero E.M.F.) A will be at F, giving again maximum E.M.F. in AA′, but in the reversed (negative) direction, etc.

To sum up, BB′ reaches any particular position a quarter of a revolution later than AA′, and so the E.M.F. in BB′ passes through the same cycle as that of AA′, but a quarter of a period later, as shown in Fig. 6.01 (*b*). The vector diagram for the two E.M.F.s,



(*a*) CIRCUIT DIAGRAM. (*b*) VECTOR DIAGRAM.

Fig. 6.02.—TWO-PHASE THREE-WIRE SYSTEM.

therefore, consists of two lines [OP, OQ in Fig. 6.01 (*c*)], of equal length, and at right angles to each other.

If the end (A′) of one coil is joined to the end (B′) of the other, then the E.M.F. across the terminals AB (Fig. 6.02) is the vector difference of the separate E.M.F.s, and so is ($\sqrt{2} =$) 1·41 times the magnitude of these, and is 45° out of phase with one of them.

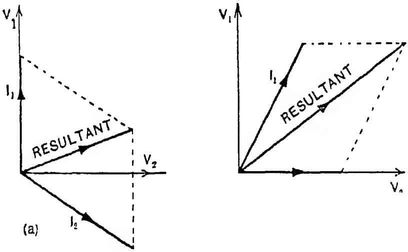


Fig. 6.03.—UNBALANCED TWO-PHASE SYSTEM.

If the currents in the two phases are equal and lag behind (or lead) their respective voltages by equal angles, the current in the common wire (*i.e.* that going to A′B′) will likewise be 1·41 times the current in either of the separate phases, being their vector sum (see Fig. 6.02).

Example 1. *In a two-phase interconnected supply the load on the leading phase is non-inductive, and amounts to 7 amperes, while that on the other phase takes a current of 9 amperes, lagging by 30°.*

(*a*) *Find the current in the common wire in magnitude and phase.*

(*b*) *Find this current if the loads are reversed.*

(*a*) The current in the common wire is the vector sum of the currents in the separate phases as shown in Fig. 6.03 (*a*).

To find its value resolve I_2 along V_2 and perpendicular to this:—

$$\text{Component along } V_2 = 9 \times \cos 30° = 7{\cdot}79 \text{ A.}$$

$$\text{Component perpendicular to } V_2 = 9 \times \sin 30° = 4{\cdot}50 \text{ A.};$$

$$\therefore \text{ resultant component perpendicular to } V_2 = 7 - 4{\cdot}50 = 2{\cdot}50 \text{ A.};$$

$$\therefore \text{ total resultant} = \sqrt{\{(2{\cdot}50)^2 + (7{\cdot}79)^2\}} = 8{\cdot}19 \text{ A.},$$

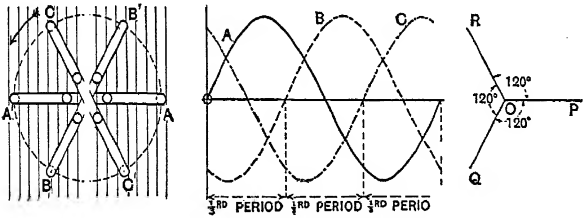
and this lags behind V_1 by angle $\tan^{-1}\frac{7{\cdot}79}{2{\cdot}50} = \tan^{-1} 3{\cdot}12 = 72°$.

(*b*) In this case resolve I_1 [Fig. 6.03 (*b*)] along and perpendicular to V_1, the components being as before, 7·79A and 4·50A respectively;

$$\therefore \text{ resultant component perpendicular to } V_1 = 4{\cdot}50 + 7 = 11{\cdot}50 \text{ A.};$$

$$\therefore \text{ total resultant} = \sqrt{\{(11{\cdot}50)^2 + (7{\cdot}79)^2\}} = 13{\cdot}9 \text{ A.},$$

and this lags behind V_1 by an angle $\tan^{-1} \frac{11{\cdot}50}{7{\cdot}79} = \tan^{-1} 1{\cdot}48 = 56°$.



(*a*) ROTATING COILS. (*b*) E.M.F.S PLOTTED AGAINST TIME. (*c*) VECTOR DIAGRAM.

Fig. 6.04.—THREE-PHASE SUPPLY.

3. Three-Phase Supply

If three similar coils, AA′, BB′, CC′ [Fig. 6.04 (*a*)], fixed at equal angles of 120° between every pair, are rotated at constant speed in a uniform field the E.M.F.s generated will all be sinusoidal, and of the same frequency and of equal maximum (and equal effective) values. But the E.M.F. of BB′ will lag behind that of AA′ by ⅓ of a period, and that of CC′ will lag by a further ⅓ of a period (⅔ in all), or, to put it in another way, the E.M.F. of CC′ will lead that of AA′ by ⅓ of a period. The vector diagram, therefore, consists of three lines [OP, OQ, OR, Fig. 6.04 (*c*)] of equal lengths at 120° to one another.

It should be noted that if the E.M.F. of B′B (instead of BB′) is considered this leads the E.M.F. of AA′ by $\frac{1}{6}$-period (= 60°), and the vector diagram becomes OP, OQ′, OR; where OQ′ bisects the angle POR, and is equal and opposite to OQ. The corresponding (E.M.F. ∼ time) graph is obtained by reversing the curve for B in Fig. 6.04 (*b*) while leaving those for A and C as they are.

If the ends of two of the coils are joined together, the magnitude of the resultant E.M.F. across the remaining two ends differs according to which ends are joined. This is unlike the case of two-phase supply, in which an alteration in the joined end of one coil alters the phase but not the magnitude of the resultant E.M.F. The differing magnitudes of the resultant E.M.F.s in the three-phase case gives rise to two different methods of connexion, called respectively **delta (△) or mesh, and Y or star.**

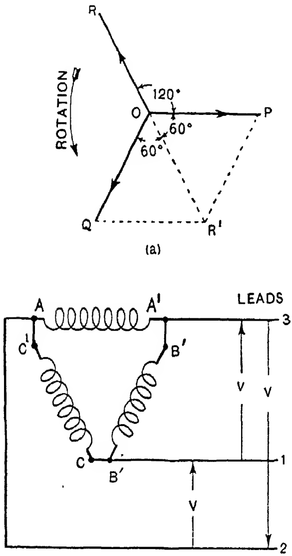


Fig. 6.05.—DELTA-CONNECTED THREE-PHASE SYSTEM.

4. Delta Connexion

If the end A′ [Fig. 6.04 (*a*)] of one coil is joined to the beginning B of another coil, the total E.M.F. across the other two terminals AB′ is the vector sum of the separate E.M.F.s. Thus, if OP [Fig. 6.05 (*a*)] is the vector representing the E.M.F. of AA′, and OQ the E.M.F. of BB′, the E.M.F. of AB′ is represented by the diagonal OR′ of the parallelogram OPR′Q.

Since OP = OQ, OR′ bisects the angle POQ, and so is opposite in direction to OR, which represents the E.M.F. of CC′. Moreover POR′ is an equiangular triangle, therefore OR′ = OP = OR.

Therefore the E.M.F. of AB′ is equal in magnitude and opposite in phase to the E.M.F. of CC′. Consequently, if C is joined to B′ and C′ to A (*not* C to A and C′ to B′), the resultant E.M.F. round the mesh AA′BB′CC′, formed by the three coils, is zero. The P.D.s between each pair of leads have the same value [Fig. 6.05 (*b*)].

If the E.M.F.s are not sinusoidal the resultant E.M.F. round the mesh may not be zero, and then a current is circulated round the mesh. The circulating E.M.F. always has a frequency three times (or a multiple of three times) that of the main E.M.F.s (see Ex. 3).

If the currents in the coils are sinusoidal the current in any lead is the vector *difference* of the currents in the two coils to which the lead is connected: *e.g.* the current flowing in lead No. 1 from left to right in Fig. 6.05 (*b*) is equal to the current flowing from B to B′ minus (vectorially) the current flowing from C to C′. Thus, if OS (Fig. 6.06) represents the current in BB′ and OT that in CC′, then TS represents that in lead No. 1. And similarly for the currents in the other two leads.

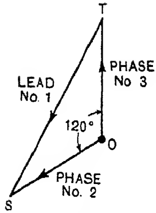


Fig. 6.06. CURRENTS IN Δ-CONNEXION.

If these currents lag behind their respective voltages by equal angles the angle TOS is 120°; and if the phase currents are each equal to I amperes, the lead current is equal to $2I \sin \left(\frac{120}{2}\right)^\circ = \sqrt{3}I = 1{\cdot}73I$ amperes, and this is 30° ahead of OS in phase.

Example 2. (*a*) *If the currents in the phases of a delta-connected system are* (i) 8 *amp. in phase with voltage;* (ii) 8 *amp. lagging* 30°; (iii) 10 *amp. lagging* 30°; *find the current in each lead.*

(*b*) *If the 2nd and 3rd loads are interchanged, what are the lead currents?*

(*a*) Let OP, OQ, OR represent the E.M.F.s of the three phases in Fig. 6.05 (*b*). Then in Fig. 6.07 (*a*), OD, OE, OF represent the currents in the three phases.

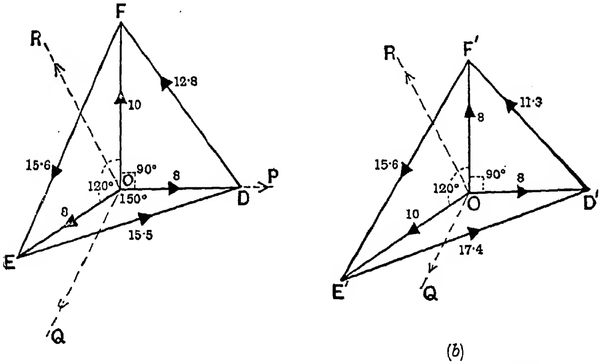


Fig. 6.07.—CURRENTS IN UNBALANCED Δ-CONNECTED SYSTEM.

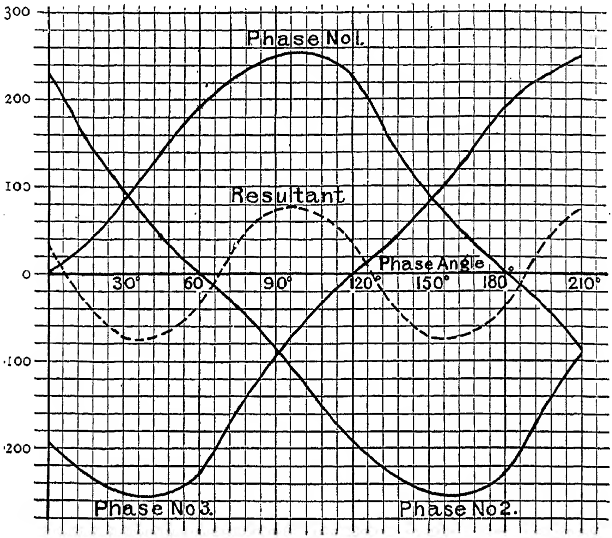


Fig. 6.08.—RESULTANT E.M.F. ROUND MESH.

The current in lead No. 1 is represented by FE, that in No. 2 by DF, and in No. 3 by ED. This last is equal to $2 \times 8 \times \sin (\frac{150}{2})^\circ$

$$= 2 \times 8 \times \cdot 966 = 15 \cdot 5 \text{ amperes;}$$

leading OD by 15°.

Current in lead No. 2 $= \sqrt{\{8^2 + 10^2\}} = \sqrt{164} = 12 \cdot 8$ amperes;

leading OF by $\tan^{-1} 10 = 39^\circ$, *i.e.* leading OR by 9°.

To find FE resolve OF along and perpendicular to OE;

$\therefore$ component along OE $= -10 \cos 60^\circ = -5$ A.

$\therefore$ total component of FE along OE $= 8 + 5 = 13$ A.

total component of FE $\perp$ to OE $= 10 \sin 60^\circ = 8 \cdot 66$ A;

$\therefore$ current in lead No. 1 = FE $= \sqrt{\{13 \cdot 0^2 + 8 \cdot 66^2\}} = 15 \cdot 6$ A.

leading OE by $\tan^{-1} \frac{8 \cdot 66}{13 \cdot 0} = \tan^{-1} 0 \cdot 666 = 34^\circ$, *i.e.* leading OQ by 4°.

(*b*) Current in lead No. 2 = D′F′ $= \sqrt{(8^2 + 8^2)} = 11 \cdot 3$ A.

Components of F′E′ along OE′ $= 10 + 8 \cos 60^\circ = 14 \cdot 0$ A.

Components of F′E′ perpendicular to OE′ $= 8 \sin 60^\circ = 6 \cdot 9$ A.;

$\therefore$ current in lead No. 1 = F′E′ $= \sqrt{\{14 \cdot 0^2 + 6 \cdot 9^2\}} = 15 \cdot 6$ A.,

leading OE′ by $\tan^{-1} \frac{6 \cdot 9}{14 \cdot 0} = \tan^{-1} \cdot 493 = 26^\circ$, *i.e.* lagging behind OQ by 4°.

[Note that the magnitude is the same as in (*a*), but the phase different.]

Components of E′D′ along OD′ $= 8 + 10 \cos 30^\circ = 16 \cdot 66$ A.

Components of E′D′ perpendicular to OD′ $= 10 \sin 30^\circ = 5$ A.;

$\therefore$ current in lead No. 3 $= E'D' = \sqrt{\{(16{\cdot}66)^2 + 5^2\}} = 17{\cdot}4$ A.,

leading OD′ by $\tan^{-1}\frac{}{16{\cdot}66} = \tan^{-1}{\cdot}300 = 17°$.

Example 3. *The E.M.F. in each phase of a Δ-connected system, taken at intervals of $\frac{1}{24}$ of a period, has the values:*—0, 35, 84, 140, 194, 225, 251, 253, 230, 156, 94, 42, 0, − 35, − 84, *etc. Find the resultant E.M.F. acting round the mesh.*

The E.M.F.s in the three phases are $\frac{8}{24}$ of a period out of phase with one another. The resultant E.M.F. at any instant is the algebraic sum of the separate E.M.F.s at that instant. Hence:—

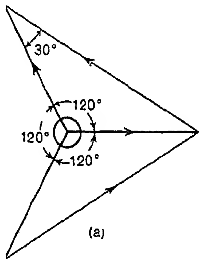
Phase No. 1	0	35	84	140	194	225	251	253	230	etc. volts.
Phase No. 2	230	156	94	42	0	−35	−84	−140	−194	etc. volts.
Phase No. 3	−194	−225	−251	−253	−230	−156	−94	−42	0	etc. volts.
Resultant	36	−34	−73	−71	−36	34	73	71	36	etc. volts.

The above gives a complete period of the resultant E.M.F. since the figures to be added repeat their former values, except that they occur in different phases. Thus the frequency of the resultant is three times that of the phase voltages.

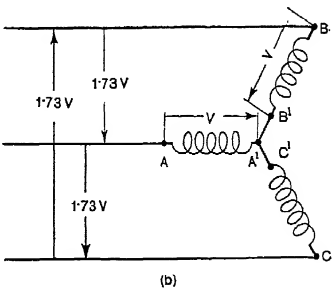
The E.M.F.s are plotted in Fig. 6.08. The ordinate of the resultant E.M.F. at any point is the sum of the ordinates of the phase voltages, due regard being given to whether these are positive or negative.

5. Y-Connexion

If the end A′ of one coil [Fig. 6.04 (*a*)] is joined to the *end* B′ of another coil, the total E.M.F. across the two terminals AB is the vector difference of the separate E.M.F.s (instead of the sum as in Δ-connexion, Art. 4). Thus if as before OP represents the E.M.F. of AA′ and OQ that of BB′ then QP [Fig. 6.09 (*a*)] represents that of AB. Similarly if the end C′ of the third coil is joined to the ends of the other two the E.M.F. of CA is given by PR, and that of BC by RQ. These three E.M.F.s are



(*a*) VECTOR DIAGRAM.



(*b*) CIRCUIT DIAGRAM.

Fig. 6.09.—Y-CONNECTED THREE-PHASE SYSTEM.

equal and are 120° out of phase with each other, *i.e.* they form a 3-phase system just as the original E.M.F.s do.

But it follows from Fig. 6.09 (*a*) (just as for the currents in Δ-connexion, Fig. 6.06) that the E.M.F. across any pair of terminals is ($\sqrt{3} =$) 1·73 times the E.M.F. of any one coil. Moreover the three terminal E.M.F.s are 30° out of phase with the three coil E.M.F.s. The common point A′B′C′ is called the **star point** (or neutral point).

The current in each line is the same as that in the coil to which it is connected. Since there is no other lead to the star point the vector sum of the three currents (reckoned positive when flowing towards the star point) must be zero. This condition is satisfied when the currents are equal and lag behind the corresponding voltages by equal angles, as the current vectors then form the sides of an equilateral triangle.

Example 4. *Find the current in the third phase of a Y-connected system when the currents in the other two phases are:—*

(*a*) 8 A. *in phase,* 8 A. *lagging* 40°.
(*b*) 8 A. *lagging* 40°, 8 A. *in phase.*
(*c*) 8 A. *lagging* 40°, 10 A. *lagging* 40°.
(*d*) 8 A. *lagging* 10°, 10 A. *lagging* 40°.

In Fig. 6.10 let OP, OQ, OR represent the voltages of the three phases; (AO the current in phase No. 1 [AA′ in Fig. 6.09 (*b*)] and OB that in phase No. 2 (BB′). Then OC is the vector sum of these two currents, and so the current in phase No. 3 is given by CO, or by OC′ where COC′ is a straight line and OC′ = CO.

(*a*) $\angle$ AOB = 160°; $\quad\therefore \angle$ OAC $\quad$ 20°

$$\therefore \text{ current in phase No. 3} \quad CO = 2 \times 8 \sin \frac{20°}{2} = 2{\cdot}8 \text{ A.}$$

$\angle$ AOC = 80°; $\quad$ AOC′ = 100°;

$\therefore$ OC′ lags 20° behind its E.M.F. (OR).

(*b*) $\angle$ AOB = 80°; $\quad\therefore \angle$ OAC = 100°;

$\therefore$ current in phase No. 3 = 2 × 8 sin 50° = 12·3 A.

$\angle$ AOC = 40°; $\quad\therefore \angle$ POC = 80°; $\quad\therefore \angle$ POC′ = 100°;

$\therefore$ OC′ lags 20° behind its E.M.F.

(N.B.—With two currents equal in magnitude the angle of lag of the third is always the mean of the angles of lag of the other two.)

(*c*) $\angle$ AOB = 120°; $\quad\therefore \angle$ OAC = 60°.

Component of OC along OA = 8 − 10 cos 60° = 8 − 5 = 3 A.

Component of OC perpendicular to OA = 10 sin 60° = 8·66 A.;

$\therefore$ magnitude of OC = $\sqrt{\{3^2 + 8{\cdot}66^2\}}$ = 9·2 A.

$$AOC = \tan^{-1}\frac{8{\cdot}66}{3} \quad \tan^{-1} 2{\cdot}89 = 71°$$

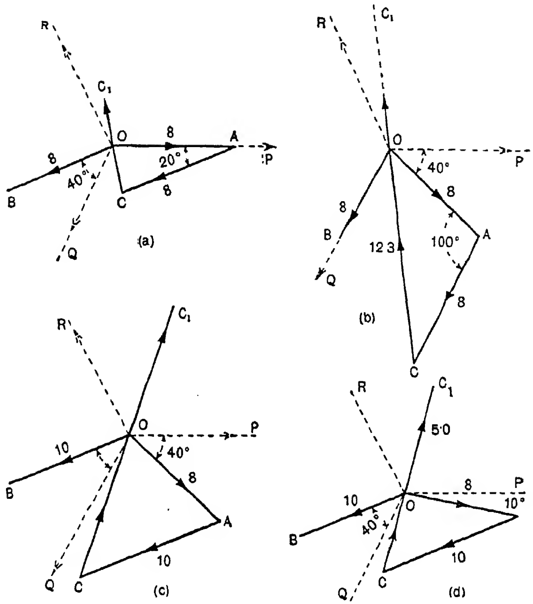


Fig. 6.10.—CURRENTS IN UNBALANCED Y-CONNECTED SYSTEM.

$\therefore \angle \text{POC} = 111°; \qquad \therefore \angle \text{POC}' = 69°;$

$\therefore$ current lags 51° behind its E.M.F.

(Note that this differs from the angle of lag of the other two.)

(*d*) $\angle \text{AOB} = 150°; \qquad \therefore \angle \text{OAC} = 30°.$

Component of OC along OA $= 8 - 10 \cos 30° = 8 - 8{\cdot}66 = -0{\cdot}66$ A.

Component of OC perpendicular to OA $= 10 \sin 30° = 5$ A.;

$\therefore$ magnitude of OC $= \sqrt{\{5^2 + (0{\cdot}66)^2\}} = 5{\cdot}0$ A.

$$\angle \text{AOC} = \tan^{-1}\frac{}{-0{\cdot}66} = \tan^{-1} - 7{\cdot}58° = 98°;$$

$$108°; \qquad \therefore \angle$$

$\therefore$ current lags 48° behind its E.M.F.

6. Power in Polyphase Circuits

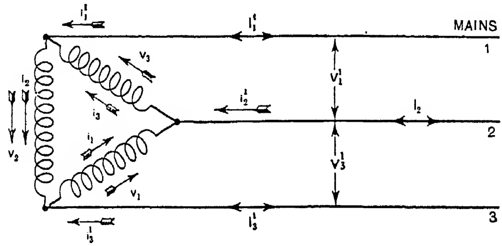
In a two-phase circuit the mean total power is $(V_1 I_1 \cos \phi_1 + V_2 I_2 \cos \phi_2)$. When, as usual, the two voltages are equal this simplifies to $V\,(I_1 \cos \phi_1 + I_2 \cos \phi_2)$. With balanced loads $I_1 = I_2$ and $\cos \phi_1 = \cos \phi_2$, therefore the power is $2VI \cos \phi$. There are

other methods of expressing the total power when the two phases are inter-connected, but these are of little utility.

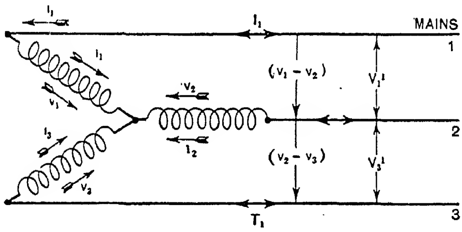
Similarly in a three-phase circuit the mean total power is

$V_1I_1 \cos \phi_1 + V_2I_2 \cos \phi_2 + V_3I_3 \cos \phi_3$, which becomes
$V (I_1 \cos \phi_1 + I_2 \cos \phi_2 + I_3 \cos \phi_3)$ if the voltages are equal, and
$3VI \cos \phi$ if the loads are balanced.

If the three phases are connected together either $\triangle$ or Y, so that there are only three mains, the total power for either balanced or



(*a*) Δ-CONNECTED.



(*b*) Y-CONNECTED.

Fig. 6.11.—POWER IN THREE-PHASE CIRCUITS.

unbalanced loads is equal to $(V_1'I_1' \cos \phi_1' + V_3'I_3' \cos \phi_3')$

where V_1' = potential of main No. 1 above main No. 2,
I_1' = current in main No. 1,
ϕ_1' = phase difference between V_1' and I_1',

and V_3' = potential of main No. 3 above main No. 2,
I_3' = current in main No. 3,
ϕ_3' = phase difference between V_3' and I_3' (see Fig. 6.11).

The proof of the above expression for the case of a $\triangle$-connected load, *i.e.* a system receiving power, is as follows:—Let i_1, i_2, i_3 be

the instantaneous values of the currents in the three phases of the load, reckoned positive when flowing anti-clockwise round the mesh; and v_1, v_2, v_3 the instantaneous values of the corresponding voltages [see Fig. 6.11 (*a*)]. And let i_1', i_2', i_3' be the instantaneous values of the currents in the mains, reckoned positive when flowing towards the load.

Then $\qquad i_1' = i_2 - i_3$, and $i_3' = i_1 - i_2$;

and $\qquad v_1 + v_2 + v_3 = 0$ at every instant;

$$\therefore\ v_2 = -(v_1 + v_3).$$

Thus, the instantaneous value of the total power

$$\begin{aligned} &= v_1 i_1 + v_2 i_2 + v_3 i_3 \\ &= v_1 i_1 - (v_1 + v_3)\, i_2 + v_3 i_3 \\ &= v_3 (i_3 - i_2) + v_1 (i_1 - i_2) \\ &= -\, v_3 i_1' + v_1 i_3'. \end{aligned}$$

But in this case $-v_3$ is the potential of main No. 1 above main No. 2, and $+v_1$ is the potential of main No. 3 above main No. 2.

Therefore $V_1'I_1' \cos \phi_1' + V_3'I_3' \cos \phi_3' =$ mean value of total power.

Note that this proof holds good whatever the wave forms of the voltages and currents.

The same proof holds good for a source of power, *e.g.* an A.C. generator. The only difference is that the currents in the mains must be reckoned positive when flowing away from the source. It gives the power delivered by the source to the external circuit and does not include internal losses.

A similar proof can be given for a Y-connected system. In this case the fact upon which the proof depends is that if i_1, i_2, i_3 are the instantaneous values of the currents in the three arms of the Y (and in the mains), reckoned positive when flowing inwards towards the star point, then their sum $(i_1 + i_2 + i_3)$ is zero at every instant see [Fig. 6.11 (*b*)].

7. Constancy of Power in Balanced Polyphase Circuits

In a monophase circuit the power varies between a maximum and a minimum, and (except in the special case of voltage and current being in phase) this minimum is negative. Consequently the maximum is more than double the average power.

In a polyphase circuit the negative portions of the power waves will occur at different times in the different phases, and so the total power usually will not reverse even with unbalanced loads. In any

case the variation of power will be reduced by the time differences between the phases.

If the load is balanced the total power becomes constant with sinusoidal waves of voltage and current.

It has been shown (Chap. V., Art. 9) that the power in a monophase circuit is given by:—

$$VI\,\{\cos\phi - \cos(2\omega t + \phi)\},$$

the graph of which is OABCDEFGH in Fig. 6.12.

In a two-phase circuit with balanced loads the power in the second phase is given by the same graph shifted $\frac{1}{4}$-period (90°), viz. KMNPQRSTU in Fig. 6.12.

Since the power wave goes through two cycles in a period of the voltage its maximum and minimum (greatest negative) values are 90° apart. Hence the maximum of the power in either phase

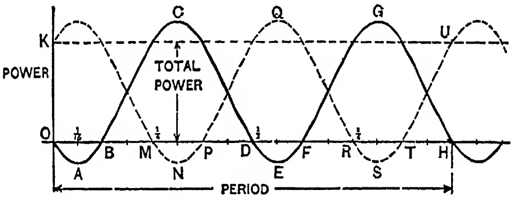


Fig. 6.12.—POWER IN BALANCED TWO-PHASE SYSTEM.

coincides with the minimum in the other phase, and the increase of power in one phase occurs during the same interval as the decrease in the other.

That the increase and decrease are always exactly at the same rate can be seen by noting that the graphs are sinusoidal (but with their middle lines above the time axis) and of the same amplitude.

This can be proved by writing the expression for power in the second phase, which is—

$$VI\left\{\cos\phi - \cos\left(2\omega t \pm \frac{\pi}{2} + \phi\right)\right\}$$

$$= VI\,\{\cos\phi + \cos(2\omega t + \phi)\}.$$

Adding the expressions for the power in the separate phases the total power is seen to be $2VI\cos\phi$ at every instant, the two variable terms cancelling each other out.

Similarly, in a balanced three-phase load the total power is constantly equal to $3VI \cos \phi$, the sum of the three variable terms being always zero. Whether Y or delta connexion is used, this equals $\sqrt{3}V_L I_L \cos \phi$, where V_L and I_L refer to line values.

8. Resistance and Reactance of a Three-Phase System

In a Y-connected system with similar phases the resistance per phase is evidently half the resistance measured between any pair of terminals. Since the line and phase currents are equal, the total resistance loss, denoting line amperes by I

$$= 3 . I_L^2 . \frac{R_t}{2} \text{ watts,}$$

R_t = resistance between terminals in ohms.

Since the power is given by $\sqrt{3}V_L I_L \cos \phi$, the quantity $\sqrt{3}I_L$ may be called the "*equivalent current.*" Thus:—

$$\text{resistance loss} = (\text{equivalent current})^2 \times \frac{\text{terminal resistance}}{2}$$

In a Δ-connected system there are between any pair of terminals two paths in parallel, one through a single phase of resistance R, and the other through two phases in series of total resistance 2R.

Whence $$R_t = \frac{R \times 2R}{R + 2R} = \tfrac{2}{3}R.$$

$$\text{The current per phase} = \frac{1}{\sqrt{3}} \times \text{line current} = \frac{I_L}{\sqrt{3}};$$

$$\text{total resistance loss} = 3 . \left(\frac{I_L}{\sqrt{3}}\right)^2 . R \text{ watts}$$

$$= I_L^2 R \text{ watts}$$

$$= I_L^2 \tfrac{3}{2} R_t \text{ watts}$$

$$= (\text{equivalent current})^2 \times \frac{\text{terminal resistance}}{2},$$

i.e. the same expression as above.

Thus it is unnecessary to know whether the system is Y- or Δ-connected in obtaining the resistance loss, and in both cases half the resistance between terminals may be taken for the purpose of this calculation.

Since the laws connecting the combination of reactances are the same as for resistances a similar method may be adopted in obtaining the combined reactance of balanced three-phase systems.

And, finally, the same method can be used for impedances, always assuming that the three phases are similar.

9. Capacitance of Three-Phase Cables

In a multicore cable the capacitances between each pair of cores and the capacitances between each core and the sheath of the cable have to be taken into account in calculating the capacitance current taken by the cable. The usual case is that of a three-phase three-core cable with cores of the same size and arranged symmetrically.

In this case the three capacitances between different pairs of cores will all have the same value, say C. Similarly, the three capacitances between a core and the sheath will all have the same value, say S. These two, C and S, will not be equal; S is usually in the neighbourhood of three times C.

If it is desired to find the values of C and S separately for a cable, at least two tests are necessary, and more may be employed as a check of accuracy. For instance, if the capacitance is measured between the three cores connected together and the sheath, this gives three capacitances of value S in parallel. Thus by dividing the result by three the value of S is obtained. The value of C cannot be obtained by direct measurement, but by measuring the capacitance with different connexions the value of C can be calculated from the known value of S.

There are many ways of making this second test,* but the following is the simplest to calculate. Connect two of the cores to the sheath and measure the capacitance between these and the third. This gives three capacitances in parallel, two of value C and one value S, *i.e.* a total capacitance of $(2C + S)$, see Fig. 6.13 (*a*), from which C can be found when S is known.

For example, if the test between the three cores and the sheath gives a capacitance of 0·528 μF per mile, the value of S is 0·176 μF per mile. If the second test gives 0·304 μF per mile, the value of C must then be 0·064 μF per mile.

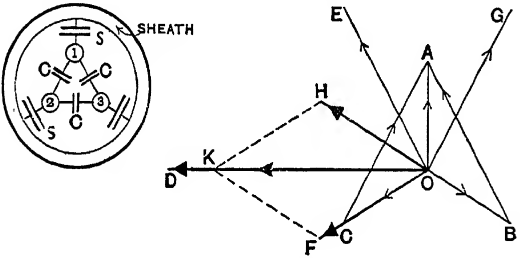
When the values of C and S are known the capacitance current taken by the cable in service can be calculated as follows. In the vector diagram [Fig. 6.13 (*b*)] OA, OB, and OC are three lines of equal lengths at 120° representing the P.D.s between the cores and the sheath, of R.M.S. value V'. The current in the capacitance S between core No. 1 and the sheath is represented by OD at right angles to OA, and of R.M.S. value ωS

* See "The Inter-relation of Capacity in Three-phase Three-core Cables," by A. B. Clark, *El. Rev.*, Vol. 70, p. 532; and *ibid.*, by W. T. Maccall, p. 996.

The P.D. of core No. 1 above core No. 2 is given by BA, or by OE equal and parallel to BA, and its R.M.S. value, V, is $\sqrt{3}$ times V′. The current flowing by core No. 1 to capacitance C between it and core No. 2 is, therefore, given by OF at right angles to OE, and its R.M.S. value is ωCV, or $\sqrt{3}\omega CV'$.

Similarly, the P.D. of core No. 1 above core No. 3 is given by CA or OG. The current flowing by core No. 1 into the capacitance C between it and core No. 3 is given by OH, at right angles to OG and of the same length as OF.

The angle EOG is 60°, therefore the angle FOH is 60°. Hence the vector sum, OK, of the two core-capacitance currents, OF, OH, is $\sqrt{3}$ times OF, and so its R.M.S. value is $\sqrt{3}\omega CV$, or $3\omega CV'$. Moreover since OK bisects the angle FOH, the angle AOK is $(60° + 30°)$, *i.e.* a right angle. Hence OK and OD are in phase,



(*a*) CAPACITANCE DIAGRAM. (*b*) VECTOR DIAGRAM.

Fig. 6.13.—CAPACITANCE CURRENT OF 3-CORE 3-PHASE CABLE.

and the total capacitance current in core No. 1 is the arithmetical sum of OK and OD. Thus its R.M.S. value is $\omega V' (S + 3C)$ or $\omega V (S + 3C)/\sqrt{3}$. Hence the "equivalent" capacitance current (see Art. 8) is $\omega V (S + 3C)$.

The value of the capacitance current can be obtained from a *single* measurement of the capacitance of the cable. For if the capacitance is measured between two cores, *e.g.* Nos. 1 and 2, there are three capacitance paths in parallel. These consist of (*a*) the capacitance C between No. 1 and No. 2; (*b*) two capacitances C in series, viz. those between Nos. 1 and 3 and between Nos. 3 and 2, giving a combined capacitance of $\frac{1}{2}C$; (*c*) two capacitances S in series, viz. those between No. 1 and the sheath and between this and No. 2, giving a combined capacitance of $\frac{1}{2}S$. The total capacitance measured is, therefore, $\frac{1}{2}S + 1\frac{1}{2}C$, *i.e.* $\frac{1}{2}(S + 3C)$.

If the capacitance measured in this way is denoted by C′, the equivalent monophase capacitance is 2C′, *i.e.* (S + 3C). In other words, the "equivalent" capacitance current is the same as would flow in a capacitance of double the measured value, if a voltage equal to the P.D. between cores and of the same frequency were applied to this capacitance.

If the capacitance is measured between one core and the other two cores connected together, there are two capacitances C in parallel with a third path. This consists of two capacitances S in parallel with each other and in series with a third capacitance S, giving a combined capacitance of 2S/3. The total capacitance is, therefore, 2S/3 + 2C, *i.e.* 2 (S + 3C)/3. Thus one and a half times the capacitance measured in this last way gives the equivalent monophase capacitance.

QUESTIONS ON CHAPTER VI.

1. In a two-phase system with a common return the P.D. between the mains of one phase is 3 300 volts. Find the P.D. between the outer mains.

If the current in one phase is 60 A., and the current in the other phase is 80 A., calculate the current in the common return,

(*a*) if both currents are in phase with their P.D.s;

(*b*) if the 60 A. current lags 45°.

Does it make any difference to the answers if the loads on the two mains are interchanged?

2. A mesh is made up of three non-inductive resistances having values of 1·2 ohms, 1·0 ohm, and 0·8 ohm respectively. If three-phase P.D.s of 250 volts are applied to the corners of the mesh calculate the currents in the resistances and in the lines.

If the resistances were inductive would the magnitudes of the currents be altered?

3. If the 1 ohm resistance in question No. 2 is replaced by a 1 ohm impedance of 0·6 ohm resistance, find the currents in each line for both orders of phase sequence.

4. Draw an E.M.F. wave with ordinates at 15° intervals of:—0, 40, 80, 120, 150, 170, 180, 170, 150, 120, etc.

Plot two other similar waves ⅓ of a period out of phase with the first. Plot the sum of the three waves.

How does this show the inadvisability of connecting three-phase alternators in delta?

5. Plot the following alternating voltage:—

Phase angle ..	0	15	30	45	60	75	90 degrees
Volts	0	84	188	312	460	506	502
Phase angle ..	105		120	135	150	165	180 degrees
Volts	450		388	280	168	70	0 etc.

Plot a similar wave ¼ period out of phase with this, and a third wave equal to the sum of the two.

Determine the R.M.S. values and the form factors of each wave, and the ratio of the R.M.S. values of the first and third waves. What would be the values of this ratio and of the form factors if the waves were sinusoidal ?

6. Plot the following alternating voltage:—

Phase angle ..	0	15	30	45	60	75	90	105
Volts ..	0	35	84	140	194	225	251	253
Phase angle ..	120	135	150	165	180	195 degrees		
Volts	230	156	94	42	0	− 35 etc.		

Plot a similar wave $\frac{1}{3}$ of a period out of phase with this, and a third wave representing the difference between the two.

Determine the R.M.S. value and the form and peak factors of each wave; and the ratio of the R.M.S. values of the first and third waves. What would be the value of this ratio and of the form and peak factors for sine waves ?

7. Plot the following alternating voltage:—

Phase angle ..	0	15	30	45	60	75	90	105 degrees
Volts	0	56	106	150	170	180	182	180 etc.

each half of the wave being symmetrical.

Find its R.M.S. value, and form and peak factors. What would be the values of all these for a sinusoidal wave of the same amplitude ?

Plot two other similar waves differing in phase by $\frac{1}{3}$ period and plot the sum of the three. What would this sum be for sinusoidal waves ?

8. Plot the following voltage wave:—

Phase angle ..	0	20	35	50	65	80	100 degrees
Volts	0	92	200	316	410	470	520
Phase angle ..		120	130	140	150	165	180 degrees
Volts		464	362	264	183	78	0 etc.

Plot a second wave equal to this but lagging $\frac{1}{6}$ of a period, and a third wave equal to the difference between the two.

Determine the R.M.S. values and the form and peak factors of each wave, and the ratio of the R.M.S. values of the first and third waves.

What would the value of this ratio be if the waves were sinusoidal ?

9. Find the current in the third phase of a Y-connected circuit when the currents in the other two phases are:—

Phase I. (*a*) 10 A. lagging 30°, (*b*) 10 A. lagging 30°, (*c*) 14 A. lagging 60°.
Phase II. (*a*) 14 A. lagging 30°, (*b*) 14 A. lagging 60°, (*c*) 10 A. lagging 30°.

10. Deduce an expression for the power in a single-phase alternating current circuit in which a phase difference exists between voltage and current, and give the power in a three-phase circuit with balanced load in terms of the line voltage and current. [C. & G., II.

11. Find the horse-power transmitted by a three-phase overhead system with 10 000 volts between the lines, a current of 350 amperes in each of the three lines, and a power-factor of ·8.

12. If the resistance between terminals of a balanced three-phase load is 3·7 ohms and the current in each line is 120 A., find the total resistance loss.

What is the resistance of each phase if the connexions are (*a*) Δ, (*b*) Y ?

If 125 kVA. are supplied, what are the values of the impedance and the reactance between terminals ?

13. A three-phase alternator is joined up to an unloaded lead-sheathed cable, having its cores symmetrically arranged in regard to the sheath. Show by a vector diagram the exact phase relationship of the line voltages, the currents between core and core, and the currents between core and sheath. The alternator windings and the cable cores are all insulated, but the lead sheath is well earthed.

14. The capacitances per mile of a three-phase cable are 0·63 μF between the three cores bunched and the sheath, and 0·37 μF between one core and the other two connected to the sheath. Calculate the capacitance current per phase of 8 miles of this cable when supplied at 6 300 volts, 50 $\sim$.

What would have been the result in the second capacitance test if the cores had not been connected to the sheath?

15. The capacitance of a three-core cable is found to be 2·4 μF between two of the cores.

Find the capacitance current when supplied with three-phase current; 2 200 volts between lines, 50 $\sim$.

CHAPTER VII

MEASURING INSTRUMENTS

1. Classification

Ammeters (or ampere-meters) are, as their name implies, instruments for measuring electric currents in amperes. Current measuring instruments in general may be classified according to the effect utilised in obtaining the measurement. Those used for direct currents are shown in the appended table. In every case the action depends on an electromagnetic effect of the current.

	Fixed Portion	Moving Portion	Name of Class
(a)	Coil	Magnet	Moving magnet
(b)	"	Soft iron	Moving iron, or attraction
(c)	"	Two pieces of soft iron	Repulsion, or moving iron
(d)	Magnet ..	Coil	Moving coil
(e)	Coil	Coil	Dynamometer

The first class includes various galvanometers, *e.g.* the tangent, Thomson, and Broca, but this method is not employed in ammeters.

In addition to the above, " hot wire " ammeters may be used for D.C. but are used chiefly for A.C. measurement. Current is measured in these instruments by the expansion of a wire which is heated by the current passing through it. A further class is the " induction " type, which is electromagnetic but can be used only for A.C. (see Vol. II.).

All classes of current-measuring instruments can be used for measuring p.d., *i.e.* as voltmeters, by suitable modifications (see Art. 5).

2. Zero and Deflexional Instruments

In some standard and laboratory instruments the reading is obtained by bringing the moving coil or iron back to its original position. The force required to effect this is measured by a spring or weight. Such instruments are known as *zero* or *null* instruments. An example is the Kelvin Ampere Balance (Chapter II., Art. 6).

In all instruments for ordinary use the force due to the current alters the position of the moving parts until it is balanced by an opposing force (see Art. 3) caused by the change of position. The magnitude of the current is then obtained by observing the amount of the change of position. Consequently such instruments may be distinguished by the name *deflexional.*

The advantage of the latter type is that readings can be obtained immediately, without any time spent in bringing the moving parts to the correct position. On the other hand, for standard purposes the uniform relative position of the fixed and movable parts of the zero type has advantages.

3. Controlling Force in Instruments

The controlling force is the force which balances the deflecting force due to the electromagnetic action of the current. In deflex-

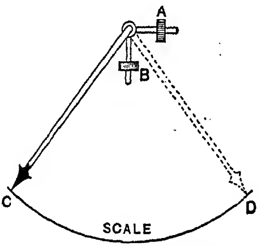


Fig. 7.01.—GRAVITY CONTROL.

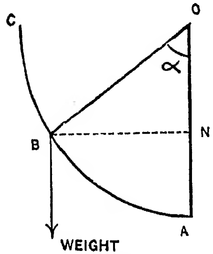


Fig. 7.02.—GRAVITY CONTROLLING FORCE AND ANGULAR DEFLEXION.

ional instruments the controlling force is exerted either by the weight of the moving parts (*gravity control*), or by one or more springs (*spring control*).

In gravity control there are usually two weights adjustable along screw-threaded pins (see Fig. 7.01). Of these one, A, is to counterbalance the weight of the pointer P and other moving parts, *i.e.* to bring the centre of gravity into the axis or else into the vertical plane through the axis. The other, B, does not affect the zero position at all, but its position affects the controlling torque, to increase which B must be lowered.

The torque in all cases of gravity control varies as the sine of the angle of deflexion, being equal to

$$W \times BN = W \times OB \sin BOA \text{ (in Fig. 7.02)},$$

where W = weight of moving parts, A is zero position of their C.G., B its new position, and BN is $\perp$ to OA.

Thus the torque increases rapidly at first and more slowly as the angle of deflexion approaches 90°.

With spring control, on the other hand, the torque varies directly as the angle. A comparison between the two is shown in Fig. 7.03 for two instruments with the same torque at a full scale deflexion of 80°.

The spring control gives a smaller controlling torque at all other angles, or, in other words, a larger deflexion for all currents below the maximum; this results in a more satisfactory scale, especially for very low readings (see also Art. 8). Again, with spring control "set-up" zeros can be used, *i.e.* the first reading on the scale may be (say) $\frac{2}{3}$ of the maximum, thus giving a very open scale over the working range.

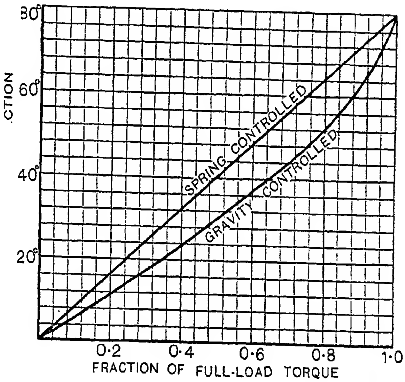


Fig. 7.03.—COMPARISON OF GRAVITY AND SPRING CONTROL.

A further advantage of spring control is that levelling of the instrument is unnecessary if the moving parts are balanced. In moving coil (Art. 9) and dynamometer (Art. 10) instruments the springs also serve as current leads for the moving coil.

Their main disadvantages are liability to change with time, and sometimes a failure to restore the pointer to zero after a prolonged large deflexion. This latter fault is due to overstressing of the material of the spring, and can be avoided by proper design.

4. Damping

When current is switched on or changed rapidly the pointer of an ammeter tends to overshoot the mark, owing to the momentum of the moving parts. If the electromagnetic and controlling forces

(gravity or spring) alone act it will then oscillate about its position of rest for a long time before coming to rest, and if the current changes frequently it may be impossible to get any accurate readings. To avoid this trouble all except the cheapest class of instruments are damped.

The term "damping" denotes the application of a force which acts only when the pointer is moving, and always opposes the motion. The position of the pointer when stationary is therefore not affected by damping, but the time taken to become stationary is diminished. If the damping is just sufficient to prevent any overshooting of the final position the instrument is strictly "dead-beat," but the term is usually applied to one which makes only one or two oscillations before settling down.

The methods employed for damping are—

(*a*) By liquid friction, glycerine or oil being used generally, for example see Art. 12.

(*b*) By air friction, see Art. 7.

(*c*) By eddy currents, see Art. 9.

The British Standards Institution requirements for damping are as follow (see B.S.S., No. 89):—A quantity corresponding to the mean of the maximum and minimum scale values (*i.e.* normally half the maximum, the exception being instruments with "set-up" zeros, Art. 3) is applied to or passed through the instrument when the pointer is standing at zero. The amplitude of oscillation of the pointer shall not exceed 2 per cent. of maximum scale value after a certain period. For moving coil instruments this period is 2 sec. for scale lengths of 6 in. or less, and is increased by steps up to 6 sec. for scale lengths of 15 in. to 18 in. For moving iron or dynamometer instruments the period allowed is about 50 per cent. longer.

5. Use of Ammeter as Voltmeter

Since volts are equal to the product of amperes and ohms they may be measured by using an ammeter, in series with a known resistance, connected to the two points the P.D. between which is to be determined.

The methods of connexion for measuring current and voltage respectively are shown in Fig. 7.04, G being the instrument in each case.

All voltmeters act on this principle, with the exception of electrostatic ones, which are used mainly for alternating pressures. The

reading is not taken in amperes and then multiplied by the resistance, but the scale is marked so as to read volts directly.

A voltmeter can be given two or more ranges by using a subdivided resistance (see Example 1).

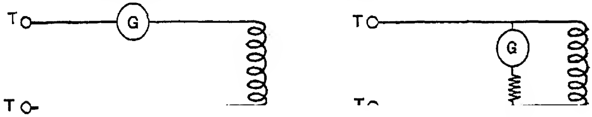
The total resistance of a voltmeter should be high so as to diminish the power absorbed by it. For power in watts = EI = E^2/R, *i.e.* the power wasted for a given voltage varies inversely as the resistance.

For similar instruments the resistance is usually increased in proportion to the maximum voltage, consequently the power wasted increases in the same proportion.

The resistance of an ammeter, on the contrary, should be low, so as to diminish the "drop" (of volts) across it and the power wasted. For the latter is I^2R watts, *i.e.* for a given current it varies directly as the resistance.

Example 1. *A milliammeter of 2 ohms resistance which gives its full scale deflexion with 150 milliamperes is to be used as a voltmeter* (a) *reading up to 3 volts,* (b) *reading up to 150 volts.*

Find the necessary resistances, and the power absorbed in each case.



(*a*) Ammeter. (*b*) Voltmeter.

Fig. 7.04—CONNEXIONS OF INSTRUMENT FOR CURRENT AND VOLTAGE MEASUREMENTS.

TT, Supply terminals.

(*a*) 3 volts must send 0·15 amp. through the instrument;

$$\text{total resistance} \quad \frac{}{0{\cdot}15} \quad 20 \text{ ohms;}$$

$$\text{resistance in series with instrument} = 20 - 2 = 18 \text{ ohms.}$$

(*b*) Total resistance $= \dfrac{150}{0{\cdot}15} = 1000$ ohms;

$$\therefore \text{ series resistance} = 1000 - 2 = 998 \text{ ohms.}$$

In each case the current at full scale reading is 0·15 amp.;

$\therefore$ in case (*a*) power absorbed = $3 \times 0{\cdot}15 = 0{\cdot}45$ watt.

In case (*b*) „ „ $= 150 \times 0{\cdot}15 = 22{\cdot}5$ watts.

6. Shunts

When large currents have to be measured only a portion is sent through the working coil, the remainder being carried by a shunt, *i.e.* a low resistance in parallel with the instrument.

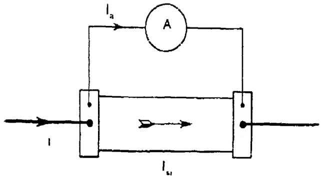


Fig. 7.05.—CONNEXIONS OF SHUNTED AMMETER.
A, Ammeter.

The multiplying power of a shunt is the ratio of the *total* current to the instrument current.

Let I_1 = instrument current in amperes. I_2 = shunt current in amperes. I = total current in amperes. G = resistance of instrument in ohms. S = resistance of shunt in ohms.

The (voltage) drop across instrument = drop across shunt,

$$\therefore\ I_1G = I_2S$$

or
$$I_2 = I_1 \times \frac{G}{S};$$

$$\therefore\ I = I_1 + I_2 = I_1\left(1 + \frac{G}{S}\right) = I_1\left(\frac{G+S}{S}\right),$$

or multiplying power of shunt $= \dfrac{I}{I_1} = \dfrac{G+S}{S} = 1 + \dfrac{G}{S},$

e.g. for a multiplying power of 10 the value of $\frac{G}{S}$ must be 9, or the shunt resistance is $\frac{1}{9}$ of instrument resistance. Or, generally, for a multiplying power, *m*, the shunt must have a resistance $= \dfrac{G}{m-1}$

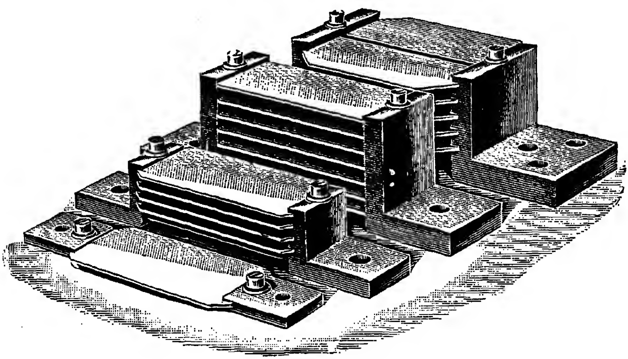
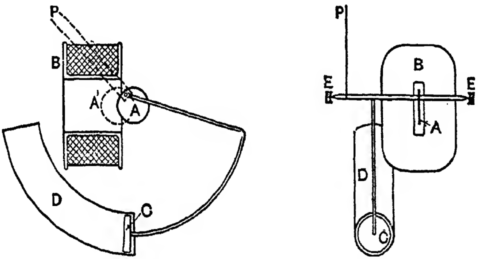


Fig. 7.06.—AMMETER SHUNTS.

Ammeter shunts are constructed of one or more strips of thin sheet-metal (about ·02 in. thick), either manganin or constantan. These are soldered into end-blocks of copper or brass arranged to be clamped to the main conductors (see Fig. 7.06). Usually the ammeter leads are connected to the end-blocks, but they are sometimes attached to the sheet-metal. Constantan is easier to solder but has a rather high thermo-E.M.F. with copper (·0037 volt per 100° C.). If there is any difference of temperature between the two shunt end-blocks this will affect the readings of the ammeter.

Another error is due to the change of resistance with temperature of the instrument affecting the multiplying power of the shunt. This is diminished by connecting in series with the copper working coil a resistance made of an alloy with a small temperature coefficient.



(*a*) ELEVATION. (*b*) SIDE VIEW.

Fig. 7.07.—MOVING IRON AMMETER WITH AIR DAMPING.

A′, Position of A with full current flowing. D, cylinder closed at upper end only. E E, Pivots.

The " drop " across the ammeter is thus increased, so this method of correction cannot be carried far.

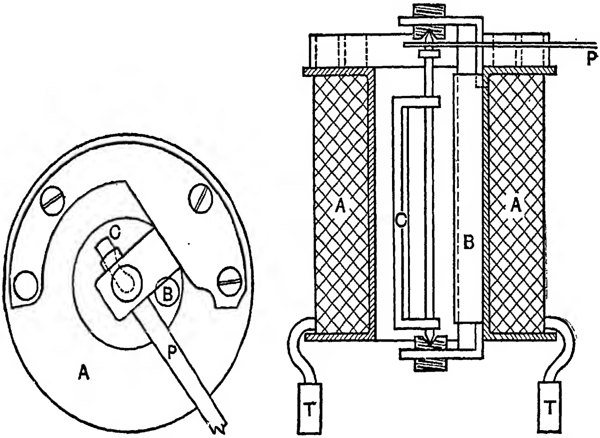
It has been suggested that the shunt might be made of copper, too, so as to have the same percentage increase of resistance as the ammeter, but this would be correct only if the temperature of the two changed together and equally, which is often not the case.

7. Moving Iron Instruments

The usual pattern of the attraction division of this class is shown in Fig. 7.07. The moving iron consists of a flat disc, A, pivoted at a point outside a flat coil, B. When the current passes through the coil, A is attracted into it and so moves the pointer P over the scale. The control may be gravity (as shown) or spring (Art. 3).

The damping is by means of a circular piston, C, which moves inside the curved cylinder D without touching it at all. This device was first used by Messrs. Siemens and Halske.

The repulsion type, as made by Nalder Bros. & Thompson, is shown in Fig. 7.08. The current is sent through a coil A, inside which are two cylindrical pieces of iron, one (B) fixed, the other (C) attached to the pointer. When current passes, B and C are magnetised similarly and therefore repel each other, thus moving the pointer P across the scale. The control may be by gravity or spring. Damping is by means of an air piston.



(*a*) Elevation. (*b*). Inverted plan (partly sectional).

Fig. 7.08.—REPULSION AMMETER.

T T, Terminals (N.B.—Controlling weights or springs omitted).

Both classes of moving iron ammeters tend to have very crowded scales at low readings. Consequently no readings can be relied on below $\frac{1}{10}$ of the maximum, and the effective range extends down to only $\frac{1}{4}$ of the maximum (B.S.S., No. 89 definition). For higher readings the scale is fairly uniform, but it becomes crowded again towards the end (see Fig. 7.09).

For voltmeters this matters less, particularly for those used to measure a fairly steady voltage with set-up zeros (see Art. 3).

In modern patterns, such as the "Superscale" of Everett, Edgcumbe & Co., suitable shaping of the fixed and moving irons,

particularly the former, has increased the scale length by a half without increasing the overall size of the instrument. The pointer moves through 125° instead of the usual 80° to 85°. The scale thus becomes as long as the diameter of the case.

A further disadvantage is that hysteresis (see Chapter IV.) causes the reading with a given current to be lower when the current is increased to that value, than it is when the current is reduced to it from a higher value. This applies to D.C. measurements only.

They are affected by external fields due to neighbouring magnets or currents, but can be shielded from these to a large extent by the use of cast-iron cases. Care must be taken that only a few of the lines set up by the coil of the instrument pass through this case, otherwise the hysteresis error will be increased.

Most moving-iron instruments are of the repulsion type since this gives a robust cheap instrument of good accuracy. The main use of the attraction type is for measurements at various frequencies,

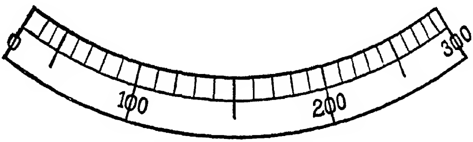


Fig. 7.09.—Scale for Moving Iron Ammeter.

e.g. in portable testing instruments. By using Mumetal for the moving vane a coil with very few turns suffices. Hence its inductance is low and so the variation due to frequency is small. Moreover the consumption of an ammeter can be reduced to about half a volt-ampere compared with 2VA for one of the repulsion type.

8. Moving Iron Instruments for A.C.

Some types of D.C. ammeters and voltmeters can be used for A.C., and others cannot. The distinction depends on whether a reversal of current causes a deflexion in the same or in the opposite direction. Thus permanent magnet instruments cannot be used for A.C. Whereas soft-iron instruments, including the repulsion type, can be used for A.C., but require some modifications.

The chief cause necessitating modification is that alternating currents induce eddy currents in any piece of metal in their vicinity. It is therefore necessary to sub-divide any metallic piece near the coil in such a way as to reduce these eddy currents. Otherwise

too much heat is produced and the insulation suffers, besides which the magnetic action of the coil is reduced greatly.

A simple example of such sub-division is that of the repulsion type of instrument. When this is to be used for A.C. the metal bobbin [Fig. 7.08 (*a*), A] on which the coil is wound is cut completely through along one radius. This leaves it in a single piece and still a satisfactory support for the coil, but interferes with the currents which otherwise would flow circumferentially in the bobbin. Alternatively a non-metallic support may be used. The fixed cylinder of iron is made up of a number of wires instead of being a single solid piece. The moving piece of iron may likewise be subdivided, but as it is small this is not essential.

Since the deflecting force *in a given position of the moving parts* varies as Bi in a moving iron instrument, and as B^2 in a repulsion instrument (where i = current, B = flux-density in the iron) the deflexion depends on the mean value of Bi and of B^2 in the respective types. If B were exactly proportional to i this would make the deflexion dependent on the mean value of i^2 in both cases. The instrument would then, apart from the effect of eddy currents, indicate the R.M.S. value of the current or voltage on the same scale as that which is correct for D.C.

The effect of the variation in the permeability of the iron is to make this only approximately true, so that for accuracy these instruments require calibration with A.C. of the same frequency and wave-form as that which they are to measure. The differences between the two scales, generally speaking, is that the deflexions with D.C. are smaller than those for A.C. of the same virtual value at the top of the scale: at smaller values they may be smaller or greater according to the conditions of the particular instrument. In a badly designed instrument calibrated with a sinusoidal wave the reading may be 6 per cent. low with a peaked wave, and 3 per cent. high with a flat wave.

A further possible source of difference for voltmeters is that the impedance exceeds the resistance, and so the current per volt is less with A.C. than with D.C. This effect can be made negligible by keeping the coil (which is necessarily inductive) a small fraction of the total resistance and by winding the series resistance non-inductively.

9. Moving Coil Instruments

These are an inversion of the moving magnet instruments of class *a*), Art. 1. A typical example is shown in Fig. 7.10.

A permanent magnet, M, M, has soft-iron pole pieces P, P, attached to it. These are bored out cylindrically. A soft-iron cylinder or core, I, is held concentrically by the bridge piece, D, of gun-metal or other non-magnetic material. This cylinder serves to guide the magnetic lines to where they are required (see also end of this Art.).

In the narrow air-gap (say ·07 in.) thus formed move the sides of the moving coil, C, of fine copper wire. The control is by means of a spiral spring, S, of phosphor bronze, attached to the axle. This further serves to take the current from the coil to the circuit, leading-in being effected either by another similar spring, or by a thin strip of copper attached to the other axle. The two axles have pointed ends which rest in jewelled bearings.

On current being passed through the coil a deflecting torque is exerted proportional to the current and to the strength of field in which it lies. The controlling torque of the spring is proportional to the angle of deflexion. Hence the scale is *an evenly divided one*, provided that the magnetic field is of the same strength throughout the path of the coil.

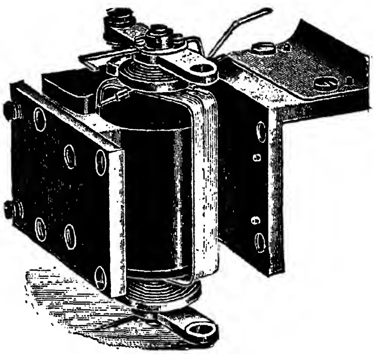


Fig. 7.10.—MOVING COIL INSTRUMENT.

Another advantage is that external fields have little effect on the readings, since the working field is much stronger than in soft-iron instruments. Except for very small currents (say a quarter of an ampere) shunts must be used, with the consequent introduction of temperature errors (see Art. 6). The standard drop across the shunt is ·075 volt.

The accuracy further depends on the magnet remaining of constant strength. To ensure this as far as possible—

(*a*) The magnet is made of special tungsten (or molybdenum) hard steel.

(*b*) It is artificially "aged" by subjecting it to heat and to mechanical vibration. This reduces the strength of the magnet but makes it more truly permanent.

(c) The air-gap is made narrow and the polar surface large. This reduces the self-demagnetising effect (see Chapter IV.).

Damping is effected by winding the moving coil on a frame of copper or aluminium. The eddy currents produced in this when it moves in the magnetic field cause forces opposing the motion, and produce very satisfactory damping.

10. Dynamometer Instruments

These are used mainly for alternating currents, but can be made to read correctly on both D.C. and A.C. circuits without separate scales. The Kelvin balance (Chapter II.) and the Siemens dynamometer are zero instruments of this class.

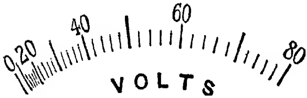


Fig. 7.11.—SCALE OF DYNAMOMETER TYPE VOLTMETER.

The moving coil is similar to that employed in moving coil instruments, but the field is due to another coil instead of to a permanent magnet.

The deflecting torque in a given position depends on the product of the currents in the two coils, *i.e.* on (current)^2 when the coils are connected in series. Hence the scales of these instruments are crowded in their lower parts and spread out at the top (see Fig. 7.11). They avoid any hysteresis error, but are very liable to errors caused by stray fields since the strength of the working field is low.

In the case of ammeters they suffer from the disadvantage of a large drop (about 0·3 volt), which must be further increased by a ballasting resistance of low temperature coefficient if a shunt is used. For moderate currents it is preferable to shunt the moving coil only, and not the whole instrument (Fig. 7.12).

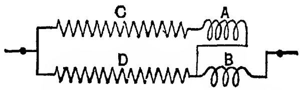


Fig. 7.12.—CONNEXIONS OF DYNAMOMETER AMMETER AND SHUNT.

A, Moving coil. B, Fixed coil. C, Series resistance. D, Shunt.

Their chief use is as wattmeters (Art. 12), but in addition they are very suitable as alternating voltmeters when very high accuracy ("substandard") is desired. For this purpose B.S.S., No. 89 requires an error not exceeding 0·3 per cent. of maximum scale value. This is surpassed only by moving coil voltmeters for D.C. from which 0·2 per cent. is demanded. For all other types of voltmeters, and for ammeters of all types except moving-coil with separate shunts, only 0·5 per cent. is required for substandard accuracy.

For ordinary A.C. purposes, even for First Grade accuracy (1 per cent. of *reading* down to half load), the improved repulsion instrument is preferable; but when a large range of frequencies has to be covered the dynamometer type has advantages.

With hot-wire instruments (Art. 11) it is difficult to obtain even First Grade accuracy.

11. Hot-Wire Instruments

Since the rate of heat production of an alternating current of any given effective value is the same as that of a D.C. of the same value (Chap. V., Art. 7), an instrument whose indications depend on the heating effect of the current will show the R.M.S. value correctly on the same scale as is used for D.C. This has the great advantage that it is sufficient to calibrate the instrument with D.C., which can be done more easily and more accurately than when A.C. has to be used. Moreover the instrument will be correct on any wave form and with any frequency. This is limited only by the condition that the frequency must not be so high that the resistance is sensibly increased (see Chap. V., Art. 16), in which case the heat produced per second per effective ampere is correspondingly increased.

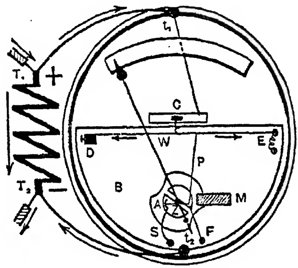


Fig. 7.13.—DIAGRAM OF HOT-WIRE AMMETER.

The commercial form of instrument working on this principle is shown in Fig. 7.13.

The current is passed through the platinum-silver wire, W, 16 cm. long, entering at the centre, C, by a flexible strip, and leaving at both ends, or vice versa. Near the centre of W a phosphor bronze wire, P, is attached, pulling on W approximately at right angles. When current passes through W it expands, and the point of attachment of P moves downward by a much larger distance than the expansion of W.

A second multiplication is obtained by a cotton thread Z, pulling on P at right angles, the lower end F of P being fixed. This thread, Z, passes partly round and is attached to a small grooved brass pulley, about 4 mm. minimum diameter, fastened to the axle. A second similar pulley on the same axle has another thread attached

to and passing partly round it, and then going to a flat steel spring, S. This spring keeps the threads and wires in tension. A long light aluminium pointer is attached to the same axle as the brass pulleys, and gives a third magnification of the original extension of the platinum-silver wire W.

The exact strength of the spring is immaterial as the position of the pointer in this instrument depends on the extension of W and not, as in other types, on the point of balance between a deflecting force and a controlling force.

To prevent the pointer being deflected by changes in the room temperature all the working parts are mounted on a base plate, B, of an alloy whose coefficient of expansion with temperature is very nearly the same as that of the platinum-silver wire. In addition a zero adjuster is fitted. This consists of an arm fixed to the base plate and a hinged arm to which the left-hand end of the wire D is attached. The position of this hinged arm is adjusted by means of a screw, backlash being prevented by a spring between it and the arm fixed to the base plate. A hole is made in the instrument cover so that t screw can be adjusted from outside. An aluminium disc, A, is fastened to the spindle and moves between the poles of a small electro-magnet, M, and so damps the motion. This is not necessary in the case of varying currents as the wire takes a little time to reach a steady temperature, and so avoids any overshoot of the pointer: but it is useful in steadying the pointer when the instrument is subjected to vibration.

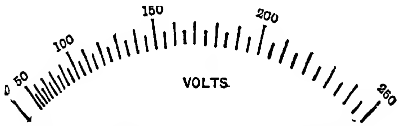


Fig. 7.14.—SCALE OF HOT-WIRE VOLTMETER.

The great advantage of hot-wire instruments is that the deflexion depends on the R.M.S. value of the current in the hot-wire whatever the wave-form and frequency. Thus a hot-wire ammeter can be calibrated with D.C. and used for measuring A.C. of any wave-form and of any ordinary frequency. The same applies to a hot-wire voltmeter provided its series resistance is non-inductive.

Their disadvantages are:—

(*a*) they cannot stand much overload without burning out;

(*b*) their zero position often requires adjustment;

(*c*) their scales are crowded in the lower portions (Fig. 7.14);

(*d*) an ammeter cannot be used with several different shunts and only one scale; since the increased resistance of the hot-wire when current flows increases the multiplying power of a given shunt;

(*e*) the power absorbed is higher than for other types.

12. Electrostatic Voltmeters

These can be used for either A.C. or D.C., but are especially suitable for the former, since the indications are independent of wave-form and of frequency. When used for D.C., a second reading with reversed poles is desirable where great accuracy is required. It can be calibrated with D.C. for use with A.C.

The action employed is the electrostatic attraction exerted between two bodies at different potentials.

The simplest type, and that used for very high potentials is the *attracted disc* (see Fig. 7.15). A flat plate, B, supported on varnished glass pillars, P, P, is connected to one terminal by a wire in an insulating tube C. The other terminal is connected by the wire A to the support of the moving system, and so to the plate V. The deflecting force is caused by electrostatic attraction between B and V, and the controlling force is due to the gravity of the moving parts. Since the attractive force varies as $(P.D.)^2$, and further increases with decrease of the distance between B and V, the scale is very open at the top and crowded at the lower readings. This type is suitable for P.D.s up to 100 000 volts (R.M.S.), and can be used for higher pressures still by modifying the insulating arrangements.

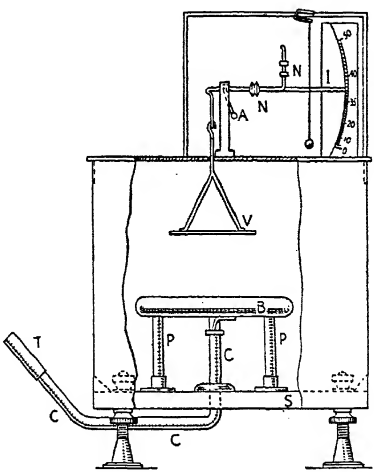


Fig. 7.15.—KELVIN ATTRACTED DISC VOLTMETER.

Other makers use vertical plates and the moving one surrounded by a large disc, which shields it from electrostatic disturbance.

For rather lower pressures the moving part (an aluminium vane, see Fig. 7.16) is suspended about a horizontal axis, and moves between and parallel to two fixed double quadrants (of brass). By using three different weights hung on the moving vane each instrument can be given three different ranges with readings in the ratios 1 : 2 : 4. With different patterns a total range of about 1 000 volts to 20 000 volts can be obtained.

Fig. 7.16.—KELVIN VERTICAL ELECTROSTATIC VOLTMETER.

The next step for lower pressures is to employ the multicellular type. With a horizontal axis a switchboard pattern (see Fig. 7.17) is obtained with an open scale of short range. This can be made suitable for P.D.s from 1 500 volts to 5 000 volts.

By using a vertical suspension so as to avoid pivot friction, and by increasing the number of moving vanes (V), and of fixed brass

cells (Q) [see Fig. 7.18 (*a*)], much lower P.D.s can be measured. The smallest range made reads from 30 volts to 120 volts, and higher ranges up to about 2 000 volts are obtained easily by modifying the suspension or the vanes. Damping is effected by a disc, attached to the bottom of the spindle, moving in a dashpot filled with oil.

The instrument shown is for attachment to a switchboard, but this type is more suitable for laboratory use.

A further great advantage of the electrostatic type is that it absorbs no power, and takes only a very small capacitance current. Its disadvantages are that it cannot be used for small P.D.s, and is sluggish on its lower possible ranges owing to the necessary weakness of the control. Moreover it is costly, but this disadvantage becomes less pronounced as the pressure increases. The scale is crowded in its lower portion.

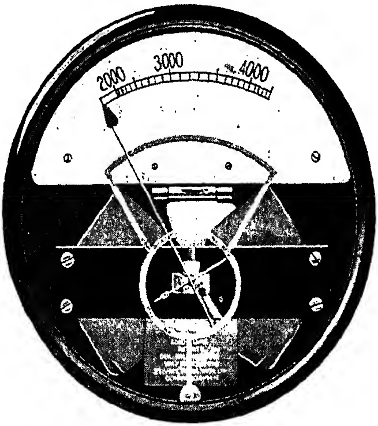


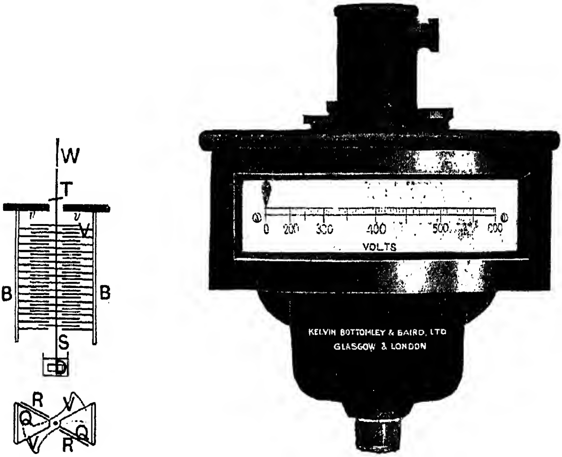
Fig. 7.17.—KELVIN HIGH PRESSURE MULTICELLULAR VOLTMETER.

To increase the range of a low-reading electrostatic voltmeter a *multiplier* is used. This consists of a high resistance with tappings taken off at intermediate points. The P.D. to be measured is connected across the whole resistance, and the voltmeter across part of it. Since the voltmeter requires no current the P.D. across it is the same fraction of the total P.D. as the resistance across it is of the whole resistance; *e.g.* if the voltmeter is connected across $\frac{1}{5}$ of the whole resistance (the fractions available are usually $\frac{1}{5}$, $\frac{1}{4}$, and $\frac{1}{2}$), the total P.D. is five times the reading of the voltmeter.

If such a multiplier were used with a voltmeter of any other type, the multiplying factor would be greater owing to the shunting effect of the voltmeter (see Question No. 25). Besides this it would take more current from the mains than the usual arrangement of a set of series resistances (see Art. 5).

The drawback to the use of a multiplier is that it loses the advantages of taking no power and very little current.

An alternative method is to connect a number of condensers of equal capacitance in series across the mains, and the voltmeter in parallel with one of them. Provided the capacitance of the voltmeter is negligible in comparison with that of the condensers, the P.D. will be divided equally between the condensers, and so any desired multiplying factor can be obtained.



(*a*) INSIDE VIEW. (*b*) OUTSIDE VIEW.

Fig. 7.18.—KELVIN MULTICELLULAR ELECTROSTATIC VOLTMETER.

B. B. Supporting plates for Q, Q.
D. Damping vane in oil.
R. R. Repelling plates to limit motion of V.
S. Spindle to which V, V are fixed.
T. Spring to protect suspension.
v. v. Vulcanite support.
W. Phosphor bronze strip suspension.

This method retains the advantage of taking no power, but the capacitance current taken must be increased greatly.

For high pressure electrostatic voltmeters a condenser of small capacitance is sometimes connected in series with the instrument. The latter must be calibrated with the condenser connected, because the variation in the capacitance of the instrument makes the ratio of division of the total P.D. between the two variable. This arrangement diminishes the crowding of the lower part of the scale,

as the instrument then obtains a larger proportion of the total P.D. than on the upper part of the scale when its capacitance is increased.

13. Wattmeters

A wattmeter, as its name implies, measures the electrical power supplied to, or delivered by, any apparatus or a circuit made up of any number of apparatuses. The word apparatus is here used to include impedances, choking coils, transformers, motors, generators, etc. Such an instrument, while hardly ever required in D.C. circuits, is of great use in A.C. supply since the power is not as a rule equal to the product of volts by amperes. Its indication should give the mean value of the power (see Chap. V., Art. 12) not the R.M.S. value as in the case of an ammeter or voltmeter.

The two main types of wattmeter are:—

(*a*) Dynamometer,

(*b*) Induction.

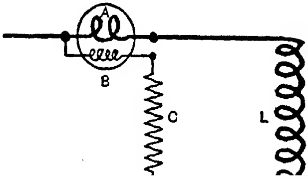


Fig. 7.19.—CONNEXION OF WATTMETER.

A. Current coil. B. Pressure coil.
C. Series resistance. L. Load.

A hot-wire type has been devised by M. B. Field, and an electrostatic one by G. L. Addenbrooke,* but these have not come into general use.

When a dynamometer instrument is used as a wattmeter the fixed coil (or coils) carries the main current, or a definite fraction of it, and the moving coil (or coils) carries a current proportional to the P.D. (see Fig. 7.19).

Owing to the absence of iron in the neighbourhood of the coils an alternating field is produced in phase with, and proportional to, the main current at any instant. Thus the torque exerted on the moving coil is proportional to (instantaneous current) $\times$ (instantaneous P.D.), *i.e.* to the instantaneous value of the power.

The pointer cannot follow the rapid changes in the instantaneous power, but takes up the position in which the controlling torque is equal to the mean deflecting torque (inclusive of negative values). Thus the position of the pointer indicates the mean power, which is the quantity required.

* Described at the International Electricity Conference, Paris, 1900.

Fig. 7.20 shows the interior of a switchboard pattern of this type of wattmeter. In use the axle is horizontal, not vertical as in the figure. There are two fixed current coils so as to produce a more uniform field than with a single coil: the result is that a uniformly divided scale is obtained. The moving coil is single, and the current is led into and out of it through the two spiral controlling springs as in the case of a moving coil instrument (see Art. 9). The series resistance is wound on a number of sheets of mica, and this construction makes it very nearly non-inductive, which is necessary for accuracy.

The movement is damped by a double vane attached to the

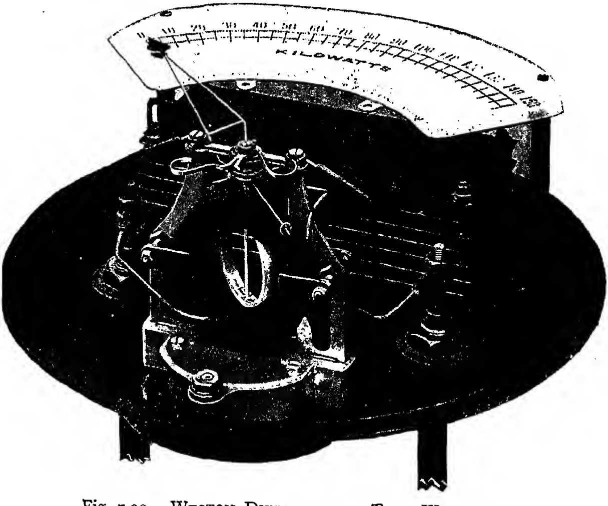


Fig. 7.20.—WESTON DYNAMOMETER TYPE WATTMETER.

axle. This moves in a double-sector shaped box, seen below the coils in Fig. 7.20. The vanes have a small clearance between them and the sides and edges of the box, and thus rapid movements are opposed by the compression of the air on the forward sides of the vanes and its rarefaction on the hinder sides.

The pointer is a triangular truss with a thin tip mounted at the end. This construction makes the pointer strong without making it heavy, which would increase friction. At the same time the natural time of vibration is kept well outside that of the power at any commercial frequency, and so steady readings are always obtainable.

14. Supply Meters

Supply meters are for the purpose of measuring the total quantity of energy supplied during a certain period, *i.e.* they are *integrating* instruments. They are of two main classes:—

A. Energy, or watt-hour, meters.

B. Quantity, or ampere-hour, meters.

The latter depend on the fact that the usual system of supply is at approximately constant voltage. Thus the energy supplied in watt-hours can be obtained by multiplying the ampere-hours by this known voltage. Such instruments do not record the ampere-hours but the corresponding watt-hours at the nominal supply voltage (see Ex. 2).

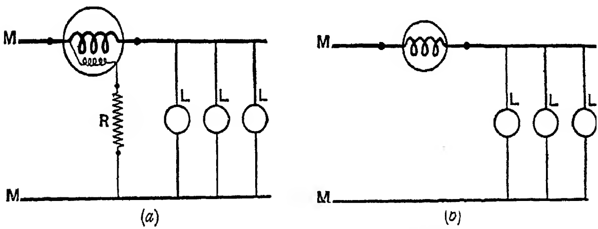


Fig. 7.21.—CONNEXIONS OF SUPPLY METERS.

(*a*) Watt-hour meter. (*b*) Ampere-hour meter.

LLL, Lamps or other loads. MM, Mains. R, Resistance in series with pressure coil

Their relative advantages are:—

WATT-HOUR METERS.	AMPERE-HOUR METERS.
Accurate on varying voltage.	Greater simplicity.
Less liable to change of accuracy.	No shunt circuit to absorb power.

The power absorbed in the shunt is not as great a disadvantage as appears from calculations of the energy used, for the load is a 24-hours one, and therefore the cost per kWh. is small (see Chapter XVIII.). The simpler construction of D.C. quantity meters, however, leads to their use in houses, factories, etc., the D.C. watt-hour type being employed mostly on switchboards. The methods of connexion for the two cases are shown in Fig. 7.21.

Subdividing both forms according to their methods of action gives the following types in actual use:—

(*a*) Electrolytic. (*c*) Clock (Aron) } see

(*b*) Motor. (*d*) Induction (for A.C. only) } Vol. II.

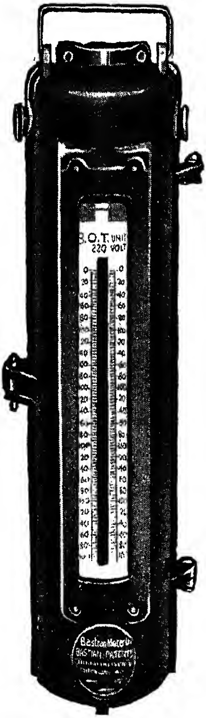


Fig. 7.22.
BASTIAN METER.

Electrolytic meters are necessarily quantity meters, but the motor type may be of either the quantity or the energy class of meter.

Example 2. *An ampere-hour meter has been adjusted to read kilowatt-hours correctly on a 220-volt circuit. It is used on a 250-volt circuit and records* 873 kWh. *What is the energy actually supplied?*

On a 220-volt circuit 1 kWh. requires a quantity $= \frac{1000}{220}$ ampere-hours;

$\therefore$ the metre registers 873 kWh. when $873 \times \frac{1000}{220}$ ampere-hours have passed through it.

At 250 volts this quantity $= 873 \times \frac{1000}{220} \times \frac{250}{1000}$ kWh.,

$= 993$

15. Electrolytic Meters

Electrolytic meters depend on the fact that the amount of chemical action of an electric current is proportional to the quantity (*i.e.* coulombs or ampere-hours) which has passed through the liquid. The chemical actions employed are:—

(*a*) Decomposition of water (Bastian meter).

(*b*) Decomposition and dissolution of copper in copper sulphate solution (Long-Schattner meters).

(*c*) Deposition and dissolution of zinc in zinc sulphate solution (Edison meter).

(*d*) Deposition and dissolution of mercury in mercurous nitrate solution (Wright meter).

In the Bastian meter the whole current passes through dilute caustic soda by means of nickel electrodes, the hydrogen and oxygen produced being allowed to escape. The liquid is covered

with a layer of oil to prevent evaporation. The general appearance is seen in Fig. 7.22. The measurement is made by observing the fall in the level of the liquid, which depends on the electrical quantity.

Its advantages are:—

(1) Accuracy at all loads however small.

(2) Cheapness.

(3) Simplicity, hence few repairs.

Its disadvantages are:—

(1) A drop of potential of about 2 volts, due mainly to polarisation.

(2) The gases form an explosive mixture.

(3) It is difficult to read while current is passing, owing to the froth formed by the bubbles of gas.

Example 3. *If the polarisation back E.M.F. of a Bastian meter is 1·7 volts and its resistance is 0·02 ohm, find the P.D. across its terminals with 25 A. and 5 A. respectively flowing through it.*

With 25 A. the resistance " drop " is $25 \times \cdot 02 = 0{\cdot}5$ volt.
With 5 A. " " " is $5 \times \cdot 02 = 0{\cdot}1$ volt.

In each case the polarisation E.M.F. has to be overcome;

$\therefore$ terminal P.D. with 25 A. $= 0{\cdot}5 + 1{\cdot}7 = 2{\cdot}2$ volts.
" " " 5 A. $= 0{\cdot}1 + 1{\cdot}7 = 1{\cdot}8$ volts.

16. The Wright Electrolytic Meter

Most electrolytic meters are shunted, and care must then be taken to make the temperature coefficients of the meter and shunt as nearly alike as possible. Also the back E.M.F. of polarisation must be only a small fraction of the total drop, otherwise the recording of the meter will be poor at low loads.

In the Wright meter (type (*d*), Art. 15) the polarisation E.M.F. does not exceed ·0001 volt, and the full load drop is about 1 volt. Hence the error from this cause amounts to only 1 per cent. at $\frac{1}{100}$ of full load, and correspondingly less as the load increases. The arrangement of the electrolytic portion of this meter is shown in Fig. 7.23. It consists of a hermetically sealed glass tube containing a saturated solution of a mixture of potassium and mercurous nitrates. Current is passed through this from the anode of pure mercury to the iridium ring which forms the cathode. The mercury is held in position by a glass " fence," and its level is kept constant by a reservoir. This fence is built up of a number of glass rods placed in a ring with an inner ring of smaller rods.

This allows free circulation of the electrolyte, and does not become choked with mercury as sometimes happened to the platinum gauze formerly used. The chemical action of the current deposits small globules of mercury on the cathode. These fall off by gravity into a narrow tube, alongside of which is a scale of kilowatt-hours at the supply voltage.

In large meters this tube forms a siphon so that after reaching 100 kWh. the mercury siphons over into a larger tube. The hundreds of units are read on a scale beside this larger tube; and the tens and units on another scale, near to the siphon tube.

Mercury is dissolved at the anode as fast as it is deposited on the cathode: consequently the liquid remains of constant strength; and so its resistance is constant, apart from changes due to alteration of temperature. The level of the mercury in the anode is kept constant by means of the reservoir. The space above the mercury in this is filled with liquid; and when mercury leaves the reservoir it is replaced by more liquid. A small amount of air is left in to allow for differences between the expansion of the liquids and of the glass containing vessel. The connexions of the meter are as shown in Fig. 7.24.

The shunt is of platinoid or of some other alloy with a low

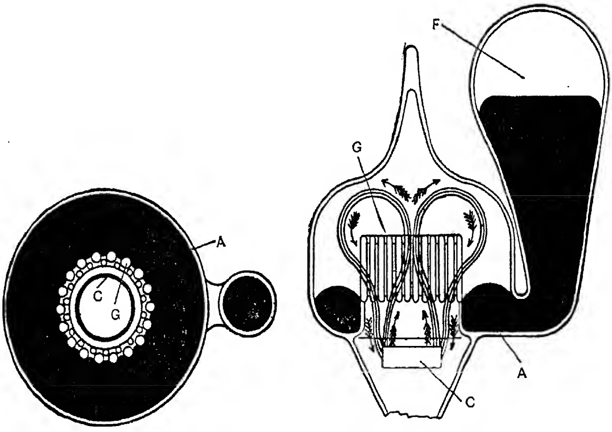


Fig. 7.23.—ELECTROLYTIC CELL OF WRIGHT METER, REASON CO.'S PATTERN.

A, Glass vessel. C, Iridium cathode. F, Mercury reservoir. G, "Fence" of glass rods. Circulation of electrolyte shown by arrows.

temperature coefficient. The series resistance is made, partly at least, of iron wire which has a large positive temperature coefficient and so compensates for the negative coefficient of the electrolytic cell itself. The two together are thus made to have nearly constant resistance over a wide temperature range.

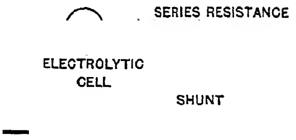


Fig 7.24.—DIAGRAM OF CONNEXIONS.

To reset the meter it is unfastened from a catch at the bottom, and the cell tilted about hinges at the top, till the mercury runs back into the upper parts.

Its advantages are:—

(*a*) Great accuracy down to very small loads.

(*b*) Small liability to break down or become inaccurate.

Its disadvantages are:—

(*a*) Length of time required for calibration (but this seldom needs repetition). This is due to the absence of any means of reading small fractions of a unit, as is possible in motor meters (cf. Arts. 18 and 34).

(*b*) Absence of check on accuracy of reading in case of dispute after reading and re-setting.

(*c*) Failure to record if connected wrong way.

17. Motor Meters

Motor meters usually act on the following principles. If of the ampere-hour type a constant field is provided by permanent magnets, and the current in the armature is proportional to the main current. Thus the torque exerted (see Chapter XI.) is proportional to the main current.

In the watt-hour type the field is produced by coils carrying the main current (or a definite fraction of it), and the armature carries a current proportional to the voltage. Therefore the torque is proportional to current × voltage, *i.e.* to watts. In both types an eddy current brake is provided. This consists of a disc of copper or aluminium rotating between the poles of permanent magnets (see Figs. 7.27, 7.29). The eddy currents produced in the disc cause a retarding torque proportional to the speed (see Chapter XII., Art. 3). When the speed is steady the two torques must be

equal. Therefore the speed must be proportional to the amperes or the watts, according to the type of meter. Now the total number of revolutions of the brake is equal to speed × time, and is, therefore, proportional to the ampere-hours (or the watt-hours) which is the quantity to be measured.

This class of meter may also be subdivided into—

(*a*) mercury motor meters,
(*b*) commutator motor meters, } according to the means adopted for leading current in and out of the armature.

Examples of both classes are given below.

Their relative disadvantages are:—

(*a*) (1) The fluid friction increases as the *square* of the velocity* (or even more rapidly) and so must be compensated (see Art. 18).

(2) Solid friction causes an error which is only important at very small loads.

(3) If part of the mercury is spilt, or it becomes dirty, the accuracy is impaired (see further Art. 18).

(*b*) (1) Solid friction is usually considerable, and must be compensated (see Art. 19).

(2) Owing to variation in the driving torque with the position of the armature the starting current may be high.

(3) The brushes and commutator are liable to cause trouble.

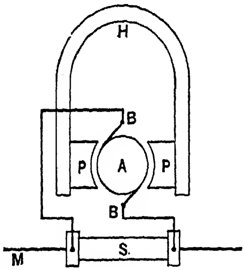


Fig. 7.25.—CONNEXIONS OF O.K. AMPERE-HOUR METER.

A, Armature. BB, Brushes. H, Permanent magnet. MM, Mains. PP, Pole pieces. S, Shunt.

Both types can be made to work with a small drop. The commutator type can be used for A.C., while the mercury type is suitable for D.C. only.

The " O.K. " ampere-hour meter, made by the British Thomson-Houston Co., used a different principle. The armature (see Fig. 7.25) is in parallel with a platinoid shunt whose resistance is such that with full load current the drop is about half a volt. There is no brake or metallic armature core, consequently the armature

* To some extent this balances the reduction of the effect of solid friction and so diminishes the average error, see " Errors of D.C. ampere-hour meters," *El. Rev.*, 2nd Feb. 1917.

runs at a speed which generates a back E.M.F. nearly equal to the "drop." Since the field is constant the back E.M.F. is proportional to the speed, and so the speed is very nearly proportional to the current.

It has now been replaced by their D.M. type with a mercury motor (cf. Art. 18) acting on the same principle (Fig. 7.26).

There is no compensation for friction, hence the meter under-registers at small loads, but the error is under 2 per cent. at of full load.

18. The Ferranti-Hamilton Meter

This (Fig. 7.27) is of the mercury motor ampere-hour type.

The mercury chamber is formed by two brass plates A, A, bolted together with an intermediate fibre ring B. The brass plates have presspahn on their inner surfaces, thus producing an insulated and leakage-proof chamber. Current is led into the mercury by the contact C, which passes through the fibre ring. It then enters a copper disc D, which it leaves at the centre by the supporting contact E. The disc is platinum-plated and enamelled to protect it from the mercury. The edge and a small portion at the centre are, however, left unplated, but are amalgamated so as to ensure good contact with the mercury.

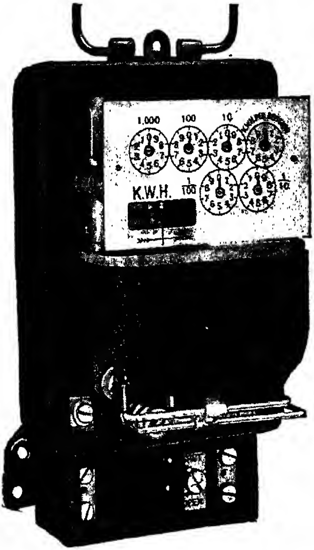


Fig. 7.26.—TYPE D.M. AMPERE-HOUR METER (COVER REMOVED)

Two mild steel pole pieces, NS, NS, are forced into each of the brass plates and are rolled over to form a tight joint. Two C-shaped permanent magnets are clamped to these between the mild steel bars F, F, and brass bars placed behind the magnets (not shown in the figure).

When current passes through the disc it is driven as a motor because of the effect of the field of the right-hand magnet. The

left-hand magnet sets up eddy currents in the disc and so opposes the motion. Thus the rate of revolution is proportional to the current.

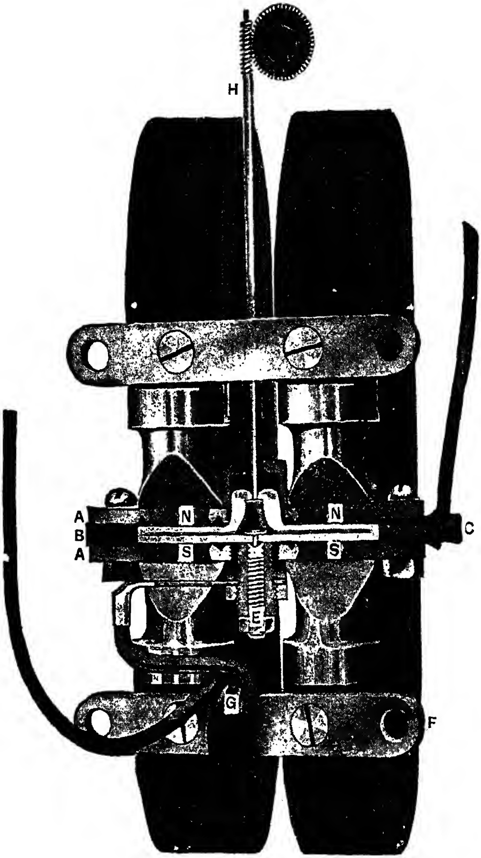


Fig. 7.27.—FERRANTI MERCURY MOTOR METER.

Owing to mercury friction the speed would not increase in the proper proportion for large currents, but for the effect of the compensating coil, G. The current passes through this after leaving

the disc, and the effect is to increase the strength of the right-hand poles, *i.e.* of the driving field, and to weaken the left-hand ones, *i.e.* the retarding field, by an amount proportional to the current. The speed is thus kept proportional to the current within .1 per cent. up to full load.

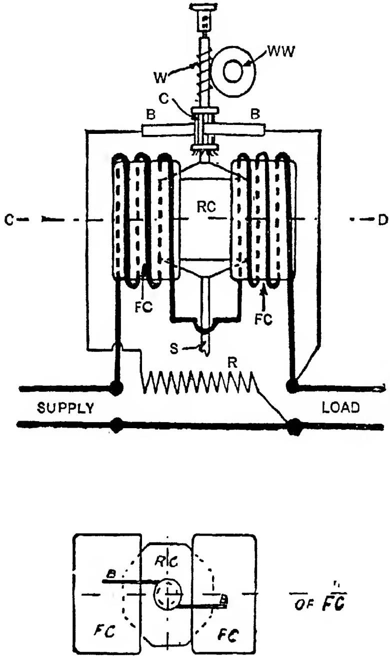


Fig. 7.28.—ELIHU THOMSON MOTOR METER.

R, Series resistance. S, Spindle. W, Worm. W W, Wormwheel.

The disc is attached to a spindle, H, on which is cut the worm which drives the recording train. The weight of the disc with its attached parts is adjusted so that it just sinks in the mercury, and

thus keeps the friction on the lower bearing at a very small value. This is important because there is no compensation for bearing friction. The bearings at each end of the spindle are jewelled. Three small nuts are provided on the spindle so as to balance it and avoid side pressure on its bearings. One of these is painted white to facilitate testing (see Art. 34).

19. The Elihu Thomson Meter

This meter is of the commutator motor watt-hour type. It is shown in Fig. 7.28. The main current is passed through the coils, FC, of copper strip, thus producing a horizontal field. In this rotates an armature, RC, of eight coils of fine silk-covered copper wire wound on a light non-magnetic octagonal frame. These are connected together (see Fig. 8.10) and to an 8-part silver commutator, C. Current is taken in and out of the armature by two silver-tipped brushes, B, B. Silver is employed to reduce brush friction.

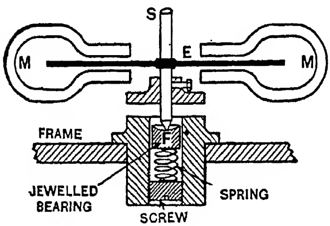


Fig. 7.29.—BRAKE AND SPINDLE SUPPORT.

E, Copper disc attached to spindles.
MM, Permanent magnets.

A compensating coil of fine wire is wound inside one or both of the coils FC. This is in series with the armature and a high resistance, across the mains. The object of the compensating coil is to exert a torque equal to that due to friction, independently of the torque due to the current. If the friction torque were constant it could be exactly compensated for in this way on a steady voltage; but it is greater at starting than when the armature is running, and it is also dependent on the amount of vibration. Consequently the compensation is insufficient at starting, and at very small loads the meter does not register.

The amount of compensation can be altered slightly by changing the position of the compensating coil inside the main coil. The registration as a whole can be changed by shifting the brake-magnets (Fig. 7.29), which are clamped to the base by two screws passing through slotted holes.

The resistance of the armature is about 600 ohms and the armature current is about ·05 amp., less in the smaller sizes. The series resistance is necessarily varied according to the supply voltage.

20. Maximum Demand Indicators

These indicators are required in certain systems of charging for electrical energy (see Chapter XVIII.). Their object is to register the maximum power (kilowatts) taken at any time during a period. Two main classes of such instruments are possible (cf. Art. 14)—

A. Maximum power indicators.

B. Maximum current indicators.

All the indicators in use, except the Merz indicator (Art. 21), are in effect wattmeters or ammeters with some device for recording the maximum current or the maximum power taken. The instruments are always made sluggish in action so as not to record momentary large currents, *e.g.* those due to the starting of motors or to short circuits.

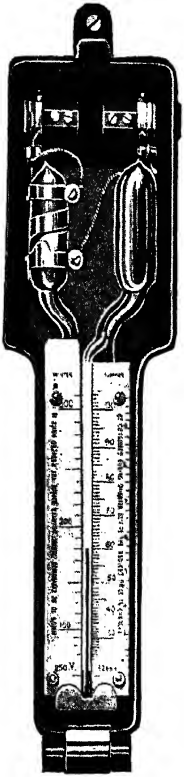


Fig. 7.30.—REASON CO.'S THERMAL TYPE OF DEMAND INDICATOR (WRIGHT'S SYSTEM).

The Wright demand indicator is illustrated in Fig. 7.30. Its principle is that of the differential thermometer. It consists of a glass U-tube containing a hygroscopic and somewhat viscous liquid (*e.g.* strong sulphuric acid). This terminates in bulbs of equal capacity, containing air which is kept dry by the hygroscopic nature of the liquid. The left-hand bulb has wound round it a spiral or strip of a high resistance alloy through which the main current passes. A narrower tube, called the reading tube, branches off from the top of the right-hand limb of the U-tube.

When current passes it gradually warms the air in the left-hand bulb, causing it to expand and force the liquid down on that side so that it overflows into the reading tube. If the current diminishes the liquid falls in the right-hand limb, but that in the reading tube remains there. A fresh increase of current makes no alteration until the liquid has been forced lower on the left side than at the previous time. Thus the amount of liquid in the reading tube depends on the maximum current sent through the heating coil. This current can be read off on a scale placed behind the reading

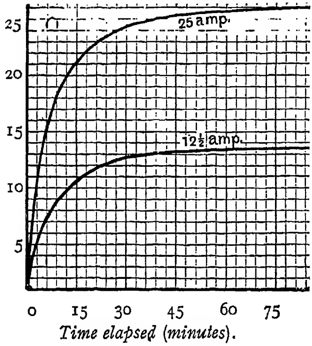


Fig. 7.31.—READINGS OF REASON DEMAND INDICATOR.

tube, which scale also gives the corresponding kilowatts at the supply voltage.

When greater sluggishness is required a cast-iron cylinder is sealed into the left-hand bulb, and protected from the air in it by a covering of glass. For still greater sluggishness a cast-iron hollow cylinder is interposed between the bulb and the resistance strip, and is insulated from the latter.

The extent of the sluggishness in attaining its full reading is shown by the curves in Fig. 7.31, drawn from readings for the intermediate pattern with steady currents flowing of 25 A. (full load) and $12\frac{1}{2}$ A. respectively. To reset the instrument it is tilted about the hinges, so as to empty the reading tube.

21. The Merz Demand Indicator

This indicator can be attached to any motor having a recording train of wheels, *i.e.* any motor meter or the Aron meter. It indicates the maximum watt-hours taken during any one hour (or other period) within the time it is in use.

It consists of a wheel A (Fig. 7.32) driven through change wheels from some convenient wheel of the meter counting train. This carries a pawl B and a spring C through which it drives the wheel D, which is mounted on a sleeve on A's spindle. On the same sleeve is another wheel E with a driving pin F. This pin engages with a similar pin G at the back of a further wheel H, on the front of whose spindle the pointer P is mounted. The spindle of H is in line with that of A and in front of it, but they are shown separately in the figure for clearness.

A pawl K engages with H so as to prevent the pointer moving back when the resetting mechanism works. This consists of a toothed quadrant Q, engaging with the wheel E; a resetting arm R with a roller on the end, actuated by a helical spring S; and a cam T. The cam is driven by clockwork so as to make one revolution in an hour (or other desired period), and is free to move ahead of the wheel driving it. Thus once every hour R is released

by the cam and is caused by the spring S to reset immediately Q and the wheels D and E, the pin F being brought against a stop so as to ensure accurate setting to zero.

During the subsequent hours the pointer will not be moved unless a larger amount of energy (kWh.) is used than in any previous hour. Thus the reading at any time indicates the maximum energy used in any one hour from the start, and heavy loads of short duration are averaged over the hour. At the end of the month or quarter the reading is noted, and the pointer then set to zero by hand after, releasing it by raising the pawl K by means of the knob W.

The change wheels are so chosen that the pointer P will make nearly a complete revolution if full load lasts throughout the hour.

The cam is shaped so that with continuous full load R is kept just ahead of Q, thus the resetting mechanism introduces no friction.

This demand indicator is used chiefly in connexion with "bulk" supply, viz. supply of energy by one company or municipality to another, which latter distributes it to the consumers in its own district.

Its main disadvantage is that it requires careful adjustment to ensure satisfactory working.

22. The Potentiometer

The potentiometer in its modern form is employed widely for the measurement of resistances, the calibration of ammeters, voltmeters, and wattmeters, and in connexion with electrical pyrometry.

As its name implies it is essentially an instrument for the measurement, or more strictly the comparison, of potential differences. With the assistance of suitable accessories it can, however, be adapted to the measurement of resistances, currents, and D.C. electric power.

The supply of current for the various tests is best obtained from accumulators. They have the advantage of supplying a current which alters only very slowly if the cells are in good condition.

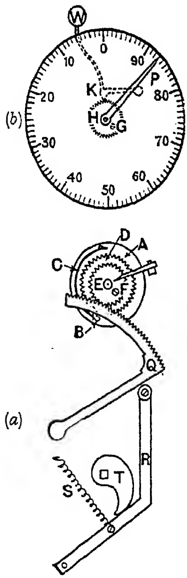


Fig. 7.32.—MERZ DEMAND INDICATOR.

(a) Mechanism.
(b) Recording dial.

23. Principle of the Potentiometer

In Fig. 7.33 let AB be a uniform wire stretched on a scale, and let a constant current flow through it. Then the P.D. between any two points C and D in AB is proportional to the length CD. For this P.D. is the product of the current and the resistance of CD; the former is constant, and the latter is proportional to the length of CD, since the wire AB is uniform.

Now let a cell, of E.M.F. E, be connected by key T through a sensitive galvanometer G to the points C and D by sliding contacts. Adjust the positions of C and D till no deflection of the galvanometer is produced on closing K: this is the position of " balance." In this position the E.M.F. of the cell must be equal to the P.D. between C and D, otherwise a current would flow through the galvanometer in a direction dependent on which of the two were the greater. A second cell, of E.M.F. E′, may then be connected to

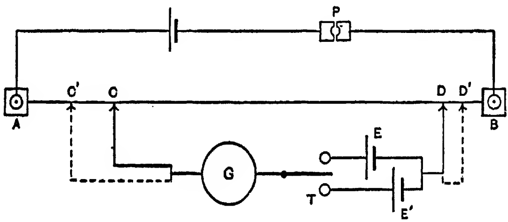


Fig. 7.33.—PRINCIPLE OF POTENTIOMETER.

A B, Wire on scale between terminal blocks. C C′, Two positions of sliding contact maker. D D′, Ditto for second sliding contact maker. E, Standard cell. E′, Cell under test. P, Plug key. T, Two-way switch.

the galvanometer by the other contact of T, and balance again obtained by moving C or D or both, *e.g.* to C′ and D′. Thus:—

E.M.F. of first cell = " drop " from C to D,

„ of second cell = „ „ C′ to D′,

and the ratio of the two drops equals the ratio $\frac{CD}{C'D'}$ provided the current in the wire has not altered between the times of obtaining balance in the two cases. This can be tested by repeating the first observation;

$$\therefore \frac{E}{E'} = \frac{CD}{C'D'}.$$

Consequently if E is known (*e.g.* if it is the E.M.F. of a standard cell) E′ can be calculated. *E.g.*—

Clark standard cell at 15° C., E.M.F. = 1·433 volts.
Length (CD) for balance on potentiometer = 77·8 cm.
Daniell cell, length (C′D′) for balance on potentiometer = 58·3 cm.;

$$\text{E.M.F. of Daniell cell} = 1{\cdot}433 \times \frac{58{\cdot}3}{77{\cdot}8} = 1{\cdot}074 \text{ volts.}$$

It should be noted that no current is flowing through a cell when its point of balance is obtained, consequently it is the whole E.M.F. of the cell and not merely the P.D. at its terminals which is measured. Thus the internal resistance of the cell does not affect the accuracy of the result.

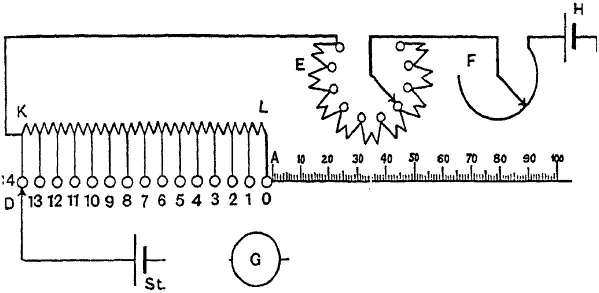


Fig. 7·34—CONNEXIONS OF MODERN FORM OF POTENTIOMETER.
E, Rough rheostat. F, Fine rheostat. G, Galvanometer. H, Accumulator.
KL, 14 equal resistances.

24. Modified Forms of the Potentiometer

Modifications in the form of the potentiometer have been introduced (*a*) to increase its accuracy, (*b*) to simplify its use.

The first step was to increase the length of the slide wire to four or even seven metres, thus reducing the percentage inaccuracy for a given error in obtaining the length. Since it is almost impossible for so long a wire to be quite uniform it had to be calibrated along its whole length, a very long and tedious process. The wire was next split up into a number of sections, only one of which required calibrating along its whole length, the resistance of the whole section alone requiring adjustment in the other cases.

In Fig. 7·34 AB represents the slide wire placed over a scale with 100 divisions (usually subdivided into tenths). In series with

this is a coil of the same wire divided into a number (say 14) of sections connected to contact studs 0, 1, 2, . . . , 14. The resistance of each section is adjusted accurately equal to that of 100 scale-divisions of the slide wire. Two contact makers are used, one, C, on the slide wire, the other, D, on the studs. In this way any length of wire up to 1500 divisions can be used to balance the cell E.M.F. or other voltage under test. Note that the zero is placed at the end of the slide wire which is joined to the other sections, and not at either end of the whole arrangement; this makes the scale direct reading; for instance, in the figure it reads 1435 divisions.

The length of the slide wire has been reduced from a metre to 25 in. or 20 in. for greater compactness, and where very high accuracy is not required to 14 in. or less. Hence the "divisions"

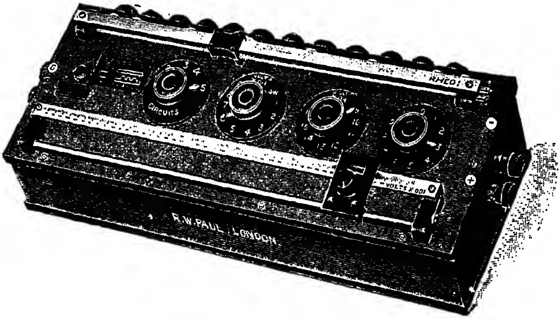


Fig. 7.35.—CROMPTON TYPE POTENTIOMETER.

are no longer centimetres: their exact length is immaterial provided they are equal and of convenient size.

The second modification consists in adding a rheostat in series with the slide wire and sections. This has two portions, one in sections connected to contact studs for rough adjustment, the other a slide wire for exact adjustment: the resistance of this slide wire is a little more than that of each of the sections. The object of this is to adjust the current supplied by the accumulator so that balance is obtained with the standard cell at a reading equal to 1000 times its E.M.F. *E.g.* with a Clark cell at 13° C. the contacts would be set at 1435 (as shown in Fig. 7.34) and balance obtained by altering the rheostat *and not moving the contacts C and D.* This is known as "standardizing the potentiometer." The unknown cell would then have its E.M.F. balanced by moving C and D

leaving the rheostat unchanged. If balance is obtained at 1074, then—

$$\text{Unknown E.M.F.} = 1{\cdot}435 \times \frac{1074}{1435} = 1{\cdot}074 \text{ volts,}$$

i.e. the potentiometer becomes direct reading, each division representing ·001 volt, and calculation is unnecessary (cf. Art. 23).

25. Modern Potentiometer

An actual *modern potentiometer* is shown in Fig. 7.35. This is the Crompton type made by R. W. Paul.

The 17 slide wire coils, the rough adjustment rheostat, and the

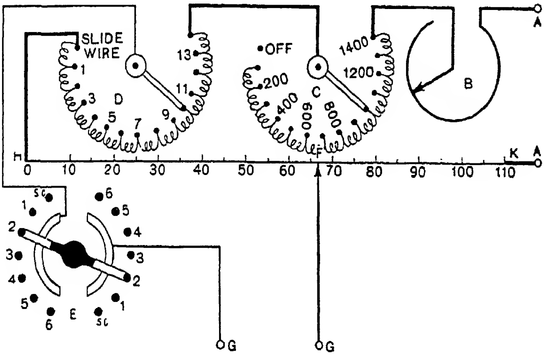


Fig. 7.36.—CONNEXIONS OF CROMPTON TYPE POTENTIOMETER.

AA, Accumulator terminals. B, Fine rheostat. C, Coarse rheostat (regulator). D, Potential switch. E, Multiple circuit switch. F, Contact maker. HK, Slide wire. GG, Galvanometer terminals.

connexions are mounted in a mahogany box $26\frac{1}{2}$ in. × 10 in. × 3 in. deep.

There are 8 pairs of terminals mounted on ebonite at the far side and two ends for connecting respectively to the galvanometer, the Clark or other standard cell, five testing circuits, and the accumulator.

By means of the multiple switch (marked "Circuits" in the figure) any pair of terminals can be connected respectively through the galvanometer to the slide wire contact maker, and to the "potential" switch. (See Fig. 7.36.)

The "rheostat" switch alters the resistance in the accumulator circuit, and along the far side is the continuously adjustable fine rheostat.

The connexions are thus equivalent to those shown in Fig. 7.34, with the addition of the multiple switch, which allows balance to be obtained with two or more potentials without any change of connexions beyond moving this switch.

The slide wire is protected by a metal bar of angle section on which the scale is marked. The sliding contact maker is adjusted by hand. It slides on the scale and on a round bar which connects it to the galvanometer. Contact is made by pressing a knob, and by turning this slightly contact is maintained.

All the switches have ebonite covers with ebonite knobs outside.

At the left-hand end is a shunt and short key for the galvanometer. When on " shunt " a resistance is connected in parallel with the galvanometer to avoid excessive deflexions in the first attempts to find the balancing point. On moving the switch to " short " the shunt is disconnected, and by pressing the top ebonite knob the galvanometer is short-circuited. This is useful in finally settling the position of balance.

26. Voltage and Voltmeter Tests

The method of measuring cell E.M.F.s, or other P.D.s, up to 1·5 volts has been already described in Arts. 23, 24. When higher voltages are to be measured a **ratio box** or **volt box** is required. This consists of a high resistance (30 ohms or more per volt) accurately subdivided. Its principle is that if a P.D. of E volts is maintained across a coil AB of R ohms resistance (see Fig. 7.37), then the P.D. across any portion, CD, of this coil is $\frac{E}{n}$ volts if the resistance of CD is $\frac{R}{n}$ ohms; *i.e.* the P.D.s are in the same ratio as the resistances. This will be true only provided no current is taken from the points C and D, which is the case when a balance is obtained with the potentiometer.

In use, A and B are connected to the two points between which the P.D. is required, and C and D are connected to a pair of test terminals on the potentiometer. The P.D. between C and D is measured by comparison with a standard cell, and that between A and B obtained by multiplying this value by the known ratio of the resistances of AB and CD. This ratio is always made a simple number. Thus if the resistance of AB is 10 000 ohms and that of CD 50 ohms, giving a ratio of 200, P.D.s up to 300 volts ($1{\cdot}5 \times 200$) can be measured. *E.g.* if the P.D. across CD is found by means

of the potentiometer to be 1·287 volts, that across AB must be $200 \times 1{\cdot}287 = 257{\cdot}4$ volts.

The resistance may be subdivided further so as to increase the accuracy at lower voltages, *e.g.* the 10 000 ohm resistance, AB, for 300 volts, may have further tappings at points such as E in Fig. 7.37. Thus if the resistance of AE is 2500 ohms the ratio is 50, and this is suitable up to 75 V. The leads to the potentiometer are taken from C and D in every case. The exact values of these resistances are unimportant provided their ratios are accurate.

In calibrating a voltmeter a suitable ratio box is required, and a supply at a voltage which can be adjusted up to the full range of the voltmeter. This is best obtained from a number of small accumulators in series, since the P.D. will then be practically constant during the test: its value may be varied by altering the number of cells in use or by connecting a high variable resistance in series with the voltmeter. The connexions for the latter method are shown in Fig. 7.37.

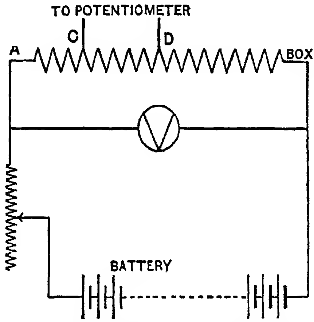


Fig. 7.37.—CALIBRATION OF VOLTMETER BY POTENTIOMETER.

The reading on the voltmeter is adjusted to some exact value. The P.D. across the potentiometer coil, CD, is measured by the potentiometer in the usual way, and multiplied by the known ratio. The difference between the P.D. across AB thus obtained and the actual reading of the voltmeter is the correction required for that reading. This process enables the corrections at all parts of the scale to be determined.

27. Resistance Measurement

Measurements of resistance by the potentiometer are made by connecting the unknown resistance R_x in series with a standard resistance R_s, and sending a suitable current through them. The P.D.s across the two resistances are compared by the potentiometer, and the ratio of these P.D.s gives the ratio of the resistances. For if I is the current through the two resistances, then

$$\frac{\text{P.D. across unknown resistance}}{\text{P.D. across standard resistance}} = \frac{IR_x}{IR_s} = \frac{R_x}{R_s}.$$

The connexions for such a test are shown in Fig. 7.38. Two accumulators in series are used for supplying the current; they must be large enough to supply it without any approach to overloading. A variable resistance, whose value need not be known, is connected in series with the others for the purpose of adjusting the current.

The procedure may be according to either of the following methods:—

(A) If the potentiometer has been "standardised" by a standard cell and it is desired not to change this,

(1) Set the potentiometer to figures which are a simple multiple of the standard resistance in ohms, *e.g.* with a 0·1 ohm standard set it to 1 000.

(2) Obtain balance on terminals *aa* by altering the variable resistance in the resistance circuit (see Fig. 7.38).

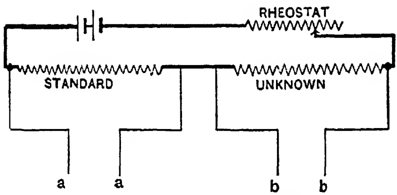


Fig. 7.38.—MEASUREMENT OF RESISTANCE BY POTENTIOMETER.

(3) Obtain balance on terminals *bb* by moving the contact maker and potential switch (see Art. 25) of the potentiometer; the position at which balance occurs gives the value of the unknown resistance directly, *e.g.* if balance is obtained at 1378 after setting as in (2), then 0·1378 ohm is the value of the unknown resistance.

(B) If the potentiometer has not been standardised proceed as in (A), but the first balance may be obtained by altering the potentiometer rheostat alone or in conjunction with the variable resistance.

28. Standard Resistances

The standards used are 1 ohm, 0·1 ohm, 0·01 ohm, 0·001 ohm, etc., and are constructed to carry without undue heating currents of 1·5 A., 15 A., 150 A., 1500 A., etc., so that the drop with the full current flowing is 1·5 volt in every case. The one used should be that with its value as close as possible to the unknown resistance. Higher resistances may be used as standards, but other methods then become available (see Art. 29).

A standard resistance for 150 amperes is shown in Fig. 7.39. It consists of a strip of manganin about 2 in. × $\frac{1}{16}$ in., which is enamelled to protect the metal from oxidisation, etc.

The current is led in and out by massive brass terminals.

The potential terminals are mounted on ebonite washers and connected to points near the edge of the strip as shown in Fig. 7.39.

The final adjustment of the resistance is made by cutting at "*a*," Fig. 7.39, till the correct drop of 1·5 volts is obtained with

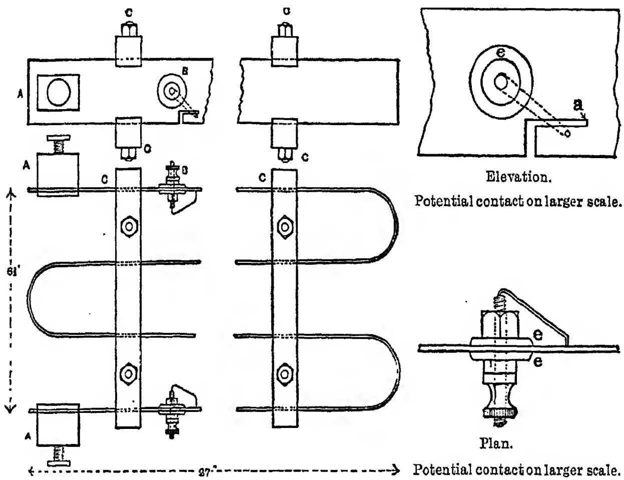


Fig. 7.39.—STANDARD RESISTANCE FOR POTENTIOMETER.

AA, Current terminals. BB, Potential terminals. cc, Wooden clamps with brass bolts. ee, Ebonite washers.

150 amperes flowing. The resistance between the current terminals is therefore slightly over ·01 ohm.

29. Drop of Potential Method for Resistance Measurement

The potentiometer method of measuring resistances is by drop of potential, but the name is applied generally to a simpler method. In this latter the unknown and standard resistances are connected in series to a suitable source of current just as in the potentiometer method. Instead of comparing the drops of potential across the two resistances by the potentiometer a galvanometer is used.

This is connected across the two resistances in turn, and the deflexions noted. The ratio of these deflexions is equal to the ratio of the P.D.s across the two resistances, and is therefore equal to the ratio of the resistances themselves, or

$$\frac{R_x}{R_s} = \frac{d_x}{d_s},$$

where R_x = unknown resistance in ohms,

R_s = standard resistance in ohms,

d_x = deflexion of galvanometer when connected to unknown resistance,

d_s = deflexion of galvanometer when connected to standard resistance.

The galvanometer must be of high resistance or have a high resistance in series with it. Otherwise the accuracy of the test is liable to be diminished by the shunting effect of the galvanometer.* For the same reason this test is most suitable for low resistances, with which this shunting action is small. The less the difference between the two resistances the less the inaccuracy from this source, because the shunting effect is then nearly the same with each of the resistances.

The standard resistance used should therefore have a value close to that of the one to be measured.

Another possible source of error is that the deflexions of the galvanometer may not be exactly proportional to the P.D.s applied. This is another reason for using a standard nearly equal to the unknown resistance.

Both the above sources of inaccuracy are absent from the potentiometer method, but it is more complicated. Consequently the above method is often employed for low resistances when great accuracy is unnecessary.

30. Wheatstone's Bridge

Wheatstone's bridge is an arrangement for measuring an unknown resistance by means of three known resistances.

* *I.e.* when the galvanometer is connected to either resistance it takes part of the current and so diminishes the drop across that resistance, or, in other words, the resistance of the galvanometer and resistance together is less than that of the resistance alone. It can be shown, however, that if the battery resistance is negligible the above relation of resistances to deflexions is still true.

The connexions are as shown in Fig. 7.40. R_1, R_2, R_3 are known resistances, R_x is the resistance to be measured. One or more of the known resistances are adjusted till no deflexion of the galvanometer occurs on closing the battery and galvanometer keys. When this is the case

$$\frac{R_x}{R_3} = \frac{R_2}{R_1} \text{ or } R_x = R_3 \times \frac{R_2}{R_1}.$$

The truth of this relation can be proved as follows:—

Let I = current through R_1, which is also the current through R_3 since no current goes through the galvanometer,

and let I' = current through R_2 = current through R_x.

Now B and C are at the same potential, otherwise a deflexion of the galvanometer would occur.

Therefore Drop across AB = Drop across AC,

or $$IR_1 = I'R_3,$$

and Drop across BD = Drop across CD,

or

$$\frac{I'R_x}{}$$

The battery and galvanometer connexions may be interchanged without affecting the result, for then

$$\frac{R_x}{R_2} = \frac{R_3}{R_1}, \quad \text{or} \quad R_x = R_2 \times \frac{R_3}{R_1},$$

which is the same value for R_x as before.

There are two varieties of apparatus employed in applying this principle. One is the *metre* (or half metre) *bridge* shown in Fig. 7.41. AD is a uniform wire of platinoid or some similar alloy, stretched on a metre scale. Copper straps and terminals are provided for making the various connexions. S is the standard resistance, R the unknown one. B is a sliding contact by which the galvanometer can be connected to any point of the wire AD.

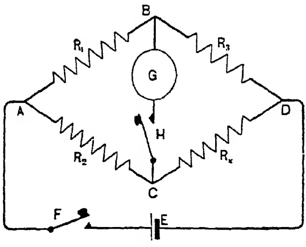


Fig. 7.40.—WHEATSTONE'S BRIDGE.

The letters A, B, C, D are placed at corresponding points in Figs. 7.40 and 7.41.

When balance is obtained—

$$R = S\,\frac{}{\text{Res. of AB}} = S \times \frac{\text{. of BD}}{\text{Length of AB}},$$

and these two lengths can be read on the scales.

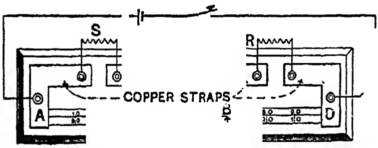


Fig. 7.41.—THE METRE BRIDGE.

The two resistances should not differ widely, otherwise an error in the scale reading, or in the coincidence of scale ends and wire ends, makes a large percentage error in the calculated resistance. This method is suitable for resistances between about ten ohms and half an ohm.

31. Post Office Box

The second type of apparatus used is the Post Office box. This consists of three variable resistances and two keys, connected together as shown in Fig. 7.42.

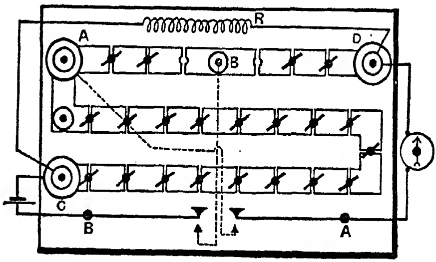


Fig. 7.42.—CONNEXIONS OF POST OFFICE BOX

R = Resistance to be measured.

The resistances consist of a series of coils connected to massive brass blocks (see Fig. 7.43). These can be short-circuited by means of brass plugs with ebony handles, or put into circuit by withdrawing the plugs. Two of the resistances, viz. AB and BD, each consist of three coils of 1 000 ohms, 100 ohms, and 10 ohms resistance

respectively. The third, AC, can be adjusted to any exact number of ohms up to 11 110 ohms (or sometimes 111 110 ohms). When balance is obtained—

$$R = \text{Resistance of AC} \times \frac{\text{Resistance of BD}}{\text{Resistance of AB}}.$$

The latter ratio is always some power of 10 (*e.g.* 100, $\frac{1}{10}$, etc.), thus no calculation is necessary beyond the adding of the resistances in use in AC, and the placing of the decimal point. The P.O. box is therefore practically direct reading.

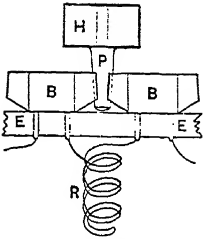


Fig. 7.43.—ARRANGEMENT OF RESISTANCE COIL IN POST OFFICE BOX.

BB, Brass blocks. EE, Ebonite base. H, Ebonite handle. P, Brass plug. R, Resistance coil.

It is suitable for resistances from 1 megohm down to about 5 ohms. For lower resistances the methods described in the previous two paragraphs are preferable. For higher resistances see Chapter III., Art. 17.

The drawback of this method when used for low resistances is that the contact resistances of the plugs and other connexions become a considerable percentage of the resistance measured. These contact resistances do not affect the accuracy of the drop of potential method appreciably, nor that of the potentiometer method at all.

32. Current Measurement by the Potentiometer

This is effected by means of the standard resistances described in Art. 28. The current to be measured is passed through the appropriate standard resistance, and the drop across this measured in volts by the potentiometer. In this case the latter must be standardised by means of a standard cell.

Then the current in amperes

$$= \frac{\text{Drop across resistance in volts}}{\text{Ohms in standard resistance}}.$$

When an ammeter is to be calibrated it is connected in series with the corresponding standard resistance and an (unknown) variable resistance and supplied with current by two or three accumulators (see Fig. 7.44). No more are required

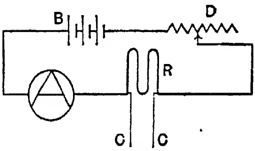


Fig. 7.44.—CALIBRATION OF AMMETER BY POTENTIOMETER.

A, Ammeter. B, Battery. C C, Leads to potentiometer. D, Variable resistance.

unless the current is too large for a single accumulator, in which case several may be connected in parallel.

The ammeter reading is adjusted to some exact value by means of the variable resistance, and the accurate value of the current obtained as above by the potentiometer, and the values compared. A series of such tests is made as in the case of a voltmeter.

33. Power Measurement by the Potentiometer

This test is merely a combination of current and voltage measurements, using the potentiometer in each case.

The standard resistance is connected so as to carry the whole of the load current, and the ratio box is connected in parallel with the load (see Fig. 7.45). Leads are taken to two pairs of potentiometer terminals, and the current and voltage measured one immediately after the other. The product of the two gives the power supplied to the load. This method is suitable for direct currents only, cf. Chap. V., Art. 9.

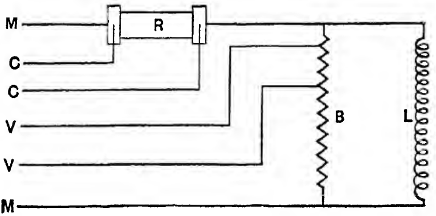


Fig. 7.45.—POWER MEASUREMENT BY THE POTENTIOMETER.

B, Volt box. C C, Potentiometer leads for current measurement. L, Load circuit. M M, Mains. R, Standard resistance. V V, Potentiometer leads for voltage measurement.

The calibration of a wattmeter may be effected by using it to measure a steady load and comparing its reading with the watts measured by the potentiometer, provided the wattmeter is of the dynamometer type (cf. Art. 10). It is preferable, except for small powers, to use a "dummy load," which saves power and reduces the amount of regulating resistance necessary.

The current coil of the wattmeter is supplied with an adjustable current, by means of two or three accumulators of large capacity and a variable resistance (D and F, Fig. 7.46). The same current is sent through a suitable standard resistance so that it can be measured on the potentiometer. The voltage coil of the wattmeter is supplied separately by a large number of small accumulators in series, a variable resistance is used if it is desired to adjust the voltage to an exact value. The ratio box is connected in parallel

with the voltage coil, so as to measure by the potentiometer the P.D. on this coil.

The saving effected by a dummy load can be seen by taking an example. Let the full capacity of the wattmeter be 50 amperes at 200 volts, *i.e.* 10 kilowatts. Then if the current coil is supplied by 3 accumulators giving 6 volts, and if the voltage coil takes ·04 ampere, the total power used is $6^{v} \times 50^{A} + 200^{v} \times \cdot 04^{A} = 308$ watts. The wattmeter (if correct) will read $50 \times 200 = 10\,000$ watts, and if loaded in the ordinary way would require further the additional 8 watts used by the voltage coil. Thus the power saved is $10\,008 - 308 = 9700$ watts.

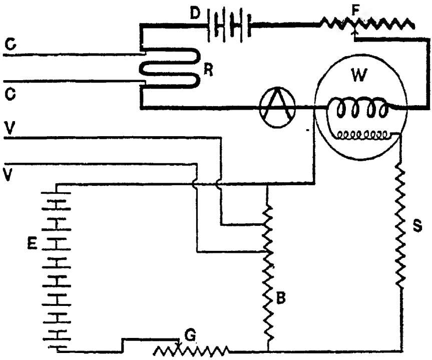


Fig. 7.46.—WATTMETER CALIBRATION BY THE POTENTIOMETER WITH A "DUMMY" LOAD.

B, Volt box. CC, Potentiometer leads for current measurement. R, Standard resistance. S, Series resistance of wattmeter (often inside instrument case). VV, Potentiometer leads for voltage measurement. W, Wattmeter.

34. Ampere-Hour Motor Meter Testing

Two tests are generally made on meters of this type—one for proportionality, the other a "dial test."

The object of the first is to test whether the rate at which the meter motor runs is accurately proportional to the current flowing through the meter. For this purpose the meter is connected, in series with an accurately calibrated ammeter and an adjustable resistance, to suitable accumulators. The current is then adjusted to the meter's full load and the time of revolution of the motor determined by a stop-watch. This is repeated at other loads, say $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, and $\frac{1}{20}$ of full load. The time of revolution should increase inversely as the current, and the variation from this can be determined.

To avoid calculation the timing is done as follows. At full load the time taken to make a convenient multiple of 20 revolutions is observed; at half load the time for half this number of revolutions,

and so on for the other loads. If the meter is accurate all these times will be equal, consequently the amount of their variation shows how much the meter departs from proportionality.

When a meter has successfully passed this test it then undergoes the *dial test.* This consists of sending the full load current through it steadily for several hours, and reading the dial before and after. The number of units registered is then compared with the number calculated from the current, the time during which it has been flowing, and the voltage for which the meter is intended. (See Example 4.)

Any number of meters of the same capacity (*i.e.* same full load current) may be tested simultaneously by connecting them all in series. This saves time in adjusting the current.

Instead of using a calibrated ammeter the current may be measured directly by the potentiometer with a standard resistance.

Example 4. *An ampere-hour meter has 25 amperes sent through it for three hours. It registers 17·5 units during this time. If it is intended for a 230-volt circuit, what is its error?*

The meter should register $\dfrac{25 \text{ amp.} \times 230 \text{ volts} \times 3 \text{ hours}}{1000}$ kWh.

$$= 17{\cdot}25 \text{ kWh.};$$

$$\therefore \text{ error} = 17{\cdot}5 - 17{\cdot}25 = +\,{\cdot}25 \text{ kWh.};$$

$$\text{percentage error} = +\frac{{\cdot}25}{17{\cdot}25} \times 100 \qquad 1{\cdot}45 \text{ per cent.}$$

35. Watt-Hour Meter Testing

Motor meters of this type are tested much as described above (Art. 34). Their pressure coils must, however, be supplied at the proper voltage and this voltage measured, most simply by a calibrated voltmeter. The supply to the current coils and that for the voltage coils are obtained from separate sources, *i.e.* a "dummy load" is used (see Art. 33).

The proportionality test is then applied as in the case of an ampere-hour meter, the voltage being kept constant and the current varied so as to obtain the desired loads. Afterwards a dial test is applied with a dummy full load.

If a number of meters are tested simultaneously with dummy loads, the voltage and current coils of all (or all but one) must be disconnected from one another (see Fig. 7.47). Otherwise the current coil of the first meter carries the current used by the voltage coils of all the rest, in addition to that recorded by the ammeter.

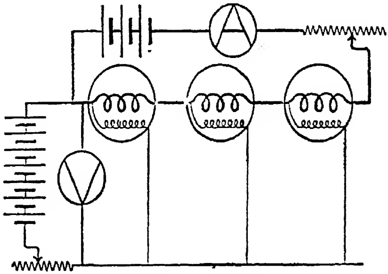
Both the current and voltage measurements may be made directly with a potentiometer if desired. It is more convenient to use an ammeter and a voltmeter, and to calibrate them from time to time.

36. Electrolytic Meter Testing

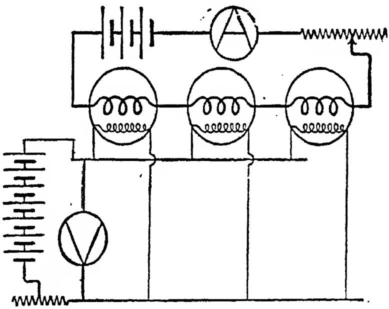
These meters can be tested only by long runs with steady currents. This is usually done at full load and at one other load (say $\frac{1}{3}$).

In the shunted type (see Art. 14) current could be saved by testing the electrolytic cell without its shunt. This would not be a satisfactory test, because the main source of error in this type is the variation in the fraction of the total current sent through the electrolytic cell.

The tediousness and large ampere-hour consumption of these tests is counterbalanced by the fact that such meters need be tested only at very long intervals. In the unshunted type a short circuit, complete or partial, is almost the only possible source of error in an originally correct meter.



Incorrect connexions.



Correct connexions.

Fig. 7.47.—SIMULTANEOUS TESTING OF WATT-HOUR METERS.

The limit of error for all types of meter allowed by B.S.S. No. 37 is $\pm$ 2 per cent. from $\frac{1}{10}$ load to $1\frac{1}{4}$ times full load when this exceeds 10A for D.C. meters, and for all sizes of A.C. meters which carry the whole current. For smaller D.C. meters, and for A.C. meters using external shunts and transformers $\pm$ $2\frac{1}{2}$ per cent. is allowed over the same range. The latter error is not to be

exceeded down to $\frac{1}{20}$ full load by the larger D.C. meters, and by all types of A.C. meters on loads of unity power factor.

QUESTIONS ON CHAPTER VII

1. A milliammeter of 3 ohms resistance reads up to 150 milliamperes. What resistance is necessary to enable it to be used (*a*) as a voltmeter reading up to 15 volts, (*b*) as an ammeter reading up to 30 amperes?

Give a diagram of connexions in each case.

2. An ammeter of 5 ohms resistance gives full scale deflexion with 0·05A.

What resistances must shunts have for ranges of 1A, 5A, and 20A respectively?

What is the resistance of the shunted ammeter in each case?

Why are the shunts made of strip, or of a number of wires in parallel?

3. What are the means employed to increase the range of instruments for measuring voltage and current? [C. & G., II.

4. What advantages has the soft-iron type of ammeter? Describe an instrument of this type.

5. Sketch the details of a moving coil ammeter, explain its action, and state its advantages and disadvantages.

6. Describe some form of ammeter suitable for a switchboard. State whether the deflexion of the pointer will be proportional to the current or not, and give reasons for your answer. [C. & G., I.

7. Why can ampere-hour meters be used to obtain the amount of energy supplied? State the relative advantages of these and of watt-hour meters.

8. An ampere-hour meter calibrated at 210 volts is used on a 230-volt circuit, and indicates 732 units in a certain period: what is the actual amount of energy supplied? If current has been taken during 200 hours, what is its average value?

9. Describe, with sketches, a meter of the motor type, and explain the method adopted to minimise the inaccuracy caused by friction.

10. Explain the advantage of making the compensating effect in a mercury motor meter less than the liquid friction.

11. Sketch and describe a maximum demand indicator, explaining its method of action and what it measures.

12. A maximum demand indicator on a 220-volt supply indicates 7A, the meter shows 360 kWh. consumption.

Calculate the quarter's (91 days) bill if the tariff is $4\frac{1}{2}$d. per unit for the first $1\frac{1}{2}$ hr. of maximum demand, and $2\frac{1}{2}$d. per unit after.

What is the equivalent flat rate in this case?

13. Sketch and give a diagram of connexions of an accurate potentiometer; and describe briefly its use for measuring the E.M.F.s of cells.

14. Show how to calibrate a voltmeter, reading to 150 volts, by means of a potentiometer and a standard cell. Give a full diagram of connexions and a list of apparatus. Upon what does the accuracy of the test depend?

15. Describe one good method for measuring low resistances.

16. What is the range of resistance measurement of the ordinary P.O. box, and why cannot it be adapted for the measurement of smaller resistances?

17. Explain, with sketches of connexions, some form of direct reading potentiometer; and show how it may be used to calibrate an ammeter. [C. & G., II.

18. Show how to calibrate a moving-coil ammeter by a potentiometer, discussing fully all the precautions which have to be taken for high accuracy. [Lond. Univ., El. Tech.

19. Draw a complete diagram of the connexions for testing a dynamometer type wattmeter by a potentiometer and accessories.

20. Describe how to test completely the accuracy of a D.C. watt-hour meter. Explain how power is saved by testing in the way adopted, and give a diagram showing how a number of meters should be connected for simultaneous testing.

21. In testing 6 watt-hour meters, 5 cells of 2 volts each are connected in series to send 20 amperes through the current coils, and the voltage coils each take 0·04 ampere at 220 volts. Compare the energy used with that recorded, and calculate the cost of the energy used in a 40-minute test at 4d. per kWh.

22. Explain the precautions that have to be taken in the construction of resistances for use with A.C. voltmeters. How is a soft-iron ammeter or voltmeter made suitable for use on A.C. circuits?

23. A unidirectional pulsating current is passed through a moving coil ammeter and a hot-wire ammeter in series. Explain why their readings differ, and state which gives the higher reading.

24. Upon what action does the working of electrostatic voltmeters depend? State their advantages and disadvantages.

Sketch such a voltmeter (not the type used for very high potentials).

25. A non-inductive coil, AB, is connected across the mains, and a voltmeter is connected across a portion, CD, of this coil. The resistance of the coil AB is 8 000 ohms, and that of the portion, CD, is 2 000 ohms.

If 440 volts is applied to AB, what is the reading of the voltmeter (*a*) if it is an electrostatic one, (*b*) if it is an electromagnetic one and has a resistance of 4 000 ohms?

26. Describe, with sketches, the construction of a wattmeter for use on direct current circuits, and state whether the reading of the instrument is likely to be affected by reversing both the current and potential circuits. Describe how such an instrument can be calibrated without using the full power which it is designed to measure. [C. & G., II.

27. Explain the principles of methods suitable for measuring the resistance of the following:—(*a*) a short copper bar; (*b*) a coil of copper wire of about 20 ohms resistance; (*c*) the insulation of an electric light installation.

CHAPTER VIII

DIRECT CURRENT GENERATORS

1. Electrical Generators

Electrical generators* are machines for converting mechanical energy into electrical energy. They depend on the fact that when a coil moves so as to alter the number of magnetic lines of force passing through it, an electromotive force is produced in it. The amount of the E.M.F. is proportional to the *rate* of cutting lines (see Appendix A) and is given by the formula

$$E = \frac{\text{Number of cuts}}{(\text{Time in seconds}) \times 10^8} \text{ volts.}$$

For a conductor moving in a *uniform* field of B lines per sq. cm. (or in a field of this *average* density) the above is equivalent to

$$E = \frac{\mathsf{B}lv}{10^8} \text{ volts,}$$

where l = length of conductor in cm., measured perpendicular to the lines (*i.e.* the distance between the ends of the conductor projected on a plane perpendicular to the lines),

and v = component velocity of conductor in cm. per sec. in a direction perpendicular to the lines and to the projected length of the conductor.

This formula is of use in designing electrical machinery. If inch units are used it remains true without any alteration.

For $\mathsf{B} = \mathsf{B}'' \div (2{\cdot}54)^2$, $l = l'' \times 2{\cdot}54$, and $v = v'' \times 2{\cdot}54$. Therefore $\mathsf{B}lv = \mathsf{B}''l''v''$, where B'', l'', v'' are the flux-density, length, and velocity respectively in inch units.

2. Essential Parts of a D.C. Generator

The essential parts of a D.C. generator are:—

(*a*) The *field-magnets*: the magnets which produce the field in which the conductors move.

* Direct current generators are frequently termed dynamos, but it seems preferable to keep the latter as a general title including motors, since the same machine may serve as a generator and as a motor at different times; and the construction of the two is identical (see Chapter IX.).

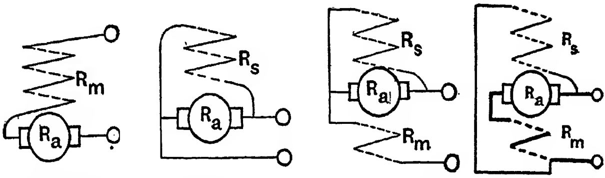
(*b*) The *field windings* or *magnet coils*: conductors to carry the current to excite the field magnets. (N.B.—These are absent in generators with permanent magnets; see Art. 3.)

(*c*) The *armature winding* or *conductors*: a connected system of conductors to cut the magnetic lines and generate the E.M.F.

(*d*) The *armature core* and *armature spider*, etc.: supports for the conductors, connecting them to—

(*e*) The *shaft*, which with the *bearings* permits of the necessary (rotary) motion of the armature.

(*f*) The *coupling*: the mechanical connexion between the shaft of the generator and that of the steam engine or other "prime mover" which drives it.



(*a*) Series-wound. (*b*) Shunt-wound. (*c*) "Short shunt" (or ordinary compound-wound). (*d*) "Long shunt" compound winding.

Fig. 8.01.—METHODS OF SELF-EXCITATION.

R_a, Armature resistance. R_m, Series (or main) field winding resistance. R_s, Shunt field winding resistance.

(*g*) Either a *commutator* or *slip-rings*: rotating conductors connected to the armature winding on which rest

(*h*) The *brushes*, supported by the *brush-gear*: electrical connexions between the external circuit and the armature winding, by way of the commutator or slip-rings.

3. Methods of Producing the Magnetic Field

The methods of producing the magnetic field required in a generator may be divided into the use of—

(*a*) Permanent magnets.

(*b*) Electromagnets excited (*i.e.* magnetized) by a current supplied from an independent source, *e.g.* another generator or a battery. The generator is then said to be separately excited. This is the general method for alternators.

(*c*) Electromagnets excited by a current obtained from the generator itself, which is therefore said to be **self-excited.**

Permanent magnets are used only for small generators, particularly those used for ignition purposes in gas, oil, and petrol engines. Such generators are called ***magnetos.*** Gradual loss of magnetism is one of their disadvantages. Electromagnets have the further advantages of lighter weight, and of ease of regulation of their magnetic strength, viz. by changing the exciting current (cf. Art. 9).

Self-excited generators may be subdivided into **shunt-wound, series-wound,** and **compound-wound** generators (see Fig. 8.01).

The first two terms are self-explanatory. The third is usually applied to a combination of shunt and series windings, though it sometimes refers to a combination of separate and series excitation. There are two slightly different forms of compound winding, named "long" or "short" shunt according to whether the shunt winding is connected across the terminals of the generator or across its brushes [see Fig. 8.01 (*c*) and (*d*)].

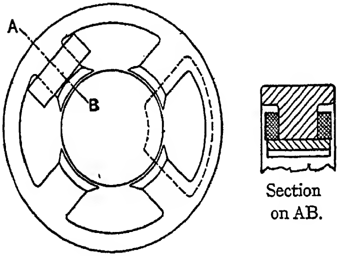


Fig. 8.02.—FOUR-POLE FIELD MAGNET.

Small dynamos are **bipolar,** *i.e.* their field magnets have only two poles, one N. and one S. For larger outputs, say over 4 kW., the number of pairs of poles is increased so that there are four, six, or more poles, alternately N. and S. These are known collectively as **multipolar** generators (or motors), and a particular machine is referred to as a "six-pole generator," an "eight-pole motor," etc.

For turbo-generators, which run at a high speed, bipolar fields are used for comparatively large outputs, even up to 50 000 kW. occasionally: and more than four poles are unusual.

A typical field magnet for a multipolar dynamo is shown in Fig. 8.02. It consists of the **field core** (or magnet core or pole) usually of cast steel but sometimes of wrought iron. Round this is placed the **field coil** of D.C.C. copper wire. The poles project radially inwards from the **yoke** of cast steel or cast iron, and they each terminate in a **pole-shoe** of cast steel, or cast or wrought iron. The consecutive pole-shoes are north and south alternately.

The narrow space between the pole-shoes and the armature core is called the air-gap, or simply the gap. The object of the pole-shoes is to spread the magnetic flux over a large area in passing across the gap, and so to diminish its reluctance (cf. Chapter IX., Art. 22). The general path of the flux is shown by the dotted lines in Fig. 8.02, from which it will be seen that the yoke carries only half the pole flux (neglecting leakage).

Further, there are as many magnetic circuits as there are poles, but the M.M.F. acting in each circuit is due to the ampere-turns of a pair of poles.

4. Armature Cores

The armature core supports the armature conductors and causes them to rotate. Its most important function, however, is to provide a path of low reluctance for the flux through the armature from the N. pole-shoes to the S. ones. It is therefore made of iron or steel of high permeability, Swedish iron being often employed.

The use of slotted or toothed armature cores (see Chapter X). has several advantages, viz.:—

(*a*) The air-gap can be reduced to what is required for mechanical clearance, thus diminishing the reluctance of the magnetic circuits.

(*b*) The conductors are positively driven, so that there is little fear of them being displaced.

(*c*) The drag on the conductors is greatly diminished, so that their insulation is in no danger of damage by pressure (see Chapter XI., Art. 1).

Such cores are therefore universal in modern machines.

Since the core rotates it cuts lines of force, and has an E.M.F. induced in it in the same direction as that in the conductors. This E.M.F. sets currents flowing in closed paths in the core, heating it and causing a waste of power. In a solid core these effects would be enormous, therefore the core must be laminated, *i.e.* built up of thin plates so as to diminish the loss due to these *eddy currents* (see further Chapter XII., Art. 3).

The direction of lamination must be perpendicular to the E.M.F., *i.e.* perpendicular to the shaft in the ordinary type of generator. These laminations (or core discs or stampings) are usually from 14 to 20 mils thick, and are insulated from each other by varnish, either shellac or preferably japan, or by oxidization of their surface. Thin paper was used, but has been abandoned owing to the expense.

5. The E.M.F. of an Armature Conductor

Since the flux-density in the air-gap rarely exceeds 10 000 lines per sq. cm., and the peripheral (or surface) velocity is not usually higher than 5 000 ft. per min. (about 2 500 cm. per sec.), the E.M.F. generated in a conductor 30 cm. (*i.e.* nearly 1 ft.) long is not more than

$$\frac{10000 \times 30 \times 2500}{10^8} = 7{\cdot}5 \text{ volts.}$$

Therefore to obtain voltages of 230 to 525, which are usual in generators, a number of conductors must be connected in series so as to add together their E.M.F.s.

The direction of the E.M.F. is given by the Right-Hand Rule, viz. Place thumb, fore-, and middle fingers of the right hand at right angles. Point thumb in direction of motion of conductor, and forefinger in direction of flux; then the middle finger gives direction of E.M.F.

Since the poles are alternately N. and S. the E.M.F. of any conductor changes its direction every time it passes from under one pole to the next. This has two results in D.C. generators:—

(*a*) A commutator is necessary to make the E.M.F. applied to the external circuit direct, *i.e.* always in the same direction.

(*b*) If a conductor is connected to another under an opposite (in the magnetic sense) pole their E.M.F.s act in the same direction round the loop (Fig. 8.03).

(*a*)

(*b*)

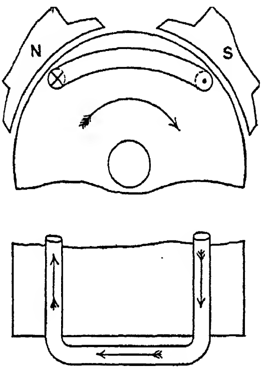


Fig. 8.03.—ARMATURE LOOP.
(*a*) Elevation. (*b*) Plan.
ownward E.M.F. ⊙ Upward E.M.F.

The way in which these loops are joined together depends on the type of armature winding adopted.

6. The Commutator

The commutator consists of a number of *segments* of hard copper usually drop-forged. They are insulated from each other by sheets of mica, and from the frame on which they are mounted by micanite rings (see Chapter X., Art. 19). The segments are connected to equidistant points of the armature winding.

The stationary brushes rest on the segments and thus take current to and from the required points of the winding, these continually changing as the commutator rotates with the armature.

The effect is the same as if the brushes rested directly on the armature and made contact with its conductors as they passed under the brushes.

7. Bar and Coil Windings

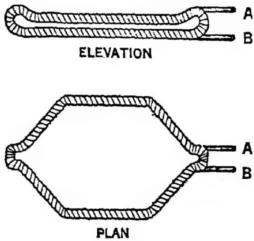


Fig. 8.04.—ARMATURE COIL.
AB, Ends of wire forming coil. (These are longer than shown in drawing, and are connected to the commutator segments.)

In place of the single loop of Fig. 8.03 a coil of two or more turns can be used giving an increased voltage (see Fig. 8.04). The problem of connecting up the various coils is the same as that of connecting up the single loops. The latter case will be taken for simplicity in the following paragraphs, but these apply to the former case also if "*coil sides*" be substituted for "conductors" and "*coils*" for "loops." The two forms are known as **bar windings** and **coil windings** respectively. The latter are usual in small dynamos where the requisite E.M.F. cannot be obtained with a bar winding without an excessive number of loops and consequently of commutator segments. They are generally made by winding D.C.C. wire on a suitably shaped wooden *former*, and binding the turns together with cotton tape.

The conductors are arranged most conveniently in two layers, and each loop comprises one conductor in each layer. Using this plan the end connexions can be arranged easily so as not to interfere with each other (see Figs. 8.06 and 8.07).

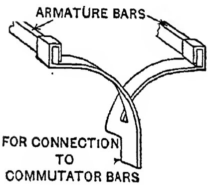


Fig. 8.05.

The simplest windings contain only two conductors (or coil sides) per slot, and this arrangement will be assumed in what follows.

The end connexions are nearly always kept at the same distance from the shaft as the active conductors. This is known as **barrel winding**. An alternative is to place them at right angles to the conductors, forming the evolute winding, so called from the usual shape of the end connectors (see Fig. 8.05).

Intermediate forms are used also, and are called **bastard windings**.

8. Lap and Wave Windings

These are the two arrangements chiefly employed, though occasionally an intermediate type is used. All are of the *closed coil* type, *i.e.* the winding forms a complete closed circuit.

If a **lap winding** starts with the top conductor in one slot and the bottom conductor in another (forming a loop as already described), it then proceeds *to the top conductor next to the first one* (Fig. 8.06). This top conductor is half of a loop, which is similar to the first loop but is one slot away from it: thus two loops have been joined in series. This method of connexion is continued until all the loops have been joined and the winding returns to the

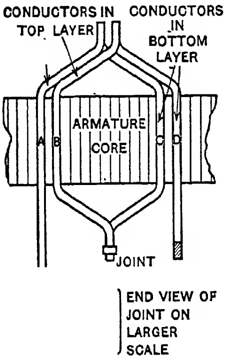


Fig. 8.06.—LAP WINDING.
AC, First loop. BD, Second loop.

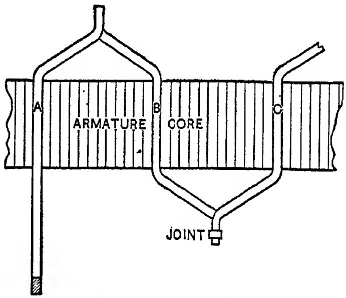


Fig. 8.07.—WAVE WINDING.
A, Top conductor of first loop. B, Bottom conductor of first loop. C, Top conductor of second loop.

conductor from which it started. The name comes from the way in which successive loops overlap the preceding ones.

In a **wave winding** the bottom conductor of the first loop is connected to a top conductor about two pole pitches away from the first top conductor (see Fig. 8.07). Thus the connexions always proceed in the same direction round the armature (instead of in alternate directions) forming a wavy winding, whence its name. It returns to A after going through all the other conductors.

9. Winding Rules

The **pitch** of a winding is the distance round the armature between two successive conductors which are directly connected.

The **back pitch** is the distance between the two conductors which form a loop (see Fig. 8.08).

The **front pitch** is the distance between the second conductor of one loop and the first conductor of the next loop, which are connected together at the front (or commutator) end of the armature.

The **resultant pitch** is the distance between the beginning of a loop and the beginning of the next loop to which it is connected.

The resultant pitch is therefore the sum of the front and back pitches. If these run in opposite directions one of them is called negative, and the resultant pitch is still their algebraic sum.

All these pitches are usually stated in numbers of conductors. Sometimes the number of slots, or the number of commutator bars (for the resultant pitch), is given instead or in addition.

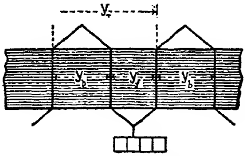


Fig. 8.08.—WINDING PITCHES.

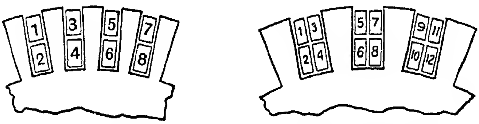
= back pitch. Y_f = front pitch.
Y_r = resultant pitch.

The following rules must apply (see Art. 8):—

In Lap and Wave Windings. (*a*) Front and back pitches are each nearly equal to the pole pitch.

(*b*) Both pitches are odd, so that all end-connexions are between a conductor at the top of a slot and one at the bottom of a slot. This is for convenience in arranging the end-connexions (see Art. 7).

In Lap Windings. (*c*) The resultant pitch is 2, *i.e.* the front



(*a*) Two bars per slot. (*b*) Four bars per slot.

Fig. 8.09.—NUMBERING OF ARMATURE CONDUCTORS.

and back pitches are of opposite sign and differ numerically by 2. The conductors are numbered round the armature. Calling (say) the top bar in one slot No. 1, the bottom bar in the same slot will be No. 2, the top bar in the next slot No. 3, and so on [see Fig. 8.09 (*a*)]. Thus all the top bars have odd numbers and all the bottom bars even ones. With more than two bars per slot this numbering is carried out on the same principle [see Fig. 8.09 (*b*)].

In Wave Windings. (*d*) The front and back pitches are of the same sign and are usually equal, and therefore half of the resultant pitch. [They may differ by 2 if it is wished to make the resultant pitch a multiple of 4.] Further, to make the winding perfectly symmetrical it must, after going once round the armature, return to one of the two top conductors next to the one from which it started. Now the pitches are nearly equal to the pole pitch, and therefore the winding wili return nearly to where it started after a number of steps equal to the number of poles.

Let $y = \text{front pitch} = \text{back pitch}.$

Then $y \times \text{No. of poles} = \text{No. of conductors} \pm 2,$

or—
$$y \times 2p = \mathrm{N} \pm 2$$
$$y = \frac{\mathrm{N} \pm 2}{2p}$$

[if the pitches differ by 2 they are respectively one more and one less than y (which is then the average pitch) and the above relation still holds].

Since y must be a whole number it follows that for a wave winding the number of conductors with 2 either added or subtracted must be a multiple of the number of poles,

or
$$\mathrm{N} = 2py \pm 2$$
and
$$\text{No. of coils} = py \pm 1.$$

In the case of a lap winding any even number of conductors may be used.

10. Winding Tables

Any method of winding may be shown by means of a table. This is illustrated for both lap and wave windings for a 4-pole armature with 182 conductors (180 would be inadmissible for a wave winding).

$$\text{Pole pitch} = \frac{182}{4} = 45\tfrac{1}{2} \text{ conductors;}$$

$\therefore$ Pitches for lap winding may be 45 and $-$ 43.

(These are taken rather than 47 and $-$ 45 so as to shorten the end-connexions.)

$\therefore$ Pitches for wave winding may be 45 and 45 (or 45 and 47).

LAP WINDING

FRONT	BACK		FRONT
I.	1	46	II.
II.	3	48	III.
III.	5	50	IV.
..	..	..	..
LXVIII.	135	180	LXIX.
LXIX.	137	182	LXX.
LXX.	139	2	LXXI.
LXXI.	141	4	LXXII.
..	..	..	..
XC.	179	42	XCI.
XCI.	181	44	I.

WAVE WINDING.

F.	B.		F.	B.		F.
I.	1	46	XLVI.	91	136	XCI.
XCI.	181	44	XLV.	89	134	XC.
XC.	179	42	XLIV.	87	132	LXXXIX.
..	..	..	..	..	..	..
LXXI.	141	4	XXV.	49	94	LXX.
LXX.	139	2	XXIV.	47	92	LXIX.
LXIX.	137	182	XXIII.	45	90	LXVIII.
..	..	..	..	..	..	..
XLIX.	97	142	III.	5	50	XLVIII.
XLVIII.	95	140	II.	3	48	XLVII.
XLVII.	93	138	I.	1		

The Roman numerals refer to commutator segments, the number of which is half that of the conductors, *i.e.* equal to the number of loops. The first table means that the winding goes from segment I by conductor 1 across the back to conductor 46 and at the front to segment II., this forming one loop. Then from segment II. through conductors 3 and 48 back to segment III., and so on until the winding returns to segment I after using all the 182 conductors.

The second table is similar except that two loops are used (three in a six-pole, and so on) before coming to a segment next to the one from which the winding starts.

In both cases the conductor numbers in each column have a common difference 2, and the segment numbers a common difference 1.

11. Winding Diagrams

An alternative method of showing a winding is by means of diagrams, several varieties of which are employed. They are best described by means of examples.

Fig. 8.10 shows the winding used in the motor of the Elihu Thomson meter (Chapter VII., Art. 17). It is a two-pole winding with 8 coils, which are not wound in slots. The circular dots represent the 16 coil-sides, the full lines the connexions at the commutator end (including the connexions to the commutator) and the dotted lines the connexions at the back end. The winding pitches are 7 and -5.

A modification of this type of diagram is the *radial diagram*,

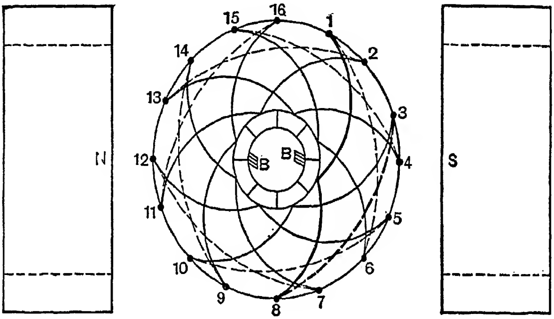


Fig 8.10.—WINDING DIAGRAM OF MOTOR OF ELIHU THOMSON METER.

BB, Brushes resting on commutator. CC, Current-carrying coils producing N. and S. poles. (N.B.—One coil, 3-8, and one end connexion, 8-1, are shown in thicker lines.)

which has the advantage that the front and back connexions do not overlap and the disadvantage of occupying more space. Fig. 8.11 is a radial diagram of a 4-pole lap winding with 36 conductors and wind-pitches of 9 and -7.

The conductors are arranged in two layers, the upper ones being shown by thick lines and the lower ones by thin lines. The two which lie in the same slot may be drawn nearer to each other than to those in other slots. The positions of the poles are indicated as shown. The commutator is drawn as two concentric circles, with the space between divided into as many equal parts as there are segments (18 in the example). The brushes are marked inside the inner circle, so as to leave the front connexions clear.

The *developed diagram* (Fig. 8.12) is obtained by imagining the cylindrical surface of the armature to be cut by an axial plane and then flattened out. The example shows a 6-pole wave winding with 44 conductors, and both winding pitches 7 $\left(= \frac{44-2}{6}\right)$.

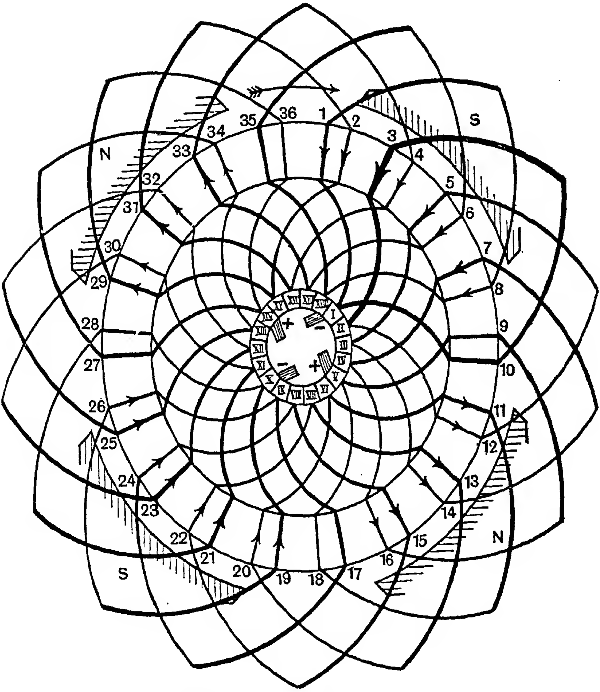


Fig. 8.11.—RADIAL DIAGRAM OF 4-POLE LAP WINDING.

NSNS, Poles. + — + — Brushes.

(N.B.—One coil, 3-10, and its end connexions are shown in thicker lines.)

The poles are indicated by section-lined areas with the lines for the N. poles drawn parallel to the diagonal of the N, and those for the S. poles at right angles to these. If the conductors move in front of the poles, then the direction of the E.M.F. produced is the same as the direction in which the lines pass along the conductors (see Fig. 8.13).

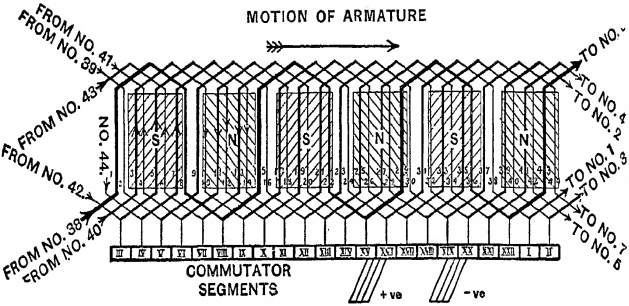


Fig. 8.12.—DEVELOPED DIAGRAM OF 6-POLE WAVE WINDING.

(N.B.—One set of 3 coils (1-8, 15-22, 29-36) is shown in thicker lines.)

The truth of this rule can be seen by applying any of the usual rules for the direction of induced E.M.F., see Art. 5. The direction is marked for some of the conductors in Fig. 8.12.

12. The E.M.F.s in a Lap Winding

In any closed coil winding the E.M.F.s must balance, otherwise the resultant E.M.F. will cause a current to circulate in the winding, causing unnecessary heating and consequent waste of power.

This condition is satisfied in both lap and wave windings, but in different ways.

Let A, B, C, D, etc. (Fig. 8.14), be points midway between the poles. Consider only the top conductors, *i.e.* treat the lower conductors as return leads (or connectors) whose E.M.F. always assists that of the top ones to which they are connected.

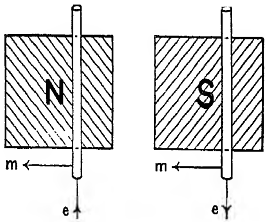


Fig. 8.13.—E.M.F. AND SECTION LINES.

mm, Direction of motion of conductors.
ee, Direction of resulting E.M.F.

In a lap winding all the (top) conductors which, at any instant, lie between A and B are connected in series, so that their E.M.F.s add up. Next, between B and C, are an equal number of conductors with equal but opposing E.M.F.s. Thus the whole winding can be subdivided into as many bands of conductors as there are poles, each band producing the same total E.M.F. in a direction which

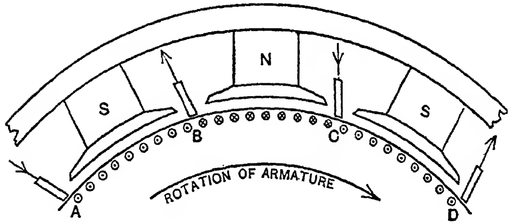


Fig. 8.14.—Directions of Currents in Armature and in Brushes.

A B C D, Brushes. ⊙ Upward current in armature conductor.

⊗ Downward current in armature conductor.

is reversed on passing from any band to the next. The total E.M.F. *round* the whole winding is therefore zero.

By placing stationary brushes so that they are connected to the conductors at A, B, C, etc., the E.M.F. of a group of conductors can be used to drive a current *in a fixed direction* through an external circuit. Further, since the E.M.F. from A to B is equal but opposite to that from B to C, A and C may be connected together and to every alternate brush. The same applies to B, D, etc.

The connexions may be compared to a ring of cells connected +ve to +ve and −ve to −ve (Fig. 8.15), with leads to the external circuit taken off at their junctions.

The following relations evidently hold in all lap windings:—

No. of armature circuits = No. of poles = No. of brushes.

Current in external circuit = (Current in each armature conductor) × (No. of poles).

It should be noted that though the current in the external circuit is always in the same direction, that in any particular armature conductor reverses each time the conductor passes the points A, B, C, etc. (Fig. 8.14).

The change from the reversing or alternating current in the armature to the direct external

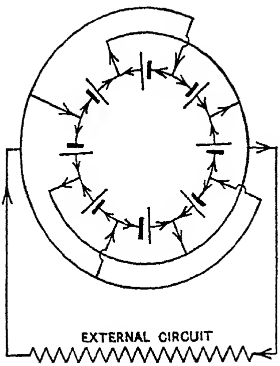


Fig. 8.15.—Cells connected to represent an 8-pole Lap Winding.

current is due to the collection of the current at points, fixed in space, but changing their position in the armature as it rotates.

13. The E.M.F.s in a Wave Winding

On applying the above method to a wave winding it will be seen that a (top) conductor between A and B is connected to one between C and D and so on to a conductor under each N. pole in turn, returning to the conductor next to the one from which the start was made. Thus all the conductors under the N. poles are connected so as to add up their E.M.F.s.

The winding then proceeds through the remaining half of the conductors, which lie under S. poles and so produce an equal but opposite total E.M.F. The E.M.F. round the whole winding is again zero. In this case therefore the winding falls into two groups whatever the number of poles. Consequently two brushes, say at A and B, are sufficient for taking the current to and from the external circuit, but a larger number (up to the number of poles) may be used. If two only are employed they are usually placed a pole pitch apart (viz. 90° in 4-pole, 60° in 6-pole, 45° in 8-pole, etc.), but they might be placed any odd number of pole pitches apart, *e.g.* 135° in 8-pole winding.

The following relations hold (cf. previous Art.):—

No. of armature circuits = 2.

Current in external circuit

= (Current in each armature conductor) × 2.

14. E.M.F. Formulae

Let n = rev. per min. of an armature,

Φ = armature flux (total lines) per pole,

$2p$ = number of poles,

$2a$ = number of armature circuits,

N = number of armature conductors,

γ = number of armature loops = N/2.

When an armature loop moves from the position in which it embraces the whole of the flux from a N. pole to a similar position opposite the next S. pole the flux changes by 2Φ, *i.e.* from $+\Phi$ to $-\Phi$; or, in other words, it cuts each of the Φ lines twice. The movement to effect this is $1/2p$th of a revolution.

Now the time of 1 revolution is $\frac{1}{n}$th of a minute or $\frac{60}{n}$ seconds, so that the time of $\frac{1}{2p}$th of a rev. is $\frac{1}{2p}\cdot\frac{60}{n}$ sec.

Therefore the average E.M.F. induced in one turn

$$= \frac{2\Phi}{\frac{1}{2p}\cdot\frac{60}{n}\cdot 10^8} \text{ volts} = \frac{2\Phi . 2p . n}{60 . 10^8} \text{ volts (see Art. 1).}$$

This is the same for each turn, and the number of turns *in series*

$$= \frac{\text{♉}}{2a} = \frac{N}{4a};$$

$\therefore$ Total (avg.) E.M.F. of armature

$$= \frac{2\Phi . 2pn}{60 \times 10^8} \times \frac{N}{4a} = \frac{\Phi n N}{60 \times 10^8} \times \frac{2p}{2a} \text{ volts;}$$

$$= \frac{2\Phi n \text{♉}}{60 \times 10^8} \times \frac{p}{a}.$$

For a two-pole or a lap-wound armature, $a = p$ (see Art. 12 and the formula becomes

$$E = \frac{\Phi n N}{60 \times 10^8} \text{ volts.}$$

For a wave-wound armature $a = 1$ (see Art. 13);

$$\therefore\ E = \frac{\Phi n N}{60 \times 10^8} \times p \text{ volts.}$$

The above are strictly only average values, but unless the number of loops is very small the total voltage is practically constant, though the voltage of any one loop varies between zero and a maximum.

Example 1. *The flux in a four-pole direct current dynamo is 2 megalines per pole. The armature has 740 conductors and is lap-wound. What is the induced electromotive force when the speed is 1 000 revolutions per minute?*

(C. & G., II.)

Here

$$\Phi = 2 \times 10^6 \text{ lines,}$$
$$n = 1000 \text{ r.p.m.,}$$
$$N = 740.$$

Therefore since the armature is lap-wound

$$E = \frac{\Phi n N}{60 \times 10^8} \text{ volts} = \frac{2 \times 10^6 \times 1000 \times 740}{60 \times 10^8}$$
$$= 247 \text{ volts.}$$

15. Homopolar Generators

Homopolar is the name given to a type in which the E.M.F. in the armature conductors is always in the same direction.

This is effected by making the poles extend completely round the armature so that every part of the winding is always under a pole of the same sort (hence the name) instead of alternate N. and S. poles. This has the advantage of avoiding the need of a commutator.

The main drawback is that, in order to connect two or more conductors in series, slip rings and brushes (or some equivalent arrangement) must be used, with consequent losses. The conditions favourable to the success of this type are—

(*a*) low voltage, to diminish the number of conductors in series;

(*b*) large current, so that the machine is large and the length of a conductor considerable;

(*c*) very high peripheral speed so as to increase the volts per conductor; this can be higher the larger the machine, since centrifugal force varies inversely as the diameter for a given peripheral speed.

By using a number of slip rings at each end the voltages of several conductors may be put in series, but this adds to the cost and the losses.

Several generators of this type have been built, *e.g.* one for 7 700 A. at 260 V. by B. G. Lamme with a peripheral speed of over 13 000 ft. per min. (see *The Electrician*, vol. LXIX, p. 662). No general use of the principle has been made successfully.

16. Multiple and Multiplex Windings

It is sometimes advisable to split the current up into more parts than there are poles, so as to use smaller armature conductors. This can be done by placing two independent lap windings on the armature. One of these will fill every other slot (with two bars per slot) and be connected to every alternate segment of the commutator. The other will occupy the remaining slots and be connected to the other segments of the commutator. Each winding then supplies half the total current, so that the current in each armature conductor is only half that in the case of a simple lap winding.

Such a winding is called a **double** winding. The principle may be extended by placing three or more independent windings on the armature. These would be called triple, quadruple, etc., windings.

The circumferential width of the brush must be at least 2, 3, 4, etc., times that of the commutator segments, so as to make proper connexion with all the independent windings at all times.

The same principle may be used with wave windings if it is desired to have a dynamo with more than two armature circuits without using a lap winding.

Multiplex (or multiply re-entrant) windings are a modification of these multiple windings, and produce the same result in a slightly different way. For example, a doubly re-entrant (or duplex) lap winding requires an *odd* number of slots. This winding starts like a double winding, *i.e.* omitting alternate slots and commutator segments. When it has passed completely round the armature it does not return to the bar from which it started, since it moves on

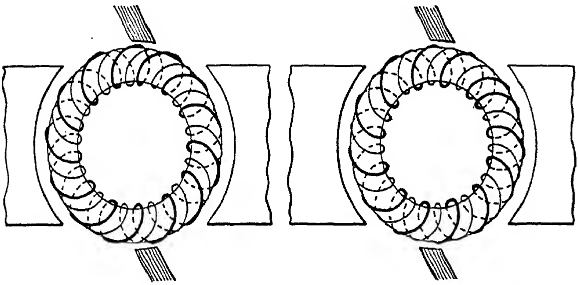


Fig. 8.16.—DOUBLE AND DUPLEX RING WINDINGS.

two slots at a time and the total number of slots is odd. Instead it comes to one of the slots omitted at first. It then passes completely round the armature again, filling the remaining slots and finally returning to the original bar. Thus there are not two independent windings, and yet there are twice as many circuits through the armature as there are pairs of poles.

The difference between the double and duplex windings is illustrated by the two-pole ring windings shown in Fig. 8.16.

In both there are four circuits through the armature, but in the double winding alone there are two independent windings.

The winding rules of Art. 9 have to be modified as follows:—

Lap winding, multiple or multiplex (multiply re-entrant).

$$\text{Resultant pitch} = 4, 6, 8, \text{etc.},$$

according as there are 2, 3, 4, etc., times as many armature circuits as poles.

If the number of bars is divisible by the resultant pitch the winding is multiple.

For a multiplex winding the number of bars ± 2 must be divisible by the resultant pitch.

Wave winding, multiple or multiplex.

$$\text{Average pitch} = \frac{\text{N} \pm \text{(number of circuits)}}{2p}$$

and $$\text{N} = 2\,py \pm \text{(number of circuits)}.$$

If the number of bars is divisible by the number of circuits and if $\text{N} = (pm \pm 1) \times \text{(number of circuits)}$* the winding is multiple.

For a multiplex winding the average pitch must have no common factor with the number of circuits: *e.g.* for a duplex winding the average pitch must be odd.

It is possible, but unusual, to have a winding combining both methods, *e.g.* a double duplex wave winding, giving eight circuits. For this particular case the resultant pitch must be divisible by four, but not by eight.

N.B.—The above nomenclature differs from that used by other writers. S. P. Thompson (*Dynamo-electric Machinery, Continuous Current Machines*) reverses multiple and multiplex, but uses multiply re-entrant in the same sense as above. Parshall and Hobart (*Armature Windings*) employ different terms again, as shown by the following examples:—

SYMBOL.	AUTHOR.	S. P. THOMPSON.	PARSHALL & HOBART.
	Single triply re-entrant (or single triplex)	Simplex triply re-entrant (or simplex triple)	Singly re-entrant triple
O O	Double singly re-entrant (or double simplex)	Duplex singly re-entrant	Doubly re-entrant double

17. Equalising Rings

Equalising rings are often fitted to lap-wound armatures. In such armatures the current flowing through each of the brush sets should be the same. If, however, the fluxes from and to the various poles are unequal this will not be the case. The conductors under the stronger poles will generate larger E.M.F.s, and therefore

* m is any whole number, and is equal to the average pitch of each winding considered by itself.

will carry larger currents (cf. the case of several cells of equal resistance but different E.M.F.s connected in parallel).

By connecting together a number of points in the winding which would be at equal potentials if the fields were equal, the differences in the brush currents are diminished. The conductors which connect these points are called **equalising rings**, and each of them can be connected to as many points in the winding as there are pairs of poles. They do *not* equalise the currents in the armature conductors under the poles, and only become effective when the conductors to which they are connected are close to the positions of commutation. They therefore have little effect on the copper losses in an armature in an unsymmetrical field; they may even increase these losses.

The advantage of their use is the avoidance of commutator troubles due to excessive currents in some of the brush sets. Several equalising rings must be fitted so as to obtain this advantage in all positions of the armature: six is the usual number, but a larger number is more effective.

QUESTIONS ON CHAPTER VIII

1. Enumerate the essential parts of a D.C. dynamo, writing them down in a table, and opposite each briefly explain the object and action of each part. [C. & G., I.

2. Draw a diagram of connexions for a drum armature with 20 conductors, showing commutator and position of brushes. Show how the current flows in the various parts of the winding, and state what changes occur as the successive sections of the commutator pass under the brushes.

3. If a 4-pole lap-wound generator driven at 950 r.p.m. has 421 armature turns and generates an E.M.F. of 115 volts, find the armature flux per pole.
If it were wave-wound what difference would be caused in the E.M.F., current, and output?

4. Draw a radial diagram for a 4-pole armature with 23 coils, simple wave-wound. Show the positions of the poles and of the brushes, and the end-connexions of at least seven coils including connexions to commutator.

5. Draw up a winding table for the above armature, including commutator segments and brush positions.

6. Draw a developed diagram for a 6-pole lap-wound armature with at least 50 coils. Show positions of poles and brushes, and mark polarity.

7. Draw a developed diagram for a 6-pole lap-wound armature with 27 coils. Show positions of poles and brushes, and the end-connexions of at least 5 coils including commutator connexions.

8. Draw up a winding table for a 6-pole lap-wound armature with between 40 and 50 conductors, and indicate one position of the brushes. Draw also a diagram showing six consecutive turns of the winding and their commutator connexions. Could the same armature be wave-wound? If not, what change is necessary?
State the pitches for the wave-winding in conductors and in slots.

9. In what way must the formula for the E.M.F. of a 2-pole simple winding be modified for the cases of multiple and multipolar windings?

Calculate the E.M.F. for a 6-pole wave (or series) wound machine with 480 conductors, $1 \cdot 6 \times 10^6$ lines per pole, at 600 r.p.m.

10. Give a winding table for a duplex 4-pole lap-winding for 62 conductors. Also draw a diagram (radial or developed) showing poles and five turns (ten conductors) of the winding.

11. A 500-volt generator running at 750 r.p.m. has 83 slots and 12 conductors per slot on the armature, whilst the field winding consists of 6 500 turns of 22 S.W.G. per pole. Can this winding be adapted so as to make the machine generate 220 volts at the same speed? If it is necessary to re-wind it, work out the new armature and field windings. [Lond. Univ., El. Mach.

12. Which of the following numbers of conductors, 204, 206, and 208, can be used for a simple wave-winding for 6, 8, and 10 poles respectively?

State the pitches for each possible case.

13. A 4-pole armature is to have over 52 conductors. Find the least possible number if it is to be—

(*a*) Simple wave-wound,

(*b*) Double wave-wound,

(*c*) Duplex wave-wound.

State the pitches in each case.

What is the difference between windings (*b*) and (*c*)?

14. (*a*) A 6-pole D.C. armature has 110 slots and 220 commutator segments. Each slot contains 4 conductors. Show how these conductors could be connected so as to form a simple wave, or 2-circuit winding.

(*b*) What are equalising, or equipotential connexions in an armature winding? What is their purpose? Can they be applied to a winding such as that in (*a*)? [Whitworth Senior Scholarship.

CHAPTER IX

ARMATURE REACTION AND COMMUTATION

1. Armature Reaction

When the external circuit of a generator is closed a direct current flows through it. The current in any particular armature conductor reverses at very short intervals, but the current in the conductor which is in any particular position (*e.g.* under the middle of a N. pole) is always in the same direction (see previous chapter, Art. 12). These armature currents produce magnetising forces which affect the magnetic field due to the magnets and their windings, and these effects are called **armature reaction.** As long as the armature current remains constant and the brushes are not moved the M.M.F. of the armature remains constant in magnitude and direction, and is unaffected by the rotation of the armature for the reason given above.

The armature reaction may be divided into two components:—

(*a*) The cross-magnetising or distorting effect.

(*b*) The demagnetising or weakening effect.

Their relative magnitudes depend on the amount of **lead** of the brushes. These are said to be in the position of *no lead* when they make contact with the commutator segments which are connected to the armature conductors lying midway between the poles: these conductors are continually changing as the armature rotates, but the brushes are always connected to those which occupy the midway position at each instant. The actual position of the brushes depends on the connexions between the armature winding and the commutator, their no lead position is usually either midway between the poles or opposite the centres of the poles, but may be anywhere.

2. The Cross-Magnetising Effect

The cross-magnetising effect of armature reaction is the only one that occurs when the brushes have no lead. Fig. 9.01 (*a*) shows the directions of the flux due to the field-magnet windings, and of the currents in the successive bands of armature conductors. The resulting M.M.F.s of the latter are as shown by the arrows in (*b*), viz. alternately inwards and outwards (or alternate S. and N.

polarity) with maxima midway between the field poles. The actual field will be the result of these armature M.M.F.s and of those due to the field windings, and it can be seen by comparing (*a*) and (*b*) that the effect of the former is to weaken one half of each pole and to strengthen the other half.

The weakened halves are those under which the conductors first come as they rotate, hence called "leading." The result may be

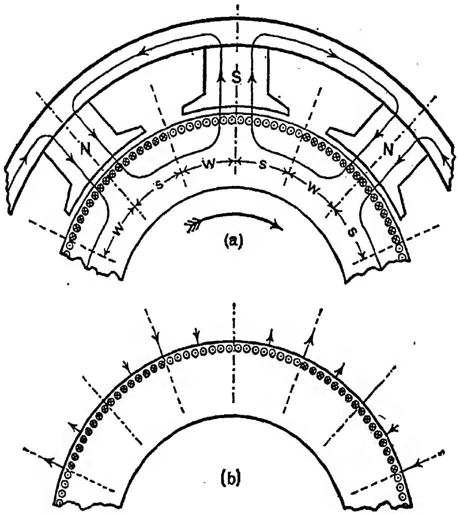


Fig. 9.01.—DISTORTING EFFECT OF ARMATURE REACTION.

(*a*) Flux due to magnet windings alone. (*b*) M.M.F. due to armature currents (shown by arrows).
N N, North poles. S, South pole. s s, Portions of field strengthened.
w w, Portions of field weakened.

stated thus:—*Armature reaction in generators weakens the leading pole tips and strengthens the trailing pole tips.*

The general truth of the above can be confirmed by noting the effect of reversing the rotation of the armature. This reverses the E.M.F. and consequently the armature current. The armature reaction is therefore reversed, those parts of the field being weakened which were previously strengthened, and vice versa. But the pole tips which were previously trailing are now leading. So, as before, the leading pole tips are weakened.

The average M.M.F. is unchanged, for the decrease over one half of each pole is exactly counterbalanced by the increase over the other half. Nevertheless there will be a slight decrease in the flux per pole because the increased permeability of the weakened half is insufficient to compensate for the decreased permeability of the strengthened half. (See Art. 5.)

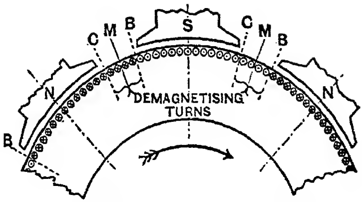


Fig. 9.02.—WEAKENING EFFECT OF ARMATURE REACTION.

M M, Midway points. B B, Brush positions. C M = M B.

3. The Demagnetising Effect

Owing to the cross-magnetisation of armature reaction the neutral points (*i.e.* the points of zero field) are shifted in the direction of rotation from the positions, midway between the poles, which they occupy when there is no armature reaction. The brushes, if they are still to be connected to the conductors in the position at which their E.M.F. reverses (see Chapter VIII., Art. 12), must be given a lead (*i.e.* shifted in the direction of rotation).

When the brushes have a lead the M.M.F.s of the armature conductors are rotated by the same amount.

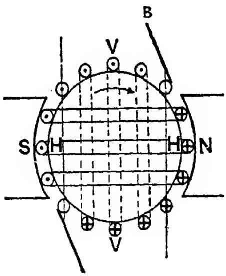


Fig. 9.03.—CROSS AND BACK AMPERE-TURNS IN 2-POLE GENERATOR.

B B, Brushes.
H H, Cross-magnetising turns.
V V, Demagnetising turns.

A reference to Fig. 9.02 will show that there is now a weakening effect in addition to the distorting effect.

To separate the two effects consider the conductors from the one directly connected to the brush to one lying the same distance behind the mid position. It will be seen that the effect of each such band is to weaken both the neighbouring poles. By combining such a band with conductors in neighbouring bands, a number of demagnetising turns are formed. This number multiplied by the current in each armature conductor gives the back (or demagnetising) ampere-turns per pair of poles. (See Art. 4.)

The intermediate larger bands form the cross-magnetising turns, from which similarly the cross ampere-turns can be calculated.

Fig. 9.03 illustrates the special case of a 2-pole generator, and shows the origin of the term cross ampere-turns.

4. Allowance for Armature Reaction

As an example consider a lap-wound armature for an 8-pole field with 264 slots and 6 conductors per slot. Total current 728 amperes. The back ampere-turns are calculated as follows:—

$$\text{Number of conductors per pole} = \frac{264 \times 6}{8} = 198;$$

$$\therefore \text{ No. of conductors in each interpolar gap} = 198 \times \frac{\text{interpolar arc}}{\text{polar pitch}}$$

$$= 198 \times \frac{10{\cdot}3}{35{\cdot}3} = 58.$$

(The numerical values are assumed here, their calculation is given in Art. 11.)

On the assumption that the brushes are shifted up to the leading pole tips this is also the number of demagnetising turns per pair of poles (see Fig. 9.04). Since the armature is lap-wound the current per conductor is 728/8 = 91 amp.;

$\therefore$ back ampere-turns per pair of poles = 91 × 58 = 5280.

The ampere-turns on the poles to compensate for this must be somewhat greater because of the increased leakage. This may be allowed for by multiplying by the leakage factor;

$\therefore$ compensating ampere-turns on poles = 5280 × 1·2 = 6340.

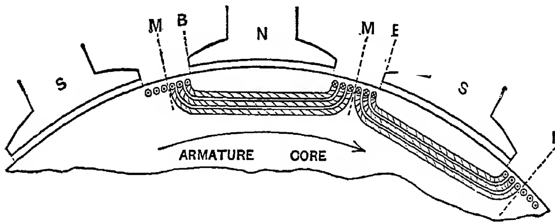


Fig. 9.04.—THE DEMAGNETISING TURNS OF THE ARMATURE.

B B, Brushes. M M M, Midway positions.
⊙ Upward current. ⊗ Downward current.

5. Reduction of Flux by Cross Ampere-Turns

The reduction of the flux by the *cross ampere-turns* arises in the following way. The flux is crowded to one side of each pole-piece (see Art. 2) and the permeability of these sides and of the portions of the armature under them is therefore reduced. The increased permeability of the parts in which the flux is diminished compensates for this only partially, as is shown below.

Let AB (in Fig. 9.05) represent the M.M.F. (in ampere-turns) applied to the air-gap and teeth under one pole when there is no armature current. The value is constant, *i.e.* AL = BM. Let AC = BD = cross ampere-turns per pole with a certain current (I_a amp.) in the armature. Then the straight line CD represents the M.M.F. when modified by the effect of the cross ATs. The value at the centre of the pole is unchanged by distortion.

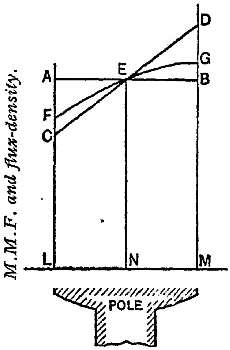


Fig. 9.05.—REDUCTION OF FLUX BY CROSS AMPERE-TURNS.

LM = Pole Arc.

LN = NM.

By choosing suitable scales AB can be made to represent also the flux-density, and the area ABML the total flux in the gap under this pole.

With a smooth core the permeability of the gap is constant. Therefore the flux-density is changed in the same proportion as the M.M.F., *i.e.* CED represents the flux-density with I_a amp. flowing in the armature. The total flux is equal to the area CDML, which is equal to ABML. Therefore in this case there is no weakening of the field as a whole by the cross ATs. The effect of change of permeability in the pole-pieces and armature is slight, since the lines of force become nearly uniformly distributed a short distance from the surfaces of these two.

When the core is toothed CD still represents the M.M.F., but the flux-density is reduced to GM on the crowded side, and increased to FL on the other. The value at the centre is still EN.

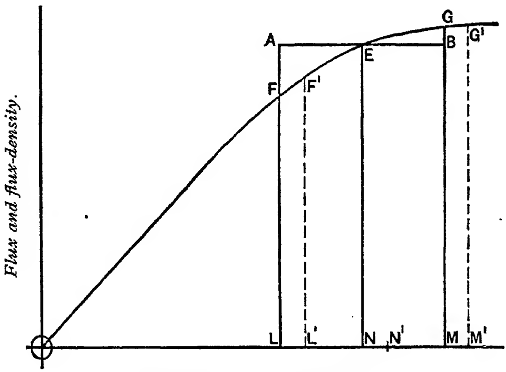
The total flux is FGML and this is less than ABML. Since the M.M.F. varies uniformly across the pole, FEG is a portion of the saturation curve for the gap and teeth (see Fig. 9.06). And as the slope of a saturation curve decreases steadily GEB is necessarily less than AEF.

To bring the flux back to its original value, FGML must be shifted to a new position F′G′M′L′, which makes F′G′M′L′ = ABML. This position can be determined by making one or two trials.

The extra ampere-turns needed to effect this = LL′ = MM′= NN′, where N′ is the middle point of L′M′. (See Art. 12.)

6. Calculation of Field Ampere-Turns

The calculation of field ampere-turns for a generator is done by dividing each magnetic circuit up into a number of portions, in each of which the flux-density is approximately constant. If the flux is known and the dimensions of each part, the ampere-turns required



Ampere-turns for gap and teeth.

Fig. 9.06.—COMPENSATION FOR DISTORTION.

for each can be calculated by the formula $\mathcal{T} = 0{\cdot}8\dfrac{\mathbf{B}l}{\mu}$ (see Chapter IV., Art. 15). The flux-density ($\mathbf{B}$) at any point is obtained by dividing the flux by the cross-sectional area of the magnetic circuit at that point. **B-H** curves or tables for the materials used may then be employed to obtain the value of $\dfrac{\mathbf{B}}{\mu}$ (= $\mathbf{H}$) corresponding to the calculated value of **B**. It is, however, more convenient if the curves, or tables give ampere-turns per cm. $\left(= 0{\cdot}8\dfrac{\mathbf{B}}{\mu}\right)$, or ampere-turns per inch $\left(= 0{\cdot}313\,\dfrac{\mathbf{B}''}{\mu}\right)$, directly instead of $\dfrac{\mathbf{B}}{\mu}\cdot$

When the ampere-turns for each portion have been calculated they are added together to obtain the total ampere-turns required. Owing to the symmetry of the magnetic circuit this may be done either for the whole circuit, or for half of it. In the former case the result is the ampere-turns per pair of poles, and in the latter case the ampere-turns per pole, so that it is of small importance which method is used.

7. Magnetic Leakage Allowance

One difficulty is the allowance for *magnetic leakage* (cf. Chapter IV., Art. 17). This is taken into account by taking two values for the total flux, one for the armature core and the air-gaps, and a larger one for the rest of the magnetic circuit. It would be more accurate to take different values of the flux for each part of the magnetic circuit, but since some of the data are necessarily inexact this is an unnecessary refinement.

The value of the leakage coefficient may be assumed from experience of machines similar to the one under consideration. If this is not available an estimate may be made in the following way.

If all the linear dimensions of a dynamo bear a constant ratio to the corresponding dimensions of another, *e.g.* if one is twice as big in every respect, they will have the *same* percentage leakage if magnetised to the same flux-density. For the *relative* dimensions of the main and the leakage paths are the same in the two machines, and so is the permeability at corresponding points; therefore the useful flux and the leakage flux are increased in the same ratio.

Usually, however, the air-gap of the larger machine will be increased in a smaller ratio than the other dimensions. In this case its percentage leakage can be obtained approximately by assuming it to vary in proportion to the air-gap; *e.g.* if a dynamo has all its dimensions double those of another, except that its air-gap is 12 mm. (instead of 16 mm.) as against an 8 mm. air-gap in the smaller one, the percentage leakage of the larger will be about $\frac{3}{4}$ ($= \frac{12}{16}$) of the percentage leakage of the smaller.

Actually it will be somewhat more than this value, for the leakage flux in a given machine is almost exactly proportional to the ampere-turns, since the leakage lines pass mostly through air. The effect of reducing the air-gap is to reduce the ampere-turns required to produce a given useful flux approximately in proportion to the gap, but not exactly so, since the rest of the circuit requires as many ampere-turns as before. Therefore the leakage is reduced by a slightly smaller proportion than the gap.

The effect of a change of flux-density is harder to estimate. A high flux-density means a high percentage leakage, because the leakage flux, as stated above, is proportional to the ampere-turns, while the main flux increases less quickly owing to diminishing permeability. To estimate the extent of the change, therefore, requires a preliminary estimate of the increase in the ampere-turns required for the increased flux-density.

8. A Typical Calculation

The magnetic circuit of a dynamo is shown in Fig. 9.07 with the main dimensions marked, and also the mean magnetic circuit ABCDEFGH.

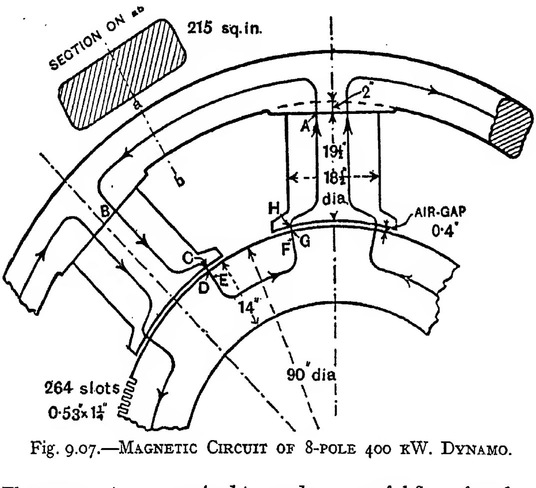


Fig. 9.07.—MAGNETIC CIRCUIT OF 8-POLE 400 KW. DYNAMO.

The ampere-turns required to produce a useful flux of $21{\cdot}6 \times 10^6$ lines will be calculated to illustrate various points in connexion with such calculations. The following further particulars are given:

Magnetic leakage factor 1·2.

Yoke, pole-cores and pole-shoes of cast steel.

Gross length of armature $18\frac{3}{4}$ in., including 6 ventilating ducts $\frac{1}{2}$ in. wide.

Pole face $18\frac{3}{4}$ in. × 25 in.

The flux in the pole-cores and shoes

$$= 21{\cdot}6 \times 10^6 \times 1{\cdot}2 = 25{\cdot}9 \times 10^6 \text{ lines.}$$

The flux in the yoke is half of this, say $13{\cdot}0 \times 10^6$ lines.

The cross-section of the yoke is 215 sq. in., therefore the flux-density is $\dfrac{13{\cdot}0 \times 10^6}{215} = 60{\cdot}5 \times 10^3$ lines per sq. inch.

From Table D (or Fig. 4·09) this requires 16·8 ampere-turns per inch, and mean length of path in yoke is 54 in.;

∴ *ampere-turns required for yoke* $= 54 \times 16{\cdot}8 = 910$ approx.

The flux-density in the pole-cores is

$$\frac{25{\cdot}9 \times 10^6}{0{\cdot}785 \times (18{\cdot}5)^2} = 96{\cdot}5 \text{ kilolines per sq. in.}$$

This requires 58 ampere-turns per inch in cast steel;

∴ *ampere-turns required for pole-cores* $= 2 \times 19\frac{1}{2} \times 58 = 2260$.

At first sight it appears as if the ampere-turns for the pole-shoes should be calculated separately. But the increased cross-section is counterbalanced by the bending of the lines: moreover, the ampere-turns needed for the shoes are so few that it would be a needless refinement to attempt to calculate them separately.

Passing over for the present the air-gap and the teeth of the core, the M.M.F. for the body of the core must be calculated. The mean length of the lines, EF, is 30 in. The cross-section is that below the bottoms of the slots made by a radial plane. The length of the armature, omitting the ventilating ducts, is $15\frac{3}{4}$ in. Part of this is occupied by the insulation of the armature laminations. An allowance of 10 per cent. may be made for this, but it will increase for specially thin plates, and decrease for thick ones: it evidently depends also on the nature of the insulation.

Thus: net length of armature iron $= 15\frac{3}{4}$ in. $\times \frac{90}{100} = 14{\cdot}17$ in.;

∴ cross-section below slots $= 14{\cdot}17 \times (14 - 1\frac{1}{4}) = 181$ sq. in.

This has to carry half the useful flux, *i.e.* $10{\cdot}8 \times 10^6$ lines;

∴ the flux-density is $\dfrac{10{\cdot}8 \times 10^6}{181} = 59{\cdot}7$ kilolines per sq. in.

This requires 9·1 ampere-turns per inch in armature iron;

∴ *ampere-turns for core (omitting teeth)* $= 30 \times 9{\cdot}1 = 273$.

It will be seen from the above that the general plan is to take the length of the mean path instead of considering all the various paths of the lines. Again, the cross-section, and therefore the flux-density is treated as uniform in each portion, whereas really the flux-density must change more or less gradually in passing (say)

from the yoke to the pole-cores. This method is sufficiently accurate for practical purposes in the portions of the circuit dealt with above.

9. Ampere-Turns for Air-Gaps and Teeth

The ampere-turns for the air-gaps and teeth form the greatest portion of the total required for the magnetic circuit of a dynamo, and therefore require more accurate calculation than those for the other parts of the circuit.

The difficulties with regard to the teeth are, firstly, that they taper from top to bottom; and, secondly, that the whole flux does not enter the tops of the teeth, some of the lines passing down the slot and entering the teeth at various points on their sides.

In the air-gap, the crowding of the lines into the tops of the teeth makes the reluctance greater than that corresponding to its width and cross-section. The effect may be considered either as an increase in the average length of path of the lines in air, or as a decrease in the effective cross-section of the gap when approaching the teeth.

In both gap and teeth the effect of "fringing" is to diminish the reluctance. The amount of this will evidently increase for wider gaps, and will depend also on the ratio of the distance between the pole-pieces to the gap. A simple method is to add to the polar arc some multiple of the gap length and use this corrected value in obtaining the reluctance of the gap. The following table (from F. W. Carter's investigation) gives the amount to be added if the pole edge is perpendicular to the armature surface:—

$\dfrac{\text{Interpolar arc}}{\text{gap length}}$	10	15	20	25	30	35	40	50	60
Multiple of gap length	2·43	2·92	3·28	3·56	3·78	3·98	4·14	4·40	4·66

E.g. if the interpolar arc is 20 times the gap length (l_a), 3·28 l_a is added to the polar arc in calculating the air-gap reluctance (see Art. 11). For other pole-edge angles the method is as follows.

Since the armature surface is curved an average angle is obtained in the following way. A chord of the pole-bore circle is drawn from the pole-tip to a point on this circle half way from the pole-tip to a point midway between the pole-tips. Calling the angle between this line and the pole-edge γ the above multiples are multiplied by the reduction factor $\{1-(\gamma-90)/200\}$.

For the teeth it is sufficiently accurate to take the number of teeth under a pole with the same fringing allowance added.

10. Calculation of Ampere-Turns for the Teeth

On the assumption that the whole flux enters the tops of the teeth the flux-density increases uniformly from top to bottom. However, **H** (or the ampere-turns per inch) increases, not uniformly, but more rapidly towards the bottom. The most accurate way of allowing for this is to calculate **B** at various points along the tooth and to plot the corresponding values of **H** at these points (see Fig. 9.08). The mean value of **H** can then be obtained, *i.e.* it is the mean breadth of the figure. The area of the figure is (mean value of **H** × tooth length), *i.e.* the M.M.F. required for the tooth. Or if *AT* per inch are used the area gives the ampere-turns needed for the tooth.

To avoid this complication an approximation may be used. *E.g.* take the value of **B** one-third (instead of a half) of the tooth length from the narrow end (*i.e.* the root) as the mean value, and the corresponding value of **H** as its mean.

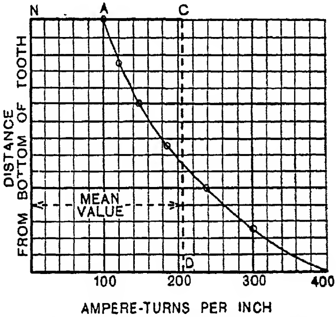


Fig. 9.08.—DETERMINATION OF AMPERE-TURNS FOR ARMATURE TEETH.

(N.B.—This figure is not for the example of § 8.)

Applying this to the example (Art. 8) the following results are obtained:—

$$\text{Tooth pitch} = \frac{\pi \times 90}{264} = 1{\cdot}071 \text{ in.};$$

$$\therefore \text{ No. of teeth under each pole} = \frac{25}{1{\cdot}07} = 23{\cdot}4.$$

As shown in Art. 11:—

$$\text{Allowance for fringing} = 1{\cdot}44 \text{ in.};$$

$$\therefore \text{ No. of teeth in fringes} = \frac{1{\cdot}44}{1{\cdot}07} = 1{\cdot}3;$$

$$\text{No. of teeth to carry total flux} = 23{\cdot}4 + 1{\cdot}3 = 24{\cdot}7.$$

$$\text{Width of tooth at top} = 1{\cdot}071 - 0{\cdot}53 = 0{\cdot}541 \text{ in.}$$

$$\text{Tooth pitch at bottom} = \frac{\pi \times (90 - 2 \times 1\tfrac{1}{4})}{264} = 1{\cdot}042 \text{ in.};$$

$\therefore$ width of tooth at bottom $= 1{\cdot}042 - 0{\cdot}53 = 0{\cdot}512$ in.;

$\therefore$ width of tooth $\frac{1}{3}$ way up $= 0{\cdot}512 + \frac{1}{3}(0{\cdot}541 - 0{\cdot}512) = 0{\cdot}522$ in.;

$\therefore$ area of teeth $\frac{1}{3}$ way up $= 24{\cdot}7 \times 0{\cdot}522 \times 14{\cdot}17^{*} = 183$ sq. in.;

$$\therefore \text{ flux-density } (\tfrac{1}{3} \text{ up}) = \frac{21{\cdot}6 \times 10^6}{183} = 118 \text{ kilolines per sq. inch.}$$

For armature iron this requires 250 ampere-turns per inch;

$\therefore$ *ampere-turns for teeth* $= 250 \times 2 \times 1\frac{1}{4} = 625$.

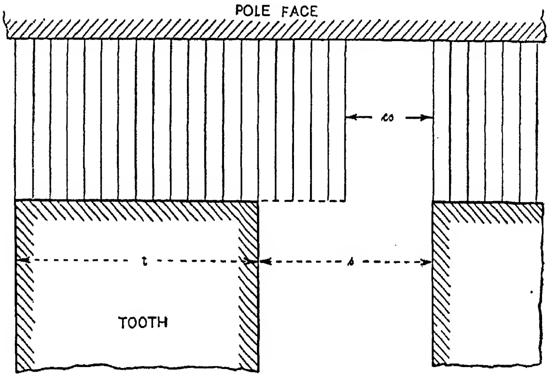


Fig. 9.09.—EQUIVALENT AIR-GAP.

11. Calculation of Ampere-Turns for the Air-Gap

Since in the example (Art. 8) the pole pitch (at the surface of the armature)

$$= \frac{\pi \times 90}{8} = 35{\cdot}3 \text{ in.},$$

$$\text{the ratio, } \frac{\text{interpolar arc}}{\text{gap}} = \frac{35{\cdot}3 - 25}{0{\cdot}4} = 26.$$

Therefore, from the table in Art. 9, the allowance for fringing in the gap is $3{\cdot}60 \times 0{\cdot}4 = 1{\cdot}44$ in.;

$\therefore$ cross-section of gap $= (25 + 1{\cdot}4) \times 18\frac{3}{4} = 495$ sq. in.

* Net length of armature iron, see Art. 8.

The crowding of the lines into the tops of the teeth can be allowed for by decreasing the actual interpolar arc, in a ratio dependent on the relative values of the gap length, tooth width, and slot width.

Consider one slot pitch, made up of one tooth top width t, and one slot width s. The area for the flux at the pole face is proportional to $(t + s)$, while that at the tops of the teeth is proportional to t (see Fig 9.09). Hence the average area is less than $(t + s)$ by some fraction c of s, and so is equal to $t + (1 - c)\, s$. Thus the actual area of the pole-face (corrected for fringing) must be reduced in the ratio

$$\frac{t + (1 - c)\, s}{t + s} = \frac{1 + (1 - c)s/t}{1 + s/t}.$$

c is called Carter's contraction coefficient, and is given for open slots by the formula

$$c = \frac{2}{\pi}\left\{\tan^{-1}\frac{s}{2l_a} - \frac{l_a}{s}\log\left(1 + \frac{s^2}{4l_a^{\,2}}\right)\right\}$$

where l_a = gap length.

Applying this to the example:—

$c = 0{\cdot}20$, and the correcting factor

$$= \left\{1 + (1 - 0{\cdot}20)\,\frac{0{\cdot}53}{0{\cdot}54}\right\} \Big/ \left(1 + \frac{0{\cdot}53}{0{\cdot}54}\right)$$

$$= \frac{1{\cdot}784}{1{\cdot}981} = 0{\cdot}901;$$

$\therefore$ corrected cross-section of gap $= 495 \times 0{\cdot}901 = 446$ sq. in.

In practice it is usual to obtain c from graphs of its value against $\left(\frac{s}{l_a}\right)$. Such graphs may be drawn from the following:—

s/l_a	1	2	3	5	7	10
c	0·15	0·28	0·38	0·50	0·58	0·66
c'	0·2	0·34	0·45	0·60	0·70	0·82

For half-closed slots c' is used in place of c, s being the slot opening, not the full width.

A second correction must be made for the ventilating ducts. The above formula may be used, substituting the distance between the ducts for t and the width of the ducts for s. Since there are 6 ducts, the laminations are divided into 7 sections, and the width of each

$$= \frac{15\frac{3}{4}}{7} = 2{\cdot}25 \text{ in.};$$ whence the correcting factor is 0·96;

$\therefore$ final corrected gap section $= 446 \times 0{\cdot}96 = 428$ sq. in.

Now in air $\quad AT = 0{\cdot}313\ \mathrm{B}''l''$ (see Chapter IV., Art. 18)

and in this case $\mathrm{B}'' = \dfrac{21{\cdot}6 \times}{428} \quad = 50{\cdot}5$ kilolines per sq. inch;

$\therefore$ *ampere-turns for the gaps* $= 0{\cdot}313 \times 50{\cdot}5 \times 10^3 \times 0{\cdot}400 \times 2$
$= 12\,650.$

The results for the whole circuit may be conveniently entered in a table as follows:—

PART OF CIRCUIT	LENGTH. INCHES	CROSS-SECTION SQ. IN.	Φ MEGA-LINES	B KILO-LINES/SQ. IN.	AT PER 1 in.	AT
Yoke	54	215	13·0	60·5	16·8	910
Pole cores	39	268	25·9	96·5	58	2 260
Armature core	30	181	10·8	59·8	9·1	270
Teeth	2·5	183	21·6	118	250	625
Air-gaps	0·800	428	21·6	50·5	—	12 650

Therefore total ampere-turns per pair of poles = 16 715.

12. Calculation of Ampere-Turns to Compensate for Distortion

Particulars as in Art. 8.

No. of conductors under each pole = 198 − 58 = 140;

$\therefore$ cross M.M.F. per pair of poles = 140 × 91 = 12 740 AT.

From the above table of results, the number of ampere-turns per pole-pair used in the gaps and teeth is 13 275 at full load.

$\therefore$ M.M.F. for gap and teeth at trailing tips
$= 13\,275 + 12\,740 = 26\,015$ AT;

$\therefore$ M.M.F. for gap and teeth at leading tips
$= 13\,275 - 12\,740 = 535$ AT.

To find the corresponding flux-densities a number of values of the flux must be assumed and the corresponding AT for teeth and gap calculated, *i.e.* the saturation curve determined.

Useful flux ..	0·4	5	10	15	25	30	megalines
AT for air-gaps ..	234	2 930	5 860	8 780	14 640	17 570	
Flux-density in teeth	2·0	27	54	82	137	164	kilolines / sq. in.
AT for teeth ..	1	10	20	40	1 660	8 650	
Total AT for teeth and gap ..	235	2 940	5 880	8 820	16 300	26 220	

The AT for air-gaps are directly proportional to the flux, since the permeability is constant. For the teeth the flux-density must first be obtained, and then the AT obtained as in Art. 11. At high densities the table given on p. 268 is employed, as Fig. 4.07 does not cover these.

Plotting these values and that already obtained in Art. 11, Fig. 9.10 is obtained. The mean height of FEGML is 18·8 megalines on the flux scale, *i.e.* the effect of distortion is to reduce the useful flux from 21·6 megalines to 18·8 megalines (cf. Fig. 9.06).

To find the AT to compensate for distortion try L′ at 4 000 and therefore M′ at 29 480. This increases the area so that it now represents 22·8 megalines, *i.e.* this is more than sufficient. One or

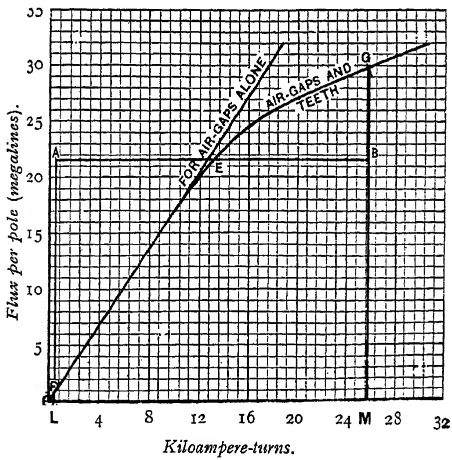


Fig. 9.10.—GRAPHICAL CALCULATION OF COMPENSATION FOR DISTORTION.

two more trials will show that L′ must be placed at about 2 725. Therefore the $\mathcal{A}$ per pair of poles to compensate for distortion $= LL' = 2\,725 - 535 = 2\,190$.

MAGNETIC PROPERTIES OF ARMATURE IRON AT HIGH FLUX-DENSITIES

B. Kilolines per Sq. Cm.	B′ Kilolines per Sq. In.	Permeability μ	$\frac{4\pi}{10}\mu$	$\frac{4\pi}{10}\mu \times 2{\cdot}54$
18	116	162	203	515
18·5	119	141	177	450
19	122½	120	152	385
19·5	126	107	134	340
20	129	94	118	300
20·5	132	81	102	260
21	135½	71	85	215
22	142	52	66	167
23	148	34	43	108
24	155	22½	28½	72
25	161	16½	21	53
26	168	13	16½	42

$$\text{N.B.}—\mathcal{A} = Bl \times 10^3 \div \left(\frac{4\pi}{10}\mu\right)$$
$$= B'l'' \times 10^3 \div \left(\frac{4\pi}{10}\mu \times 2{\cdot}54\right)$$
$$= 0{\cdot}313\,\frac{B'l''}{\mu}.$$

13. Series-Windings of Compound-Wound Generators

In a shunt-wound generator (Chapter X., Art. 4) the whole of the ampere-turns would be provided in the shunt-winding, and their value reduced at light loads by a resistance in series with them. In a compound-wound generator (Chapter X., Art. 6) the shunt-winding provides part of the ampere-turns; and the series-winding another part, which varies automatically with the load. At full load the series ampere-turns provide the turns necessary to compensate for armature reaction, *e.g.* in the example of Arts. 4 and 12,

(6340 + 2190 =) 8530 ampere-turns. In addition they provide the ampere-turns needed to raise the E.M.F. of the generator as the load rises, so as to compensate for armature " drop."

In the above example the flux is that required for an E.M.F. of 570 volts giving 550 volts at the terminals at full load. Thus on open circuit the useful flux should be reduced in the ratio $\frac{550}{570}$. To obtain the corresponding ampere-turns accurately the calculations of Arts. 21-24 are repeated with the flux values reduced in this proportion. The results are shown in the following table (cf. Art. 11):—

NO LOAD AMPERE-TURNS PER PAIR OF POLES

PART OF CIRCUIT	INCHES	KILOLINES PER SQ. IN.	AT PER INCH	
Yoke ..	54	58·4		870
Pole cores ..	39	93·1	45·7	1 780
Armature core	30	57·7	8·8	260
Teeth ..	2·5	114	202	505
Air-gaps ..	0·800	48·7		
			Total	15 615.

[Note that this is less than $\dfrac{550 \times 16715}{570} = 16130$, owing to the increased permeability of the iron.]

The difference is $16715 - 15615 = 1100$ ampere-turns.

Therefore, for level compounding, the series turns at full load must give $(8530 + 1100) = 9630$ ampere-turns per pair of poles. The full load current of the generator is 728 amp. Thus the number of series-turns to be placed on each pole is $\dfrac{9630}{2 \times 728} = 7$ nearly.

For over-compounding a similar calculation will give the additional ampere-turns required for any desired increase of terminal P.D. at full load. The corresponding number of extra series turns is thence obtained as above.

14. Commutation

As has been explained in the previous chapter, the commutator and brushes cause the alternating E.M.F.s of the armature conductors to produce a P.D. always in the same direction between the

generator terminals. Thus when a constant current is flowing in the external circuit the current in each armature coil is exactly reversed each time the corresponding commutator segment passes under a brush. This change of connexions is called **commutation.**

Fig. 9.11 (*a*) represents diagrammatically part of the armature winding and of the commutator near a positive brush.

It will be seen that the coil connected to segments A and B, and all to the left of it (up to the next brush), carry a current from left to right. Similarly the coil connected to D and E, and all to its right, carry a current from right to left. Each of these currents is half the total brush current, whether the winding is lap or wave.

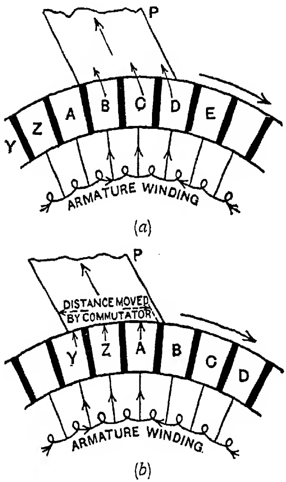


Fig. 9.11.—REVERSAL OF CURRENT IN AN ARMATURE COIL (A B).

(*a*) Before reversal. (*b*) After reversal. P, Positive brush.

In Fig. 9.11 (*b*) the commutator has rotated so that the coil AB now carries half the brush current from right to left, *i.e.* in the opposite direction to the current in the same coil in position (*a*).

When the coil passes the next brush, which will be a —ve one, the current is restored to its original direction, to be again reversed on passing the next +ve brush. These changes occur in every coil in turn.

15. Short-Circuiting of Coils during Commutation

Since the brush must be in contact with at least two segments during portions, if not all, of the time of revolution of the commutator, each coil is periodically **short-circuited.** In other words, there is a local closed circuit consisting of the coil, two commutator segments and connectors, and a portion of the brush.

Referring to Fig. 9.11 (*a*) it will be seen that the short-circuiting of one coil commences when the brush is first touched by the second of the commutator segments (A) to which the coil is connected. The short-circuit ceases when, as in (*b*), Fig. 9.11, the first of the segments, B, moves out of contact with the brush. Thus the time of short-circuit is that required by the commutator to move a

distance equal to the circumferential thickness of the brush less the thickness of one insulating plate of mica.

16. Contact Resistance

Contact resistance between the brush and the commutator is of great assistance in producing good commutation.

The resistance between any two conductors in contact does not follow Ohm's Law like conductor resistance, *i.e.* it is not constant for different currents. It usually decreases with increasing current

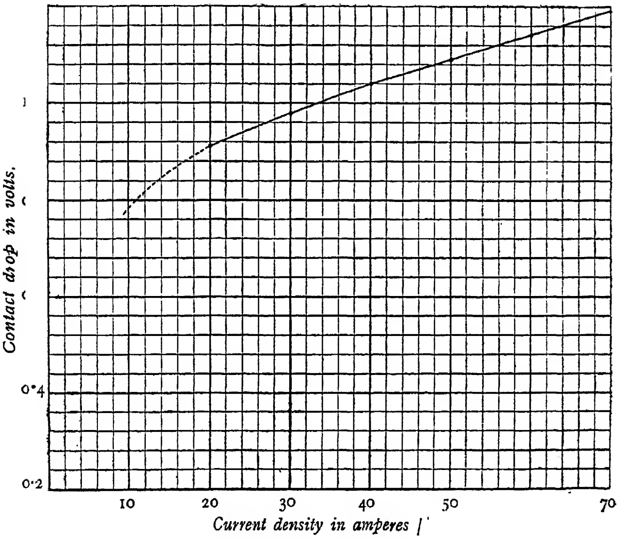


Fig. 9.12.—CONTACT DROP AND CURRENT DENSITY.

densities. For a given current density it varies inversely as the area of contact, resembling conductor resistance in this respect. It is also dependent on the pressure between the surfaces, decreasing with increasing pressure, at first rapidly and then more and more slowly. It varies with the relative speed of the surfaces, but only to a small extent with the speeds used in practice.

Contact resistance has widely different values for different materials. In the case of copper brushes at a current density of 150 to 200 amperes per sq. in., and with a pressure of $1\frac{1}{2}$ lb.

per sq. in. it is about ·0015 ohm for a square inch, *e.g.* with 160 A. per sq. in. the drop at each brush contact is about 0·24 volt.

For carbon brushes at current densities of 25 A. to 75 A. per sq. in., and with a pressure of $1\frac{1}{2}$ lb. per sq. in. the drop per contact is 0·6 volt to 1·4 volt, according to the quality of the carbon, hard carbons giving higher values than soft ones. With special carbons this may be reduced to 0·3 at 30 A. per sq. in. Between these limits of current density the drop for carbon brushes is almost constant; the variation is from 10 per cent. to 30 per cent. with different qualities. In other words, the contact resistance varies almost in inverse proportion to the current density. At higher densities it becomes more nearly constant (see Fig. 9.12). The drop is usually greater at the positive brushes of generators, *i.e.* from copper to carbon, than at the negative ones.

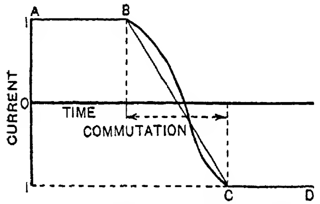


Fig. 9.13.—COMMUTATION WITH CORRECT LEAD.

17. Change of Current during Commutation

If the current in one particular coil is plotted on a time base (Fig. 9.13) it will be represented by a horizontal line AB, *i.e.* constant current, up to the beginning of commutation. From the finish of commutation it will be represented by another horizontal line CD, on the opposite side of the zero line and the same distance from it as AB is, *i.e.* the current has exactly reversed. The way in which the current changes from B to C depends on the conditions under which the coil undergoes commutation.

To simplify matters take the circumferential brush thickness equal to that of a commutator segment and two mica plates, so that only one coil is undergoing commutation at a brush at any instant, another starting as each one finishes.

As already stated, up to the start of commutation the coil carries a constant current, I, from left to right, which, together with an equal current from the right-hand coil goes to the brush by segment B, as shown in Fig. 9.13 (*a*).

At the end of commutation the current in the coil is $-$ I (the negative sign denoting from right to left), and the brush current all

comes from segment A [Fig. 9.14 (*b*)]. The commutator moves a distance equal to the pitch of one segment (= brush width — mica width, cf. Art. 15) while this change occurs. At any intermediate position let I be the current from left to right in the coil; if the current has reversed I_c will be negative.

The current flowing into the brush from segment B will be $I + I_c$, and from segment A, $I - I_c$ [see Fig. 9.14 (*c*)]. As long as I_c is positive the former is the greater, but when the current reverses the latter becomes the greater. In the particular case when I_c is zero, equal currents, I, flow into the brush from each of the segments A and B.

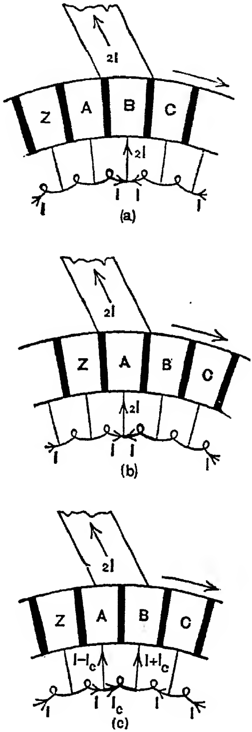


Fig. 9.14.—CURRENTS IN ARMATURE COIL DURING COMMUTATION.

(*a*) Start of commutation.
(*b*) End of commutation.
(*c*) During commutation.

(N.B.—The coil undergoing commutation is shown in thicker lines.)

18. Ideal Commutation

Consider the case in which the current change is as represented by the straight line BC in Fig. 9.13, *i.e.* a uniform rate of change of current. Let t = time of commutation in seconds. Neglect mica width.

The whole change of current in the coil is 2I, from + I to — I. Hence mt'' from the start (where m is any proper fraction), m of commutation is over and the current has changed by m of 2I,

or $$I_c = I - m.2I;$$

$$\therefore \; I_B = I + I_c = 2I - m.2I$$
$$= 2I\,(1 - m),$$

and $$I_A = I - I_c$$

consequently
$$I_A : I_B = m : 1 - m,$$
where I_A, I_B are the currents flowing into the brush from the segments A and B respectively.

During these mt seconds the commutator has moved by m of the width of a segment. Therefore m of A is under the brush. So that I_A is the same fraction (m) of the total brush current, 2I, as the surface of A in contact with the brush is to the whole surface of A. Similarly, m of B has moved from under the brush, leaving $1 - m$ in contact. Thus I_B is to brush current as surface of B in contact with brush is to whole surface of B, just as for A. An alternate way of stating the result is that the brush current is divided between A and B in proportion to their respective surfaces in contact with the brush.

This is equally true for other brush widths, but the proof is more complicated. The mica width modifies it slightly.

This is what would naturally happen if there were no E.M.F. in the coil during commutation, and if the coil and commutator connexions had resistances which could be neglected in comparison with the contact resistances between the segments and the brush. For on these assumptions the current would divide in the inverse ratio of the contact resistances (as for any two resistances in parallel), *i.e.* in the direct ratio of the surfaces. This straight line change of current may be considered as the ideal case which cannot be exactly attained in practice.

19. The Reactance Voltage

Deviations from ideal commutation are due mainly to the reactance voltage, which is the name given to the *self-induced E.M.F.* in the coil. When a current is flowing in any coil it produces magnetic lines linked with the coil. Any change in the current alters the number of lines, and an E.M.F. is produced in the coil, just as if a magnet had been put into it or withdrawn from it (cf. Chapter IV., Art. 24). This E.M.F. is called self-induced because it is caused by the change of current in the coil itself, and is not due to any movement of the coil, nor to a change of current in neighbouring coils.

The direction of the self-induced E.M.F. is always such that it opposes the change causing it.

I.e. if the current is increasing the E.M.F. is against it, but if the current is decreasing the E.M.F. acts in its direction.

This E.M.F. causes the current in an armature coil undergoing commutation to change more slowly than it would under the influence of contact resistance only. *E.g.* in Fig. 9.15 BC represents (as in Fig. 9.13) the change of current during ideal commutation, and BE the change when self-induction is taken into account. The result is that though the current in the coil is reversed it has

not reached the full value (CN) when commutation should be complete, but only some smaller value (EN). A reference to Fig. 9.14 (*b*) will show that a current equal to the difference between these values (*i.e.* CE in Fig. 9.15) is flowing from commutator segment B to the brush at the instant when they part company. This results in sparking, just as when any other current-carrying circuit is broken. The self-induction has a further bad effect in prolonging the duration of the spark and thus intensifying the damage to the commutator.

20. Commutating E.M.F. due to Brush Lead

One method of neutralising the effect of self-induction is by giving the brushes sufficient forward lead to bring the short-circuited coil into a reversing field. It has been shown in Art. 2 that one effect of armature reaction is to necessitate a forward lead of the brushes, so that commutation may take place in zero field. By moving the brushes further in the same direction the coil is commutated in a reversing field, *i.e.* while still short-circuited it is cutting lines of force in the new direction. Thus an E.M.F. is produced which tends to stop the original current in the coil and to start one in the reverse direction.

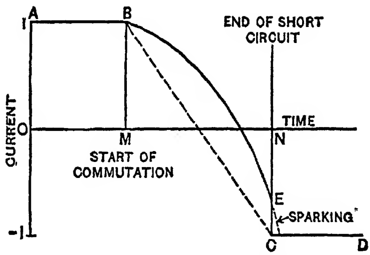


Fig. 9.15.—COMMUTATION WITH INSUFFICIENT LEAD.

If this E.M.F. is equal to the reactance voltage the effect of the latter is wiped out and commutation is perfect. Now as the brushes are advanced the position in which the coils are commutated is moved nearer the pole tips or further under them (see Fig. 9.02, Art. 3), so that the reversing field becomes stronger: consequently a brush position can be found at which the reactance voltage is balanced by the E.M.F. due to the reversing field.

If the extra lead is insufficient to effect this completely, commutation is still somewhat as shown in Fig. 9.15, but CE is reduced and so the sparking is reduced. If too much lead is given the current will be more than reversed, as shown by BF in

Fig. 9.16. This, too, results in sparking, for a current represented by CF is flowing between the brush and commutator segment when they separate: the direction of this current is the reverse of that with no lead (Art. 19).

21. Effect of Changes of Load

Though a position can be found to give sparkless commutation for any steady load, the best position changes with the load. There are two reasons for this change:—

(1) The reactance voltage increases in proportion to the current to be reversed.

(2) Armature reaction weakens the reversing field when the generator current increases.

The truth of the latter statement may be seen by noting that an increase of current causes an advance of the neutral position, viz. that in which commutation occurs in zero field.* Thus with fixed brush position an increase of load reduces the extra lead of the brushes (*i.e.* their lead ahead of the neutral position) and so diminishes the strength of the field in which each coil is while commutation occurs.

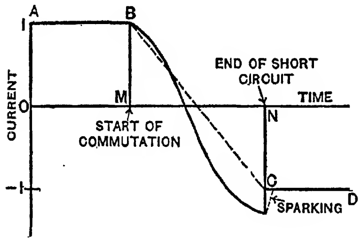


Fig. 9.16.—COMMUTATION WITH EXCESSIVE LEAD.

If then the brush lead has been adjusted so that the reactance voltage is exactly balanced by the E.M.F. due to the reversing field, any change of load will affect the latter in the wrong direction, weakening it when it should be strengthened, and vice versa.

22. Advantages of Carbon Brushes

It has been shown in Art. 18 that contact resistance tends to make the current change during commutation follow a straight line law, and prevent sparking. As long as the difference between the reactance voltage and the E.M.F. due to brush lead is small

* Lead is measured from the midway position, *i.e.* that in which commutation occurs in the coils when they are half-way between the poles (see Art. 3). The neutral position might be called the no-field position, since the former name often is used erroneously for the midway position.

compared with contact resistance drop, the departure from this law is small, and sparking is kept down. Therein lies the main advantage of carbon brushes, for since their contact resistance is high (see Art. 16) sparking can with them be kept within harmless limits even with fixed lead. Copper brushes, on the other hand, having a much lower contact drop, must have their lead increased as the load increases. The lead of carbon brushes should be adjusted to give perfect commutation at about half load: the unbalanced voltage will then be small at all loads from full to none.

Minor advantages of carbon brushes are:—

(*a*) The carbon lubricates and polishes the commutator.

(*b*) If sparking occurs it damages the commutator less than with copper brushes, and damage to the brush itself is of little importance.

Their main disadvantages are:—

(*a*) The contact resistance, which is essential to their benefits, causes a loss of about 2 volts. Hence they are unsuited to low voltage machines, in which this forms a larger percentage loss. The output of the machine does not affect this percentage.

(*b*) Owing to this loss the commutator must be made larger than for copper brushes, so as to dissipate the head produced without greater rise of temperature.

(*c*) Their lower current density necessitates larger brush-holders.

(*d*) Their want of flexibility makes them unsuitable for the high peripheral speeds of turbo-generator commutators. For these brushes of copper gauze or of brass wire are used. Special holders enable carbon brushes to be used sometimes. One successful method is the employment of pneumatic pressure instead of springs.

23. Calculation of Reactance Voltage

The reactance voltage depends upon the number of magnetic linkages (or lines $\times$ turns) with the commutated coil, and the time of commutation, its mean value being equal to

$$\frac{\text{Change of linkages}}{\text{Time of commutation in sec.}} \div 10^8\Big) \text{ volts.}$$

As shown in Art. 15, commutation lasts while the commutator surface moves a distance equal to the circumferential brush thickness less the thickness of the insulation between two segments;

$$\therefore\ t = \frac{w_b - w_m}{v},$$

where t = time of commutation in seconds,
w_b = brush thickness in cm., (or inches)
w_m = mica " " (inches)
v = peripheral speed of commutator in cm. per sec. (in./sec.)

The changes of linkages in a particular machine is almost exactly proportional to the change of current, because a great part of the path of the linked lines is in air, so that the varying permeability of the iron has no appreciable effect. Let the current in the commutated coil change from $+ \mathrm{I}$ to $- \mathrm{I}$ (amperes), and the linkages change correspondingly from $+ \mathbf{L}\mathrm{I}$ to $- \mathbf{L}\mathrm{I}$; so that $\mathbf{L}$ is the linkages per ampere.

Then average reactance voltage $= \dfrac{2\mathbf{L}\mathrm{I}}{t \times 10^8}$ volts.

Consider first the case in which the commutated coil consists of a single loop, and only one is commutated at a time. Hobart's method for obtaining $\mathbf{L}$ is as follows. Divide the loop into "embedded" and "free" portions. The former is that which lies in the slots and so is partly surrounded by iron; the latter consists of the connexions at each end, which are surrounded chiefly by non-magnetic material (see Fig. 9.17). The parts of the core not containing iron must not be included in obtaining the embedded length.

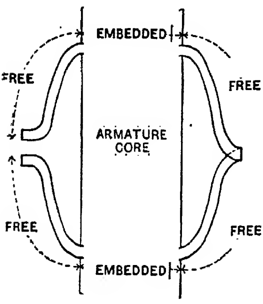


Fig. 9.17.—EMBEDDED AND FREE PORTIONS OF AN ARMATURE LOOP.

Hobart* finds that 1 ampere produces a flux of 4 lines per cm. of embedded length, and of 0·8 line per cm. of free length. But each side of the loop lies in a slot in which another conductor is undergoing commutation under a different brush at the same time. Thus the fluxes and the change of fluxes due to the embedded parts are doubled;

* *C.C. Dynamo Design*, p. 108.

$$\therefore\ \mathrm{L} = 8l_e + 0{\cdot}8l_f = 8\,(l_e + 0{\cdot}1l_f),$$

where

$$l_e = \text{embedded length of loop in centimetres,}$$
$$l_f = \text{free length of loop in centimetres.}$$

Thus the reactance voltage

$$\frac{2\mathrm{LI}}{\times 10^8}\qquad\frac{16\,(l_e + 0{\cdot}1l_f)}{t \times 10^8}$$

(If l_f cannot be obtained more exactly it may be assumed equal to 3 × pole-pitch.)

If the coil instead of being a simple loop consists of several turns (represented by $\mho$) the voltage will be (number of turns)^2 times as great. Because with a given current the flux will be $\mho$ times as great, and each line links with each of the ($\mho$) turns: thus the linkages will be $\mho^2$ as many, and the voltage will be correspondingly increased.

Similarly, if m coils are short-circuited simultaneously the flux will be increased nearly m times, since these coils overlap almost completely.

Hence, finally the approximate mean value of the reactance voltage

$$= \frac{16\,(l_e + 0{\cdot}1l_f)\mathrm{I}\mho^2 m}{t \times 10^8}\ \text{volts.}$$

This value should not exceed the total brush contact drop (for two contacts) except in large low-speed machines, when it may rise to twice the total contact drop. In motors of ordinary size about one and a half times the total contact drop is allowable.

Example 2. *Calculate the reactance voltage of a* 250 *kW.*, 525-*volt*, 6-*pole lap-wound generator driven at* 220 *r.p.m. It has* 1 200 *armature conductors and* 600 *commutator segments. The outside dimensions of the armature are* 64 *in. diam.* × 12 *in. long. Net length* 9 *in.*

As shown in Art. 14, $\dfrac{\text{Time of commutation } (t)}{\text{. of coils shorted simultaneously } (m)}$

$$\frac{\text{Commutator segment pitch } (w_c + w_m)}{\text{Peripheral velocity } (v)} = \frac{\dfrac{\pi \times \text{diam. of comm.}}{600}}{\dfrac{220}{60} \times (\pi \times \text{diam. of comm.})}$$

$$\frac{60}{220 \times 600}\cdot\ \text{c.} = \frac{}{2200}\ \text{sec.}$$

$$\text{Total armature current} = \frac{250000}{525}$$

$$\therefore\ \text{Current per conductor} = \frac{476}{6} = 79\ \text{A.}$$

$$\text{Embedded length} = 2 \times 9 \times 2{\cdot}54 \text{ cm.} = 45{\cdot}7 \text{ cm.}$$

$$\text{Free length} = 3 \times \text{pole pitch} = 3 \times \frac{\pi \times 64}{6} \times 2{\cdot}54 \text{ cm}$$
$$= 256 \text{ cm.};$$

$$\therefore \text{Inductance (L)} = 8 \times 45{\cdot}7 + 0{\cdot}8 \times 256 = 365{\cdot}6 + 204{\cdot}8 = 570.$$

There is only one turn per commutator segment;

$$\therefore \text{Reactance voltage} = \frac{2\text{LI}m}{t \times 10^8} = \frac{2 \times 570 \times 79 \times 2200}{10^8} \text{ volts}$$
$$= 1{\cdot}98 \text{ volts.}$$

24. Effect of Various Factors on Reactance Voltage

The effect of various factors on reactance voltage, and therefore on the quality of the commutation, can be seen readily by the aid of the formulae of the preceding paragraph.

(*a*) *An increase of brush thickness* increases the time of commutation, but increases equally the number of coils undergoing commutation simultaneously.

For $$t = \frac{w_b - w_m}{v} \text{ (see Art. 23),}$$

and average number of coils short-circuited $= \dfrac{w_b - w_m}{w_c} = m,$

where w_c = pitch of the commutator segments.

E.g. if the brush covers 2 segments and 3 micas there will always be 2 coils short-circuited (see Fig. 9.18), coil number 3's short-circuit starting at the instant when coil number 1's ceases;

$$\therefore \frac{m}{t} = \frac{v}{w_c},$$

i.e. the reactance voltage (see formula Art. 23) is almost independent of the brush thickness, provided this is not less than that of 1 commutator segment and 2 micas.

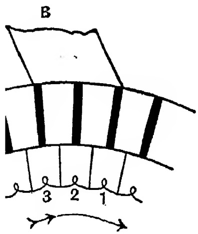


Fig. 9.18.—BRUSH WIDTH TO SHORT-CIRCUIT TWO COILS SIMULTANEOUSLY.
B, Brush.

(*b*) *An increase in the number of commutator segments* reduces the turns per coil ($\mho$) inversely. At the same time it increases the number of coils short-circuited simultaneously (m) in direct proportion. Since the reactance voltage varies as $\mho^2 m$ the combined effect is to change the reactance voltage *inversely* as the number of segments. This increase can be carried on of

course only to the point where each coil contains only one turn. In small dynamos it cannot be carried as far, because the thickness of a segment and a mica cannot conveniently be made less than 0·2 in.,* and so the number is limited by the diameter of the commutator.

(*c*) *A change in the diameter of the commutator* makes no difference, since for a given brush width the time of commutation and the number of coils shorted both vary inversely as the diameter. Or alternatively, the peripheral velocity (v) and the pitch of the segments ($= w_c$) both increase in proportion to the diameter: therefore, as shown in (*a*), $\frac{m}{t}$ is unchanged, and so is the reactance voltage.

(*d*) *Change of Speed of Generator.*—The time of commutation varies inversely as the speed (r.p.m.) of the machine, consequently the reactance voltage is directly proportional to the speed. Thus a given generator with a fixed current flowing in it is more liable to spark the higher the speed (see further Chapter XI., Art. 14).

(*e*) *Change of cross-section of the brushes* alters the mean current density inversely, and changes the contact resistance to some extent. Too low a current density is bad, because it reduces the contact resistance and, even more important, reduces the difference of the contact drops caused by any departure from ideal commutation. At the same time a high current density increases the contact resistance loss and may cause trouble through overheating of the brushes and commutator. The most suitable value depends on the quality of the carbon used and on the reactance voltage of the dynamo (cf. Art. 23).

25. Interpoles

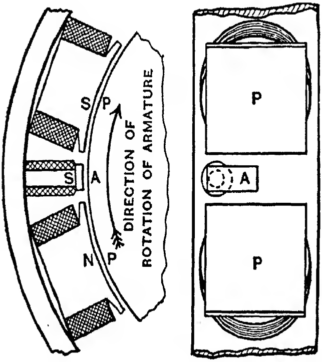
A second method of producing a commutating E.M.F. is by means of *Interpoles*, also called commutating poles. With these the brushes have no lead, but a reversing field is provided by means of special poles placed midway between the main poles. The winding on the interpoles carries the full armature current or a definite fraction of it, *i.e.* they are series wound (see Chapter X.).

The field produced is therefore nearly proportional to the armature current, and so with a suitable winding the reversing E.M.F. will nearly balance the reactance voltage at all loads, since

* Smaller sizes are sometimes used if the design requires these.

the latter is proportional to the armature current. It will likewise balance it at all speeds, since both voltages vary directly as the speed. The polarity of each interpole must be the same as that of the next main pole in the direction of rotation, so as to make the induced E.M.F. a reversing one. In the case of motors the opposite polarity is required for the interpole (see Chapter XI., Arts. 3, 12).

Such poles were patented by Menges in 1884, and suggested by various other writers at later dates, but it is only since about 1905 that they have been used extensively. They enable machines to be run at the full load which they can carry without overheating instead of being also limited by sparking considerations (see Chapter XI., Art. 14). Thus the size and the total cost of a machine of given output may be reduced in spite of the extra cost of the interpoles. Moreover, since the fringe of the main poles is not relied on for commutation, the air-gap may be reduced to the minimum determined by mechanical considerations.



Side view. *End view.*

Fig. 9.19.—MAIN POLES AND INTERPOLE OF A GENERATOR.

A, Interpole (S, polarity).
PP, Main poles (N & S).

Thus the weight of copper on the main poles is diminished, so that the total copper on a machine of a given size may require no increase or even be diminished when commutating poles are used. These considerations apply particularly to high speed (*e.g.* turbine driven) generators, and to those of large size. Nevertheless the advantages outlined above make it worth while to employ them in a very large proportion of D.C. generators. They are exceptionally useful in special designs where it is difficult to keep the reactance voltage down to a satisfactory value.

Another field for their employment is where the field ampere-turns are small compared with those of the armature. This always occurs in motors which have to run at widely varying speeds (see Chapter XI., Art. 9), so that interpoles are fitted to most motors, other than very small ones.

26. Proportions and Winding of Interpoles

The interpoles may be placed between every pair of main poles, or only in alternate spaces because a short-circuited coil has its two sides in successive spaces. When circular cores are used for the main poles those of the interpoles should be placed at one side of the yoke, so as to leave more ventilating space between their coils and the main field windings (see Fig. 9.19).

The pole-shoes preferably extend only part way across the armature so as to diminish the inductance of the shorted coil. The "effective" width of the shoe should be such that the coils are under it during the whole of commutation. *I.e.* the width = slot width + distance the armature surfaces moves during the commutation of all the conductors in a slot. It can be shown that this is equivalent to—

$$\text{Effective shoe width} = w_s + \frac{D}{d}[w_b + w_c(n - 1)],$$

where w_s = slot width, w_b = brush width (less mica width for exactness), w_c = pitch of commutator segments, n = number of commutator segments per slot = half number of "coil-sides" per slot, D = diameter of pole-shoe bore, d = diameter of commutator.

The shoes are chamfered so that the width at the face is less than the effective width by twice the air-gap. This gap is usually $1\frac{1}{2}$ to 2 times the main gap. Sometimes the air-gap is the same as that for the main poles, but additional reluctance is provided by a brass or fibre distance piece at the root of the interpole. This reduces the magnetic leakage.

The number of ampere-turns used on interpoles is obtained empirically. Ordinarily the ampere-turns per interpole are made about one and a third times the armature ampere-turns per pole. If only one interpole per pair of poles is used the ampere-turns naturally must be doubled.

27. Compensating Winding

A third method for preventing sparking is the *use of a compensating winding.* This was suggested by Ryan in 1892, and consists of a winding placed in slots in the pole faces, and carrying the full armature current. By connecting it so that the currents flow as shown diagrammatically in Fig. 9.20, it is made to balance the cross-magnetising effect of armature reaction.

Thus the neutral position is kept fixed, and the field strength is not diminished by armature reaction. This is not sufficient for sparkless commutation, since the reactance voltage increases with

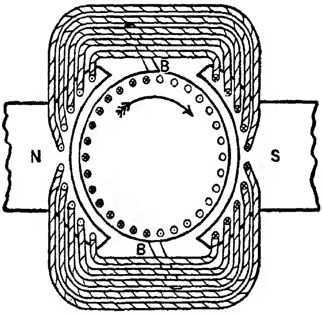


Fig. 9.20.—COMPENSATING WINDING.

B B, Brushes. N, S, North and South poles.

load. But by increasing the ampere-turns in the compensating winding beyond those necessary to balance armature reaction, a reversing field is produced. This can be increased by furnishing unwound interpoles, attached either to the yoke or (Ryan's own method) to the pole-pieces (see Fig. 9.21).

The main use of compensating windings is on turbo-generators with metal brushes.

28. Other Methods of Improving Commutation

These consist chiefly of constructional increases of the cross-reluctance. For instance, by slotting the pole longitudinally (see Fig. 9.22) the distortion of the field by armature reaction is diminished.

In the Lundell pole (see Fig. 9.22) this is combined with a long narrow leading tip in motors (in generators this should be the trailing tip). The effect of the latter is to diminish the extent to which the magnetic lines are distorted towards this tip by armature reaction: for all the lines which emerge from BC have to pass AB, and any increase in their number diminishes the permeability rapidly. The effect of the slot in the pole is somewhat similar, viz. the decreased permeability of the half into which the lines are

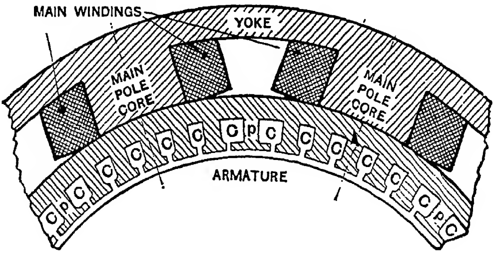


Fig. 9.21.—RYAN'S COMPENSATING WINDING WITH INTERPOLES.

C C C, Slots for compensating winding. $p\,p$, Interpoles.

crowded keeps down the amount of distortion. If the slot were absent the crowding would occur only close to the pole-face.

When laminated pole-shoes are used the tips may be omitted from alternate plates. By omitting the trailing and leading tips alternately the plates may be all made of the same shape [see Fig. 9.23 (*a*)]. This gives a wide fringe to the field, so that the reversing E.M.F. can be easily adjusted by moving the brushes.

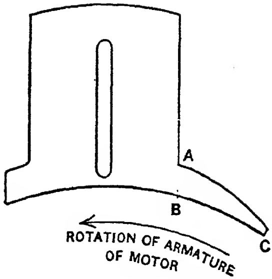
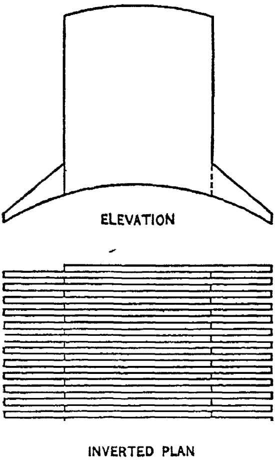


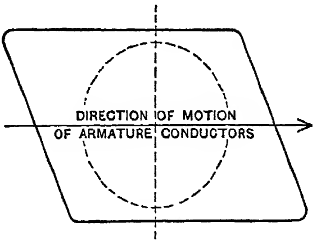
Fig. 9.22.—LUNDELL POLE FOR MOTORS.

A similar effect is obtained by "chamfering" the pole-shoes so that the air-gap gradually widens towards each tip; or by curving the pole-face to a larger radius, which produces a gap which is narrow at the pole centre, increasing all the way from there to each tip. By making the pole face polygonal instead of rectangular; or by "skewing" the pole-shoe [see Fig. 9.22 (*b*)], the field distribution can be modified in a similar manner. With laminated pole-shoes the skewing can be done by varying the positions of the bolt-holes in the plates.

None of these methods, however, produces much improvement in the commutating qualities of a machine, though they are of some use.



(*a*) *Laminated pole shoe.*



(*b*) *"Skewed" pole-shoe.*

Fig. 9 23.—POLES FOR IMPROVEMENT OF COMMUTATION.

29. Miscellaneous Points

The *flux-densities* usually employed in dynamos are as follows:—

	Kilolines per sq. inch.
Yoke, when of cast-steel	60 to 90
„ „ „ cast-iron	30 to 45
Pole-cores, cast-steel or laminated	90 to 110
Air-gap	35 to 60
Armature teeth ($\frac{1}{3}$ way up)	100 to 140
„ core	60 to 80

The *ampere-conductors* (number of conductors × current carried by each) *per inch* of armature periphery are limited by the temperature rise allowable. They vary from about 300 in small machines to 650 in large D.C. ones, and up to 1 100 in large turbo-alternators.

The *armature ampere-turns per pole* are limited by the amount of armature reaction which will not cause commutation trouble. This depends on a number of other factors, *e.g.* the speed, number of turns per commutator segment, etc. Satisfactory commutation will probably result if the armature ampere-turns per pole are about 75 per cent. of the field ampere-turns per pole for the gap and teeth. The proportion may rise to 90 per cent. or even higher when other conditions are favourable. With interpoles 130 per cent. is permissible.

The air-gap varies from $\frac{1}{8}$ in. in small machines to $\frac{1}{2}$ in. in very large ones.

The minimum width of the teeth is seldom below 0·2 in. nor above 0·8 in., and its ratio to the slot width is generally between 0·5 and 1·25. The depth of a slot is usually not more than three times its breadth. In partially closed slots the opening is about $\frac{2}{3}$ of the slot breadth.

QUESTIONS ON CHAPTER IX.

1. How and why does the field strength change if the load changes and the field current is constant?

2. Calculate the back ampere-turns of a 4-pole generator, lap-wound, with 600 conductors, giving 50 amperes, if the actual lead of the brushes is 10°.
By about how much per cent. will the armature flux be diminished if there are 8 000 ampere-turns per pair of poles in the magnet windings?

3. What is the use of the commutator? Describe its action in a simple case.

4. Calculate the reactance voltage for a 6-pole lap-wound armature 56 in. diam. × 10 in. net length, with 500 commutator segments (1 turn per segment), when it delivers 480 A. at 280 r.p.m. Assume that only one section is commutated at a time.

5. Explain why the brushes of a generator must be given a lead. How does the best amount of lead vary with the load? What other effects has armature reaction?

6. Explain how carbon brushes assist in obtaining good commutation under poor conditions.

7. Why are metal brushes generally employed on turbo-generators? What are their advantages and disadvantages?

8. If the current in the shorted coil has fallen to zero when commutation is $\frac{9}{10}$ over, find the direction and magnitude of the resultant E.M.F. in the coil required for this state. Assume peripheral width of brush equal to that of one segment (mica negligible): contact P.D. = 1 volt when whole current is uniformly distributed in brush; neglect changes in contact resistance due to change of current density.

9. Plot during one cycle of commutation the currents from each segment, and in the short-circuited coil, assuming them to be settled by contact resistance alone, for the following case. Total brush current 60 A.

Peripheral thickness of brush equal to one segment and two micas.

Mica thickness $\frac{1}{5}$th of segment.

10. Show graphically the way in which the current in the sections of an armature changes during commutation, (*a*) with the correct lead;
(*b*) with too little lead;
(*c*) with too much lead.

Give reasons for the shapes of the curves; and explain how carbon brushes improve commutation in cases (*b*) and (*c*).

11. Explain why, and to what extent the reversing voltage during commutation depends on (*a*) the length of armature iron;
(*b*) the length of each end connexion;
(*c*) the number of turns per section;
(*d*) the shape and number of slots, and the placing of a section of the winding in the same or different slots.

12. Define the term "reactance voltage" as applied to direct current machines. Calculate its value in the case of a machine for which the data are as follows:—

Net length of armature	12·5 centimetres
Length of mean turn	100 ,,
Current per section	100 amperes.
Turns per segment	1
Width of brush	3 segments
Frequency of commutation	500

[Lond. Univ., El. Mach.

13. Explain how a slot in the centre of the pole reduces the effect of armature reaction.

14. Describe, with sketches, two methods for preventing armature reaction from causing bad commutation under varying loads.

State their relative advantages.

15. What is meant by magnetic leakage? Explain how it is taken into account in calculations on generators and the meaning of a "leakage coefficient" of (*a*) 16 per cent., (*b*) 1·18. Which of these represents the larger amount of leakage?

16. Explain how you would calculate the excitation required for the magnetic field of a dynamo. The air-gap of each pole of a smooth-core 4-pole dynamo is 300 sq. cm. in area, and the distance from pole face to core is 5 mm. How many ampere-turns must be used on each pair of poles for air-gap excitation alone if the flux-density in gap is 10 000 lines per sq. cm.

[C. & G., II.

17. The pole face of a D.C. machine is 8 in. square, the armature teeth are 0·6 in. wide, slots 0·5 in. wide, 1·5 in. deep, air-gap clearance 0·30 in. Find the reluctance of the gap as a whole and calculate the ampere-turns to give a mean gap flux of 40 000 lines per sq. in. [Lond. Univ., El. Mach.

18. Calculate the cross ampere-turns of a 4 pole wave-wound generator with 350 conductors and delivering 120 amperes if the brush lead = ·08 × circumference of commutator.

19. The following figures apply to the magnetic circuit of a generator:—

Ampere-turns per pair of poles:	1200, 2000, 2400, 3000, 4000, 4800
Armature flux per pole (megalines):	1·6, 1·87, 1·96, 2·03, 2·08, 2·09

The field ampere-turns are 2 500 per pair of poles (after subtracting the back ampere-turns of the armature) and the cross ampere-turns are 1 200 per pair of poles.

Find (*a*) the percentage reduction of flux due to the cross ampere-turns,
(*b*) the additional field ampere-turns necessary to compensate for the effect of the cross ampere-turns.

CHAPTER X

CHARACTERISTICS AND CONSTRUCTION

1. Characteristic Curves

The behaviour of dynamos with the various types of field winding can be compared by means of characteristic curves (or simply *characteristics*). These are, for generators, curves connecting voltage and current when the generator is driven at a constant speed. They are of three kinds according to the voltage and current plotted, and are named according to the following table:—

Name	Volts at Constant Speed	Current in
External characteristic	At terminals	External circuit
Total ,,	Generated E.M.F.	Armature
Magnetic ,,	,, ,,	Field winding

The external characteristic is of most importance in considering the suitability of a generator for a given purpose. It can be obtained directly by a series of simultaneous measurements with a voltmeter and an ammeter.

The total characteristic is of interest chiefly for the designer. It cannot be obtained directly by experiment; since a voltmeter cannot read the E.M.F. generated on load, owing to the volts lost in the armature resistance. The necessary methods to obtain it are explained in Art. 7.

The magnetic characteristic can be obtained by measuring the armature voltage at constant speed on open circuit. The excitation is supplied from a separate source, and simultaneous readings of voltage and of field current are taken. The voltage measured is directly proportional to the magnetic flux in the armature (see formula in Chapter VIII., Art. 14), and the current is proportional to the ampere-turns, and therefore to the M.M.F. producing this flux. Thus the shape of the curve depends on the dimensions of the magnetic circuit and on the magnetic qualities of its materials.

2. Resistance and Output Lines

The information obtainable from the external characteristics of generators can be increased by drawing certain other lines on the

diagram. For instance, let it be required to find the current and voltage of a generator of known characteristic when connected to a given external resistance. Let ABC in Fig. 10.01 be the external characteristic. Since the external resistance is fixed so is the ratio $\frac{\text{terminal volts}}{\text{external current}}$. The required point must therefore lie on a certain straight line, OP, through the origin. This can be drawn by taking any convenient point giving the required ratio of volts

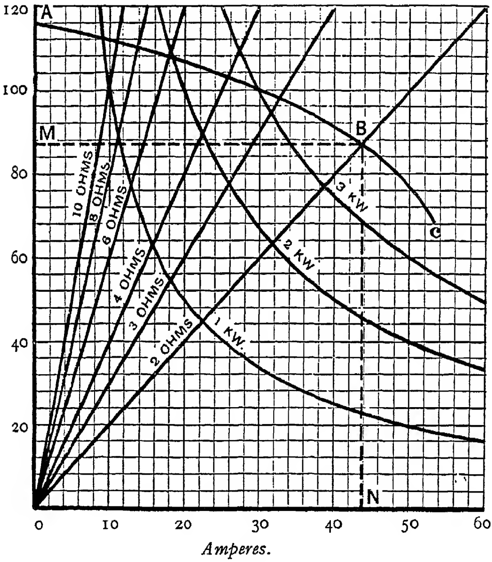


Fig. 10.01.—CHARACTERISTIC WITH RESISTANCE AND OUTPUT LINES.

to amperes, and joining it to the origin: *e.g.* if the external resistance is 2 ohms the point representing 200 volts and 100 amperes (or 100 volts and 50 amperes, etc.) may be taken.

The point, B, where this straight line cuts the characteristic then gives the terminal voltage (BN) and external current (BM) under the given conditions.

A straight line such as OBP is called a resistance line, and a number of them can be drawn readily, as is shown in the diagram.

The *output* of a D.C. generator is equal to the product of the terminal volts and the amperes in the external circuit. Hence to obtain the conditions corresponding to a given output a curve must be drawn such that (volts × amps.) is constant, *i.e.* a rectangular hyperbola. These output or kilowatt lines are not so useful as the resistance lines. The simplest way of drawing them is by obtaining a number of points giving the desired product and joining them. Three are shown in Fig. 10.01 with outputs marked. The point (or points) where they cut the characteristic shows the terminal P.D. and external current of the generator when it is giving the corresponding output.

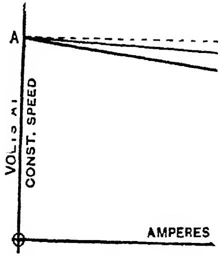


Fig. 10.02.—CHRARACTERISTICS OF SEPARATELY EXCITED GENERATOR.

A B, Total characteristic.
A C, External characteristic.
...... Horizontal line.

3. Characteristics of Separately Excited Generator

The *separately excited* generator supplied with constant field current has an external characteristic like that shown in Fig. 10.02. The fall of voltage with increase of current is due to two causes, (*a*) the "armature drop," viz. the volts used in sending the current through the armature resistance, (*b*) the weakening of the field by armature reaction.

If the former only were present the total characteristic would be a horizontal straight line, for the E.M.F. would be constant. Owing to the second cause it actually lies between such a line and the external characteristic.

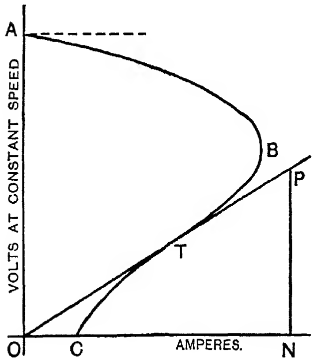


Fig. 10.03.—EXTERNAL CHARACTERISTIC OF SHUNT-WOUND GENERATOR.

4. Characteristics of Shunt-Wound Generator

The external characteristic of a *shunt-wound* generator is shown in Fig. 10.03.

The drop in volts as the current increases is more rapid than for a similar, but

separately excited machine. The reason is that, in addition to the two causes for drop in the latter, the field current of the shunt-wound machine diminishes as its terminal voltage falls, thus further weakening the field.

The resistance of the field winding is supposed to be constant in obtaining the characteristic, so the generator should be run for some time (several hours if it is a large one) before readings are taken. Neglect of this precaution will increase the voltmeter readings, the effect being greatest at first and gradually diminishing as the winding heats up.

As the external resistance is decreased, a point is reached (B on the curve) at which any further decrease of the resistance causes a *decrease* of current owing to the rapid fall in the terminal volts. If the resistance is decreased sufficiently the voltage falls to zero, and the E.M.F. to the voltage due to residual magnetism.

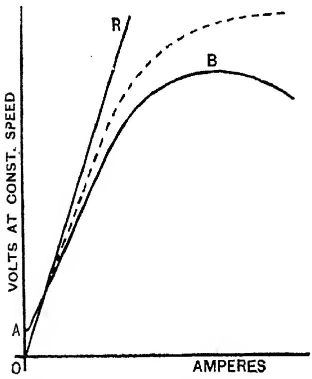


Fig. 10.04.—CHARACTERISTICS OF SERIES-WOUND GENERATOR.
......Total and Magnetic characteristic. ABC, External characteristic.

Let OTP be a tangent to the characteristic, and PN perpendicular to the current axis. Then

$$\frac{\text{PN measured on volt scale}}{\text{ON measured on ampere scale}}$$

gives the resistance in ohms at which a large decrease of current occurs for a small reduction of resistance.

The external resistance must therefore be greater than the above value or the generator will "fail to excite." And if it is less than the resistance corresponding to point B the voltage will remain on the lower branch (CTB) of the characteristic. The higher the external resistance the higher the voltage, hence it is best to start a shunt-wound generator on open circuit, and to keep the external circuit open till the machine has "built up" to full voltage.

5. Characteristics of Series-Wound Generator

In a *series-wound* generator the field current is the same as the main current, consequently the voltage increases with the current.

At first this increase is proportional to the current, but at higher values the voltage increases less rapidly (see Fig. 10.04) owing to the decreasing permeability of the iron in the magnetic circuit. The total characteristic continues to rise though more and more slowly, but the external characteristic rises to a maximum at B and then falls. The reason for this is that after a certain point the increase of E.M.F. generated becomes less than the increased drop due to armature resistance (see Art. 7).

The magnetic characteristic coincides with the total characteristic in this type if armature reaction is negligible; and the magnetic characteristics of other types are similar to this.

If a resistance line such as OR be drawn, it will be seen that the generator will fail to excite almost entirely until the external resistance is reduced *below* the value corresponding to OR. If the resistance is slightly changed from this value the current and voltage will change by large amounts, *i.e.* the generator is in a state which has very slight stability. To obtain good stability the generator must be worked beyond the steep part.

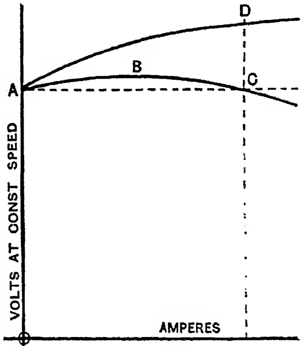


Fig 10.05.—EXTERNAL CHARACTERISTICS OF COMPOUND-WOUND GENERATORS.
ABC, Flat-compounded.
AD, Over-compounded.

6. Characteristics of Compound-Wound Generator

Since the effect of increase of load is to diminish the voltage of a shunt-wound generator and to increase the voltage of a series-wound one (except with very large currents), it is possible by combining the two methods to make the terminal voltage at full load the same as that at no load. The external characteristic of such a generator is shown by ABC in Fig. 10.05. It should be noted that at loads below full load the terminal P.D. will be slightly higher, and at overloads lower than the no load value.

The point, C, at which the generator is flat-compounded, *i.e.* has the same P.D. as at no load, can be made to occur at any one load. By choosing this point at about $\frac{2}{3}$ full load the fluctuations in the P.D. are made small over the range of full load. For certain

purposes (see Art. 10) it is desirable to make the P.D. rise when the load increases. This is done by increasing the number of series turns. The generator is then said to be over-compounded and has an external characteristic such as AD.

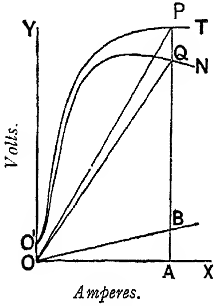


Fig. 10.06.—TOTAL CHARACTERISTIC FROM EXTERNAL FOR SERIES-WOUND GENERATOR.

O'QN, External characteristic. O'PT, Total characteristic.

7. Obtaining Total Characteristic from External Characteristic

The *total characteristic* can be obtained from the *external characteristic* if the resistances of the windings are known, since armature reaction is included in both characteristics.

In the case of a series-wound machine the total resistance $(R_a + R_{se})$ in ohms of the armature and the series winding is alone required. The difference between the E.M.F. and the terminal P.D. is the sum of the armature and series winding "drops," viz.

$$IR_a + IR_{se} = I\,(R_a + R_{se}) \text{ volts,}$$

where I = current in external circuit in amperes.

= current in armature and in series winding.

Then if BA is this difference for any load current OA, and the straight line OB is drawn, it will give the total drop corresponding to any current.

Thus if an ordinate QA be drawn from any point Q on the external characteristic cutting OB in B, the drop at that load is equal to BA. Therefore by producing AQ to P where PQ = BA, a point P on the total characteristic is obtained. A number of such points can be obtained readily and the whole curve drawn.

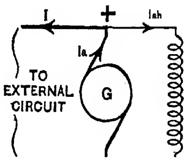


Fig. 10.07.—CURRENTS IN SHUNT-WOUND GENERATOR.

For a shunt-wound generator a double change is necessary.

Firstly— $I_a = I + I_{sh}$ (see Fig. 10.07)

where I = current in external circuit in amperes.

I_a = current in armature in amperes.

I_{sh} = current in shunt winding in amperes.

If the last has not been measured it can be obtained from the formula—

$$I_{sh} = \frac{\text{Terminal P.D.}}{\text{Resistance of shunt winding}}.$$

When the value for any one terminal P.D. has been calculated, *e.g.* FG, for a P.D. equal to OF (see Fig. 10.08) the value for any other P.D. can be obtained by drawing the straight line OG.

Secondly the E.M.F. is obtained from the terminal P.D. by adding the armature drop by means of a construction similar to that for a series-wound generator. This must be done *after* the addition of the shunt current, because the armature drop equals the armature resistance multiplied by the armature (not the external) current.

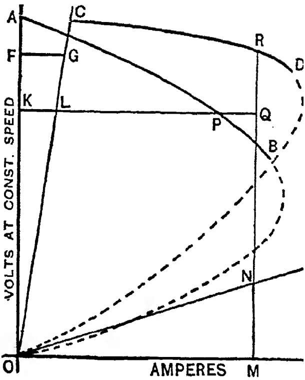


Fig. 10.08.—TOTAL CHARACTERISTIC FROM EXTERNAL FOR SHUNT-WOUND GENERATOR.

APB, External characteristic. CRD, Total characteristic.

KQ : $I_a = I + I_{sh}$ = KP + KL, ∴ PQ = KL.
RM = E.M.F. = Terminal P.D. + armature drop = QM + NM, ∴ RQ = NM.

For a long-shunt compound-wound generator a similar construction serves, but the resistance of the series turns and of the armature together must be used in the second (*i.e.* the voltage) construction.

For a short-shunt compound-wound machine (see Fig. 10.09) three steps are necessary:—

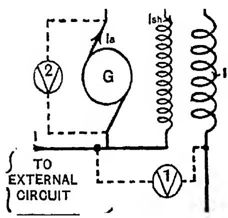


Fig. 10.09.—CURRENTS IN "SHORT-SHUNT" COMPOUND-WOUND GENERATOR.

V_1 = Terminal P.D.
V_2 = Brush P.D.

(*a*) Find the brush P.D. by adding the series drop (IR_{sh}) to the terminal P.D.

(*b*) Find the shunt current from the formula $I_{sh} = \frac{\text{Brush P.D.}}{R_{sh}}$, and add it to the external current to obtain the armature current.

(*c*) Add the armature drop (I_aR_a) to the brush voltage to obtain the E.M.F.

All these can be done graphically as in the other types.

Armature reaction can be taken into account approximately by subtracting a suitable amount from the actual excitation. The proper amount to subtract can be determined by calculating the ampere-turns to balance armature reaction (see Chapter IX.). This method is mainly applicable to the predetermination of the external characteristic of a design from its magnetic characteristic.

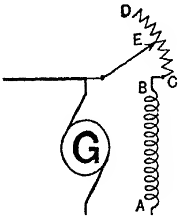


Fig. 10.10—VOLTAGE REGULATION OF SHUNT-WOUND GENERATOR.

A B, Field winding. C D, Field rheostat. E, Movable contact. C, Position of E for maximum volts.

8. Voltage Regulation

The voltage of a generator with a given load current can be altered by changing either its speed, or the strength of the field. The former can be effected by means of the steam-engine stop valve, but this method is employed only occasionally, *e.g.* in ship lighting generators.

The field strength can be altered readily by the use of a variable resistance, which can be placed for convenience close to the voltmeter and other switch gear.

In the case of a shunt-wound (or a separately excited) generator the regulating resistance is placed in series with the field, as shown diagrammatically in Fig. 10.10. A decrease of resistance strengthens the field and so raises the voltage, the maximum value occurring when the resistance is short-circuited.

For a series-wound generator the resistance must be in parallel with the field (see Fig. 10.11), so that it shunts a portion of the main current away from the latter. In this case to raise the voltage the resistance must be increased, and the maximum voltage (for a given main current) is produced when the resistance is disconnected so that the full main current goes through the field winding. If the resistance is short-circuited the voltage falls to a very low value.

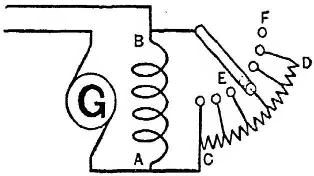


Fig. 10.11.—VOLTAGE REGULATION OF SERIES-WOUND GENERATOR.

A B, Field winding. C D, Field rheostat (or "diverter"). E, Movable contact. F, Position of E for maximum volts.

The voltage of a compound-wound generator is regulated usually by a resistance in series with the shunt winding. A resistance in

parallel with the series winding may be added if it is desirable to regulate the amount of compounding.

The regulating resistances in series with a shunt field winding have to compensate for the increase of resistance due to heating of the coils, in addition to producing the increase of field strength necessary when the load increases.

9. Field Rheostats for Shunt-Wound Generators

If the magnetic characteristic of the machine is known, the effect of resistance in the field circuit can be found by drawing resistance lines. These must correspond with the total field circuit resistance, *i.e.* that of the field winding itself together with that of the part of the rheostat in use. Conversely the resistance needed to produce a given voltage can be found.

If the speed is altered the new voltage or new resistance value can be found either (*a*) by re-plotting the characteristic with its ordinates altered in the same ratio as the speed, or (*b*) by using the old characteristic with the voltage scale (length per volt) altered inversely as the speed; *i.e.* volts per unit length changed as the speed.

Example 1. *A shunt-wound generator has the following magnetic characteristic when driven at* 400 *r.p.m.*:—

Field winding current:	1	2	3	4	5	6	*A*.
Armature E.M.F.:	92	161	205	228	242	248	*V*.

(*a*) *If the field winding has a resistance of* 50 *ohms when cold and* 60 *ohms when hot, find the open circuit voltage in each condition, and when an additional resistance of* 15 *ohms is put in series with the hot winding.*

(*b*) *Find the resistance to give* 210 *V. on open circuit.*

(*c*) *If the machine is driven at* 500 *r.p.m., find the voltage with a field circuit resistance of* 80 *ohms.*

The characteristic is plotted in Fig. 10.12.

(*a*) OA, drawn through the point 250 V.-5 A., is the 50-ohms line; it cuts the characteristic in B, showing that the machine generates 239 V. when field winding is cold. OG, through 240 V.-4 A., cutting graph in D, gives open-circuit voltage 223 V. when field winding is hot. With 15 ohms additional the total field circuit resistance is 75 ohms. OE, through 225 V.-3 A., gives this, and its intersection F with graph shows that open circuit voltage is lowered to 177 volts.

(*b*) Resistance = 210/(3·19) = 66 ohms.

(*c*) The volts scale is altered in ratio 400/500, so that what previously represented 200 V. now represents 250 V. On this scale OG (broken line) through 240 V.-3 A. gives 80 ohms, and its intersection H shows that open circuit voltage is now 266 volts.

10. Uses of the Various Types

Either separately excited or shunt-wound generators, with field regulators, are almost universal for all ordinary lighting and power

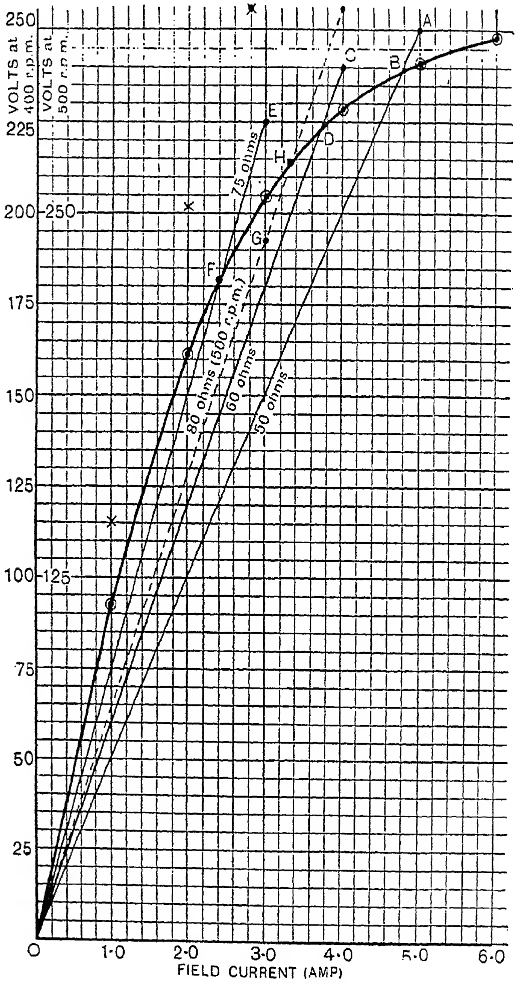


Fig. 10.12.—EFFECTS OF FIELD RESISTANCE ON VOLTAGE.

regulation (see Vol. II.). It is used also for isolated lighting plants where continuous supervision cannot be given. Over-compounding is used when the generator is some distance from its load. The rise of generator voltage then compensates for that lost in the cables. Series-wound generators are used chiefly for arc lighting, either for single lamps (*e.g.* cinematographs and searchlights) or for a number in series, or for welding. Their other use is in the Thury high voltage constant current system. They are nearly always arranged to give approximately constant current, *i.e.* to work on the drooping part of their characteristic (see Art. 5). The droop is increased by designing them with a large armature reaction.

11. Reversal of Rotation

If a separately excited generator is driven in the reverse direction its polarity is reversed, the positive terminal becoming negative,

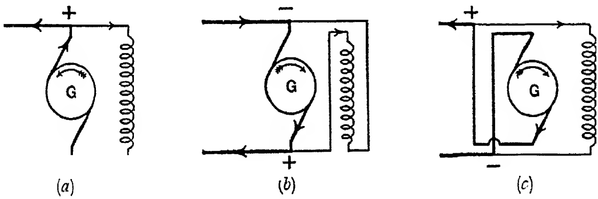


Fig. 10.13.—REVERSAL OF ROTATION OF SHUNT-WOUND GENERATOR.

(*a*) Original connexions. (*b*) Field connexions reversed (for reversed rotation). (*c*) Brush connexions reversed (for reversed rotation.)

and vice versa. The original polarity can be restored by reversing the field current, or by interchanging the connexions between the brushes and the terminals.

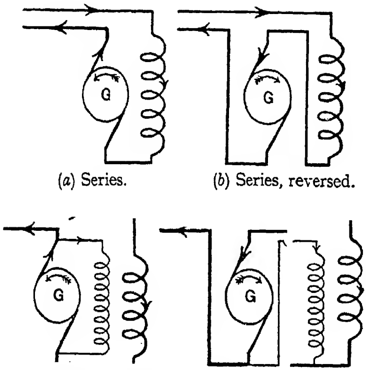
A shunt-wound generator rotating the reverse way will fail to excite. The reason is that the E.M.F. due to the residual magnetism will be reversed. This will send a current through the field windings tending to wipe out the residual magnetism, instead of increasing it and thus building up the field. To make the machine generate properly the connexions between the brushes and the field-winding terminals must be interchanged (see Fig. 10.13). Two slightly different ways of doing this are possible.

The second can be done by moving the brush rocker round in either direction by one pole-pitch (*i.e.* $\frac{1}{4}$ revolution in a 4-pole machine, and so on). If this method is adopted the terminals retain their original polarity.

The same applies throughout to series-wound and to compound-wound generators, the connexions for the original and for reversed rotation being as shown in Fig. 10.14.

12. Reversal of Polarity

The polarity of either a series-wound or a shunt-wound generator depends on the residual magnetism. If the connexions of the field winding to the brushes are correct, and if the external resistance is suitable (low for the series-wound, see Arts. 4, 5), the field will be built up gradually from the residual magnetism. Hence if it should be desirable to reverse the polarity, this can be effected by reversing the residual magnetism (*e.g.* by a field current from any independent source), and then running the generator in the usual way. For most practical purposes it would suffice to interchange the main connexions, thus leaving the polarity of the machine unchanged but reversing the current through the external circuit.



(*a*) Series. (*b*) Series, reversed.

(*c*) Compound, original connexions. (*d*) Compound reversed.

Fig. 10.14.—REVERSAL OF ROTATION OF SERIES- AND COMPOUND-WOUND GENERATORS.

The two types of winding differ, however, in their behaviour regarding undesirable reversal of polarity. If the external circuit contains an opposing E.M.F. it may accidentally overpower that of the generator and cause a reversal of the armature current.

In a series-wound generator this implies a reversal of the field current and therefore of the polarity. The two E.M.F.s will then be in the same direction, resulting in an excessive current and damage from overheating.

In a shunt-wound generator the field current remains in its original direction (see Fig. 10.15), and so the two E.M.F.s remain in opposition. The current will therefore be kept down to a reasonable value, and on the removal of the accidental cause of the

excess of the opposing E.M.F. the currents will return to their original directions.

The most usual case of this sort is in the charging of accumulators. Hence shunt-wound generators are suitable for this purpose, but series-wound ones are not. Even a compound winding is inadvisable because of this effect with its series turns. Moreover, since a series-wound generator will not excite on open circuit it is impossible to charge cells with one, except by special and troublesome arrangements.

13. Generators in Parallel

In a D.C. power station a number of generators are usually connected in parallel. This enables the number of generators in use to be adjusted to suit the load; and permits any generator to be disconnected for repairs without interruption of the supply (see further Vol. II.).

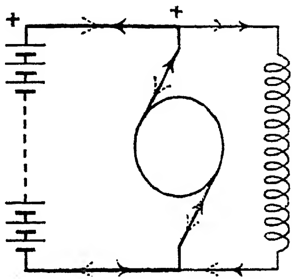


Fig. 10.15.—REVERSAL OF ARMATURE CURRENT IN SHUNT-WOUND GENERATOR.

← Orignal direction of current. ⇢ New direction of current.

The way in which two or more generators in parallel share a given load depends on the characteristics of the generating sets. These differ from the characteristics of the generators themselves in taking into account the changes of speed due to changes of load. The extent of these changes depends on the governing arrangements of the driving engine.

When these characteristics are known a combined characteristic can be obtained by adding the separate currents at a number of equal voltages (since generators are in parallel). From this the voltage for any combined load can be read off, and thence the current supplied by each machine found (see Example 2).

If the characteristics can be represented with sufficient accuracy by straight lines the results can be obtained by calculation instead of graphically (see Example 2).

The same method can be used to deal with the generators in parallel with a battery, treating the battery as an additional generator with its own volt-ampere characteristic.

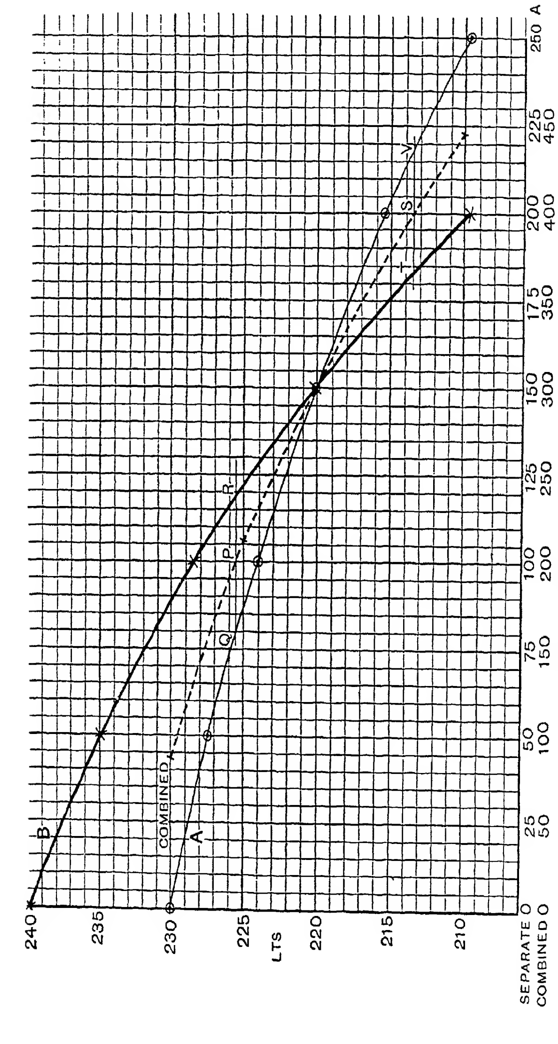


Fig. 10.16.—Characteristics of Two Generators in Parallel.

N.B.—The combined characteristic is obtained by finding the centre points of horizontal lines joining the two separate characteristics.

Example 2. *Two shunt generators, A and B, operating in parallel share equally a total load of* 300 A. *at* 220 *volts. Their load characteristics for the excitations at which they are operating are as follows:—*

A.	*Terminal volts*	..	230	227·5	224	220	215·5	209·5
	Load amperes	..	0	50	100	150	200	250
B.	*Terminal volts*	..	240	235	228·5	220	209·5	
	Load amperes	..	0	50	100	150	200.	

If the total load changes to (*a*) 200 *A.*, (*b*) 400 *A.*, *how will these new loads be shared by the machines?*

What should be done to make the machines share these loads equally?

The given characteristics are plotted in Fig. 10.16. From this the combined characteristic is drawn to half the previous current scale.

(*a*) The point P shows the terminal volts are 225½, and the horizontal line QPR shows the loads are 80 A on A and 120 A on B.

Alternatively:—For currents a small amount under 100 A.,

$$V_A = 231 - 0{\cdot}07\, I_A, \text{ approx.};$$

and for currents a small amount over 100 A.,

$$V_B = 244{\cdot}5 - 0{\cdot}16\, I_B, \text{ approx.}$$

$$\text{Since } V_A = V_B \text{ it follows that } 0{\cdot}16\, I_B - 0{\cdot}07\, I_A = 13{\cdot}5 \ldots\ldots\ldots (1)$$

$$\text{and it is known that } I_B + I_A = 200 \ldots\ldots\ldots\ldots\ldots (2)$$

$$(1) \times 100 + (2) \times 7 \text{ gives } 23\, I_B = 2\,750$$

$$\text{whence } I_B = 120 \text{ A., approx.}$$

(*b*) The point S shows the terminal volts are 213½, and the horizontal line TSV shows the loads are 217 A on A and 183 A on B.

Alternatively the method used for (*a*) may be employed. The approximations for V_A and V_B must be changed since the characteristics are not straight lines.

To make the generators share the load equally in case (*a*) the excitation of A must be decreased or that of B diminished, or both these changes made, by adjusting their field rheostats. In case (*b*) adjustments must be made in the opposite directions.

An alternative is to alter the speeds in a similar way by adjustments to the engine governors.

14. Relations of Dimensions, Speed, and Output

The output of a generator of a given size varies directly as the speed, provided the flux is unaltered. Since suitable values for the flux-density are settled approximately by other considerations (see Chapter IX., Art. 29) this relation (output $\propto$ speed) is nearly true for all usual conditions.

In comparing generators of similar design but different sizes the following approximate relations hold good:—

(*a*) Output $\propto$ length of armature core.

(*b*) Output $\propto$ (diameter of armature core)^2.

The truth of the former of these can be seen by comparing two dynamos differing only in length, the flux-density being kept

constant. The longer one will produce a correspondingly larger E.M.F. and can carry almost the same current, *i.e.* its output is increased in proportion to its length. Further, by altering its winding it may be made to carry a larger current and produce a smaller voltage, keeping the output unchanged at its increased value. The length to be used in this formula is the *gross* length of the core, although the active length of each conductor is a smaller length. The reason for this is that *ventilating ducts* (see Art. 18), which reduce the net length, should be introduced only when they increase the permissible current by improved ventilation, more than they reduce the voltage by diminishing the flux.

Relation (*b*) is approximately true because, firstly, an increase of diameter means a corresponding increase of peripheral speed if the r.p.m. are unchanged. And, secondly, an increase of diameter allows a larger number of conductors (or the same number of larger sized conductors) to be wound on the armature. Each of these changes implies a corresponding increase of output, and therefore the output increases approximately as the square of the armature diameter.

These three relations are summed up by the use of an **output coefficient**, which is defined as being equal to W/d^2ln,

where W = output of dynamo (watts),

n = R.P.M.,

d = diameter of armature core,

and l = gross length of armature core.

If the above relations were exact this output coefficient would be constant. Actually its value rises as the size of the dynamo increases, as shown in the following table for machines without fan ventilation:—

Diameter of Armature		Output Coefficient $\left(\frac{W}{d^2ln}\right)$	
		Centimetre Units	Inch Units
25 cm.	10 in.	·0015	·024
50 ,,	20 ,,	·0024	·039
100 ,,	40 ,,	·0033	·054
150 ,,	60 ,,	·0038	·062
200 ,,	80 ,,	·0042	·068

With fan ventilation the given values may be increased by 25 per cent. up to 20 in. dia., and by ·0006 (cm. units) or ·010 (inch units) for larger sizes.

The reasons for the increase of the output coefficient with size of machine are:—(*a*) The assumption of a fixed flux-density is approximate only; in larger machines a higher value may be used with advantage. For the sizes covered by the above table the average flux-density ranges from 45 kilolines per sq. in. to 65 kilolines per sq. in. (*b*) In larger machines more ampere-conductors per inch (or per cm.) may be used. The variation of this "electric loading" for the above sizes is about from 400 to 1 000 ampere-conductors per inch of armature periphery. It can be shown readily that the output coefficient is proportional to the average flux-density in the air-gap multiplied by the electric loading.

The chief use of the output coefficient is in obtaining approximate dimensions in designing a generator for a given output. It must be regarded only as a rough approximation, and a high value of it is desirable only provided the good qualities of the dynamo have not been sacrificed to it.

15. Speed and Size

The speed suitable for a generator depends partly on its size, and partly on the type of engine driving it. Very large generators are nearly always alternators driven by steam turbines. In these the maximum speed is settled by the frequency used, *e.g.* for the British standard of 50 cycles per sec. two poles require 3 000 r.p.m. This speed is used in all but the very largest machines, which have four poles and are driven at 1 500 r.p.m.

With high speed enclosed vertical reciprocating engines, which were the type most used previous to the introduction of the turbine and which are still useful for small outputs, much lower speeds are suitable. For D.C. generators standard values are 625 r.p.m. for 30 kW., 500 r.p.m. for 100 kW., 300 r.p.m. for 500 kW., and 250 r.p.m. for 1 000 kW.

With other types of steam-engine still lower speeds become necessary, *e.g.* 83 r.p.m. for 1 000 kW. and 107 r.p.m. for 100 kW. with slow speed horizontal engines. Similar speeds are used with oil engines.

The diameter of the armature is limited by the centrifugal force, and the stress caused by this depends mainly on the peripheral speed. Now the peripheral speed is proportional to (armature diameter $\times$ r.p.m.) or $d \times n$. If this is assumed to be constant

the output coefficient (Art. 13) is equal to $\frac{W}{dl \times \text{constant}}$, and therefore the output is approximately proportional to dl. This relation is made use of by means of Steinmetz's output coefficient, which is $\frac{d \times l}{\text{output}}$. If the output is in kilowatts and d and l in inches this coefficient has a value of 2 to 2·5 for dynamos of 500 kW. and upwards, rising to 3·5 for 200 kW. and to 5 for 100 kW. For high speed dynamos these values will be lower.

The uses of this coefficient are the same as that of the one defined in Art. 14, but it should be noted that a high value of the Steinmetz coefficient means a *small* output for a given size.

16. Choice of Number of Poles

The best number of poles to use in a generator of a given output depends on a large number of factors. The only really satisfactory way of determining it is to carry through alternative trial designs with different numbers of poles. The numbers adopted by different makers differ considerably, but the following figures give the general practice for D.C. generators driven by reciprocating engines.

No. of Poles	High Speed Engines	Low Speed Engines
2	Up to 4 kW.	Up to 2 kW.
4	3 kW. to 100 kW.	2 kW. to 40 kW.
6	50 kW. to 250 kW.	30 kW. to 150 kW.
8	200 kW. to 600 kW.	100 kW. to 400 kW.
12	400 kW. to 2 000 kW.	300 kW. to 1 500 kW.
16	750 kW. upwards.	400 kW. to 2 000 kW.

If d^2l is fixed (see Art. 14) the maximum value of d is settled by the permissible peripheral speed. A high value leads to good utilisation of the active material of the armature, because it makes v large in the formula $E = Blv \times 10^8$, and so reduces the total length of active conductor required. The length of the end connexions, however, increases owing to the increased pole pitch, and the magnets and the armature spider become more expensive as d increases. Consequently it is only for high speed generators that the peripheral speed ever limits the diameter.

Since a circle has a smaller periphery than any other figure of the same area it is economical to make the magnet cores circular if possible. Similarly with laminated poles a square cross-section gives a cheaper field winding than a rectangle with unequal sides. With circular poles the ratio of core diameter in length varies directly as the number of poles. It depends also on the ratios $\frac{\text{pole-arc}}{\text{pole-pitch}}$ and $\frac{\text{shoe-length}}{\text{shoe-width}}$. The former is always in the neighbourhood of 0·7. The shoe ratio is the same as that of the magnet core or nearly so, *i.e.* close to unity for cast cores, and rather more for laminated ones; and d/l is settled by these ratios, and so is approximately constant. Thus the number of poles $= \frac{d}{l} \times$ constant, approximately (see example). The same applies to square poles with a different constant. Further, a few large poles require less wire than a larger number of smaller poles. *E.g.* one large circular pole has a periphery only half that of four circular poles of the same cross-section and therefore each of half the diameter of the large pole. The general relation is that the length of wire required varies as $\sqrt{\text{No. of poles.}}$

Even if the circular (or square) shape has to be departed from there is still some saving by the use of few poles.

On the other hand a large diameter improves the ventilation and an increase in the number of poles improves commutation. This is due to the greater subdivision of the current, which diminishes both the reactance voltage and the armature ampere-turns per pole. A rough guide is to take the number of poles equal to (total amperes ÷ 200). In small machines a larger number of poles is usually necessary from considerations of heating and of armature reaction.

Example 3. *Find the overall dimensions of the armature core for a generator to give* 500 *kW. at* 525 *volts, when driven at* 300 *r.p.m.*

Using inch units and assuming an output coefficient of ·06,

i.e.
$$\frac{W}{d^2ln} = \cdot 06$$

$$d^2l \cdot \frac{500 \times 1000}{300 \times \cdot 06} = 27\,800.$$

Therefore possible values are:—

$$d = 60; \quad 55; \quad 50; \quad 45 \text{ in.}$$
$$l = 7{\cdot}7; \quad 9{\cdot}3; \quad 11{\cdot}1; \quad 13{\cdot}9 \text{ in.}$$
$$\frac{d}{l} = 7{\cdot}8; \quad 5{\cdot}9; \quad 4{\cdot}5; \quad 3{\cdot}24.$$
$$2\frac{d}{l} = 15{\cdot}6; \quad 11{\cdot}8; \quad 9{\cdot}0; \quad 6{\cdot}5.$$

(If it is desired to use circular magnet cores, then with ordinary proportions the number of poles $=2\frac{d}{l}$ approximately. For rectangular cores, $2{\cdot}4\frac{d}{l}$.)

Now $$I = \frac{500 \times 1000}{525} = 952 \text{ amperes,}$$

so 6 poles might be used (see above) but probably 8 or 10 poles will be most suitable (see p. 306).

Take 8 poles;

$$\therefore \text{ make } d = 48 \text{ in.};$$

$$\therefore \text{ output coefficient} = {\cdot}057 \quad \text{(Art. 14)};$$

$$\therefore l = \frac{500 \times 1000}{300 \times {\cdot}057 \times (48)^2} = 12{\cdot}7 \text{ in.};$$

$$\therefore \text{ make } l = 12\tfrac{3}{4} \text{ in.}$$

$$= \frac{\pi \times 48 \times 300}{12}$$

which is reasonable.

Diameters up to $(18\,000/n)$ will give peripheral speeds which are allowable.

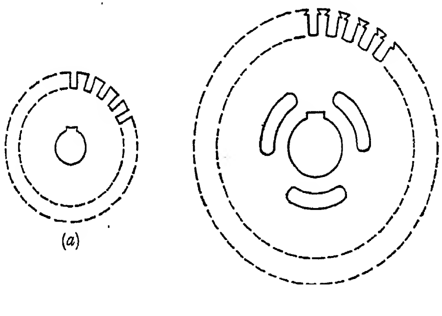


Fig. 10.17.—TOOTHED ARMATURE CORE-PLATES.

17. Armature Cores

The *armature laminations* (see Chapter VIII., Art. 4) are threaded directly on the shaft in the case of small dynamos. In the smallest sizes the iron is continued right down to the shaft [Fig. 10.17 (*a*)]. In larger sizes holes are cut in the plates [Fig. 10.17 (*b*)]; these lighten the armature and improve its ventilation. Nearly all the magnetic lines pass through the ring of iron outside these holes. Thus very few pass through the shaft, and consequently eddy currents in it are almost abolished.

In both cases the laminations are clamped between two end plates, usually made of cast iron. One rests against a collar on the shaft, and the other against a nut which is tightened so as to

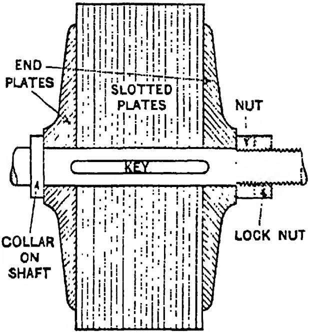


Fig. 10.18.—SECTION OF SMALL ARMATURE CORE.

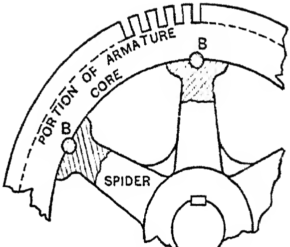


Fig. 10.19.—ARMATURE CORE AND SPIDER.

BB, Bolts.

hold the plates firmly (Fig. 10.18), or against a split-ring sprung into a notch in the shaft. Driving of the plates is effected by a feather on the shaft which fits the keyway in each disc.

In larger sizes still the core-plates become rings, which are carried on a " spider " of cast-iron or cast steel keyed to the shaft. Fig. 10.19 shows one method by which the spider drives the laminations. Bolts run from end to end of the core, each of them lying half in a recess in the ends of the arms of the spider, and half in a similar recess in the plates. End plates are used as before, but they are clamped by nuts on the bolts in this method of construction.

In the largest armatures the core-plates are divided into sections, usually of one of the shapes shown in Fig. 10.20. In pattern (*a*) the projections on the inner sides of the plates fit in corresponding grooves in the ends of the spider arms. They are thus held in position against centrifugal force by this dovetailing arrangement. End rings bolted to the spider arms are used to prevent side play. In pattern (*b*) bolts are run from end to end of the armature through the holes in the core-plates and through the end rings.

In building up an armature core with plates in sections the joints are " staggered," *i.e.* the joints in each layer are opposite the centres of the plates in

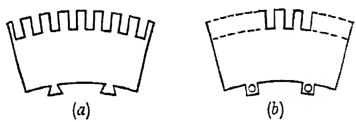


Fig. 10.20.—CORE-PLATES FOR LARGE ARMATURE.

the adjacent layers. This is shown in Fig. 10.21, the full lines denoting the joints in one layer, the dotted ones those in the layer below it. The magnetic circuit is better with staggered joints than if all the joints lay in the same plane.

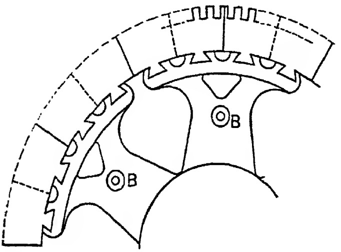


Fig. 10.21.—PORTION OF LARGE ARMATURE CORE AND SPIDER.

BB, Bolt holes and seatings for commutator spider.

18. Ventilating Ducts

Ventilating ducts are usual in all armatures except small or very short ones. One mode of providing these is to use a plate similar to the other core-plates but thicker (say 60 mils). The teeth are twisted at right angles so as to support the teeth of the plates on each side of the duct. The thick plate has also bosses formed on each side for the support of the remaining portions of the plates.

In another method brass distance pieces are used instead of the bosses; and in a third method the teeth are not twisted, but I-shaped steel or brass distance pieces are carried up nearly to the tops of the teeth, and are spot-welded to the thick plate.

The spider and clamping rings must be designed so as to allow air to come along the centre of the armature to these ducts. Sometimes a fan is fitted so as to increase the air circulation. In motors this can be effected by making the pulley spokes like fan blades, and in large generators the same can be done with the commutator spider. These methods reduce the cost, but are less effective than separate fans. These are as large in diameter as is convenient, and fitted on the end away from the commutator.

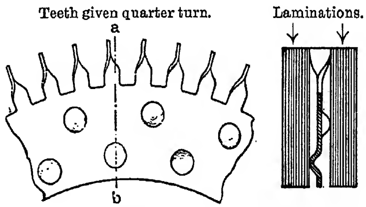


Fig. 10.22.—PLATE FOR VENTILATING DUCT.

In large armatures the ventilating ducts are usually 2 in. to 4 in. apart, and $\frac{1}{4}$ in. to $\frac{1}{2}$ in. wide, and one is placed between each end plate and the stampings.

19. Shafts

Shafts for dynamos are made of best forged mild steel. The spider (or the laminations themselves in small machines) is usually attached to it by a feather, or a keyway and a key, fitting a keyway in the spider. Sometimes two keys 90° apart are used, and occasionally a larger number.

It is important for the shaft to be stiff, since changes in the air-gaps will alter the flux and therefore the E.M.F.s in the various conductors. The forces acting on the shaft are (*a*) the torque transmitted from the steam engine in a generator, or to the pulley in a motor, (*b*) the bending moment due to the weight of the complete armature, and (*c*) magnetic forces if the field is not quite symmetrical.

The last may alter due to wear of bearings or other changes, and is in any case difficult to calculate. A large factor of safety is therefore taken in designing.

Allowing a stress of 8 000 lb. per sq. inch the diameter to transmit the torque is given by—

$$d = 3{\cdot}4\sqrt[3]{\frac{\text{H.P.}}{\text{r.p.m.}}}$$

where d is the diameter in inches.

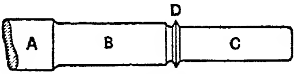


Fig. 10.23.—PORTION OF DYNAMO SHAFT.

A, Portion of armature seating.
B, Commutator seating.
C, Journal.
D, Oil thrower.

The bending moment at the centre can be calculated, an allowance made for magnetic forces, and the necessary thickness at the centre to withstand all three sets of forces obtained. It is more usual, however, to employ a semi-empirical formula, *e.g.* to make the diameter at the thickest part from 3 (in small dynamos) to 2·1 (or rather less in very large ones) times the value given by the above formula; or—

$$d = k\sqrt[3]{\frac{\text{H.P.}}{\text{r.p.m.}}}, \text{ where } k = 7{\cdot}1 \text{ to } 10.$$

Usually the diameter is reduced for the commutator seating, and further reduced for the journals. The total reduction is generally 15 per cent. to 30 per cent. In large generators (over 200 kW.) the commutator is carried on a bracket attached to the armature spider (see Art. 22). The shaft is then tapered from the armature seating to the journals.

Oil-throwers are fitted between the journals and the armature and commutator seatings, to prevent oil creeping along the shaft

and possibly spoiling the insulation of the dynamo. They consist of sharp-edged rings, shrunk on to the shaft or attached to it by screws (see Fig. 10.23).

20. Bearings

Bearings of large machines are always of the self-oiling type. Oil-rings (usually two) rest on the journal and are rotated by it. They dip into an oil well, and so continually bring a supply of oil to the top of the journal (see Fig. 10.24). The height of oil in the well is shown by a glass gauge connected to it by a pipe. A tap is also connected to this to allow the oil to be run off when it has become dirty. The oil-thrower is inside the outer casing. The bush is usually of gun-metal, or of cast-iron with a white metal

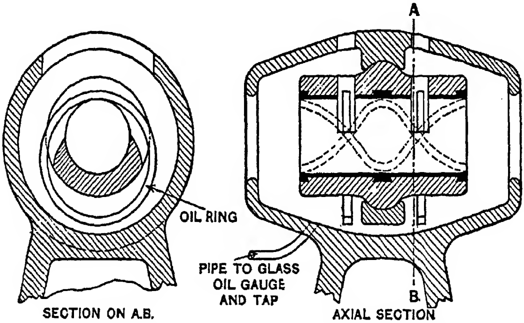


Fig. 10.24.—SELF-OILING BEARING WITH SPHERICAL SEAT.

lining cast in, but sometimes a plain cast-iron bush is used. Spiral channels are cut in the bush to assist the distribution of the oil. Openings with covers are provided in the top of the casing so that it can be seen whether the oil-rings are working properly.

The bearing, especially in high speed machines, is sometimes provided with a spherical seating so as to allow it to swivel somewhat and take up its proper alignment with the shaft.

In small machines ball bearings are often employed. This type is particularly useful for small induction motors in which the air-gaps are very small. For intermediate sizes roller bearings are used.

21. Insulation and Attachment of Armature Coils

In " former-wound " or " wire-wound " armatures (see Chapter VIII., Art. 7) each wire is double cotton-covered (D.C.C.). The

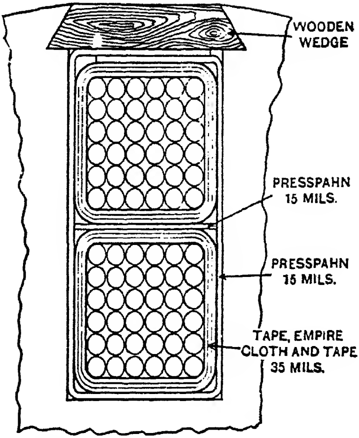


Fig. 10.25.—ARMATURE SLOT SHOWING INSULATION OF COILS.

wires forming each coil are then taped together with a cotton tape 7 mils thick, wrapped on in an overlapping helix so that it makes a covering 14 mils thick. In "bar-wound" armatures the conductors are either taped as above, or covered with a cotton braiding about 15 mils thick.

For large armatures, and sometimes for small ones, the coils are then dried for two or three hours in a vacuum at 200° F., and afterwards impregnated with a plastic insulating varnish. The portions of the coils which go into the slots are then wrapped with Empire cloth and another tape, and are then impregnated as before. The slots are lined with presspahn, or with some equivalent fibrous insulator. Up to 600 volts, the maximum usual for D.C. machines, micanite is not necessary. The coils are then placed in the slots with a strip of presspahn between the upper and lower "coil-sides" in the same slot. If there are more than two conductors (or coil-sides) in a slot it is unnecessary to place further insulation between those in the same layer because the P.D. between them is small, whereas between the upper and lower layers there is the full P.D. of the machine.

In small armatures the coils are wound before impregnation, and the whole armature impregnated when the winding has been completed.

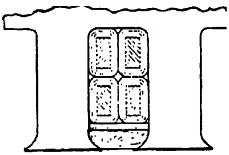


Fig. 10.26.—ARMATURE SLOT WITHOUT SLOT-LINING.

In some cases a slot-lining is not used, the coil-sides receiving thicker insulation and then being placed (see Fig. 10.26) directly in the slots. In this case the coils are often consolidated by pressure in a heated mould before winding. In both cases the coil insulation projects ½″ beyond core.

The coils are held in the slots by wedges of wood or vulcanised fibre. With open slots the wedges are held in grooves punched for the purpose at the top of the slots (see Fig. 10.25). With roofed slots the projections of the teeth serve to hold the wedges (see Fig. 10.26). When no slot-lining is used a strip of mica, presspahn, or red rope paper is placed above the coil insulation to protect it from damage when inserting the wedge.

In all cases binding wires are used to secure the coils, and sometimes these are relied on altogether, wooden wedges being dispensed with. The ends of the coils lying outside the slots are always bound, and the central parts are bound when open slots are used. The binding wire is steel, 25 S.W.G. to 16 S.W.G., and is put on in the form of a band of several turns of one continuous wire. At intervals the wires forming a band are soldered together with a tinned copper clip, and the ends are secured by a special clip of tinned copper. A band of micanite cloth is placed under the wires before winding to protect the conductors.

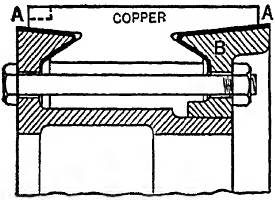


Fig. 10.27.—SECTION OF COMMUTATOR.
A, Micanite V rings. B, Clamping ring.

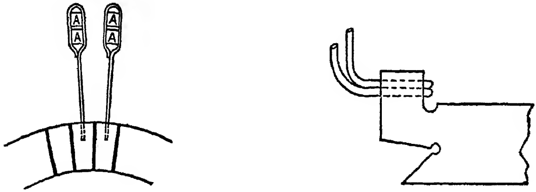
22. Commutator Construction

An example of typical commutator construction is shown in Fig. 10.27 (see also Chapter VIII., Art. 6).

The copper segments are insulated from each other by mica, 25 mils to 40 mils thick. This mica should be soft so as to wear away as rapidly as the copper. They are held between two V-rings, from which they are insulated by micanite rings $\frac{1}{16}$ in. to $\frac{1}{8}$ in. thick. The segments must be suitably tapered, since mica occurs in parallel plates. The thickness of the segments at the top is about $\frac{1}{4}$ in., varying according to the voltage of the machine. The average "volts per segment" should not exceed 15, *i.e.* the number of segments between a positive brush and the nearest negative one should not be less than $\frac{1}{15}$ of the voltage. The peripheral speed is usually not greater than 3000 ft. per min., except in turbo-generators.

The commutator in small dynamos is mounted on a sleeve which is keyed directly to the shaft, or to an extension of the armature sleeve or spider. In larger machines a commutator spider is used, and this is generally bolted directly to the armature spider, or mounted on a sleeve extending from it.

For turbo-generators the commutator has to be long, owing to the high r.p.m. It is therefore supported against centrifugal force by two or more steel rings shrunk on over mica insulation.



(*a*) AA, Ends of armature conductors. (*b*) End view. (*c*) Section. C, Commutator segment.

Fig. 10.28.—COMMUTATOR RISERS.

The radial wearing depth of commutator segments should never be less than $\frac{3}{8}$ in., increasing up to $1\frac{1}{2}$ in. for commutators over 2 ft. in diameter. For intermediate sizes the wearing depth is roughly $\left(\frac{d}{24} + \frac{1}{8}\right)$ in., where d = diameter of commutator in inches.

The connexions of the commutator to the armature conductors are made by strips of tinned copper, called risers. These are sweated into a saw-cut made in the corner of each commutator segment, and rise radially to the ends of the armature coils to which they are connected [see Fig. 10.28 (*a*)]. Sometimes a rivet is used to give additional security.

In wire-wound armatures the ends of the coils are left long enough to be brought down to the commutator and sweated to lugs on the segments [see Fig. 10.28 (*b*)].

In evolute windings (see Chapter VIII., Art. 7) the end connexions may take the place of the risers, or else short risers may be necessary depending on the diameter of the commutator and the length of the evolute connectors (see Fig. 10.29).

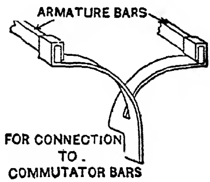
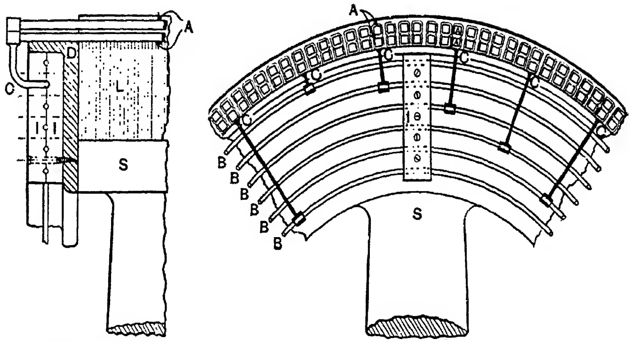


Fig. 10.29.—EVOLUTE CONNECTOR.

23. Equalising Rings

The *equalising rings* are usually mounted on the armature spider, or the extension of it which supports the coil. One form of construction is shown in Fig. 10.30.



Sectional end view. Side view.

Fig. 10.30.—EQUALISING RINGS.

A A, Armature bars.
B B B, Equalising rings.
C C C, Strip copper connexions from A A, to B,
D, Bracket end ring of armature core.
I I I, Insulating blocks.
L, Armature laminations,
S S, Armature spider.

An alternative method is to make them the same diameter as the commutator, and mount them on its spider with insulation between (see Fig. 10.31). They can then be connected to the appropriate segments of the commutator.

A third method is to use double risers for the commutator, so that each coil end is connected to two segments a field pitch (*i.e.* two pole pitches) apart. These double risers are arranged in two layers in the same way as evolute armature connectors (see previous Art.). This method gives very good equalisation, but adds considerably to the amount of copper used.

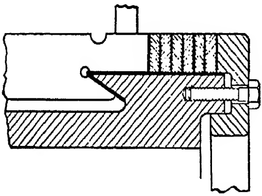


Fig. 10.31.—EQUALISING RINGS.

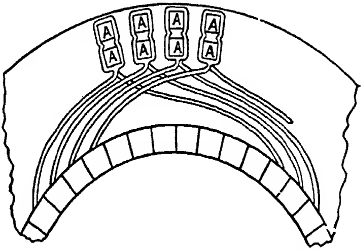


Fig. 10.32.—DOUBLE RISERS FOR "EQUALISING.

A A, Ends of armature bars.

24. Brush-Holders

Brush-holders for carbon brushes are usually of the *box type*. An example is shown in Fig. 10.33. An arm is clamped to the brush spindle. At its outer end it has a box, open at top and

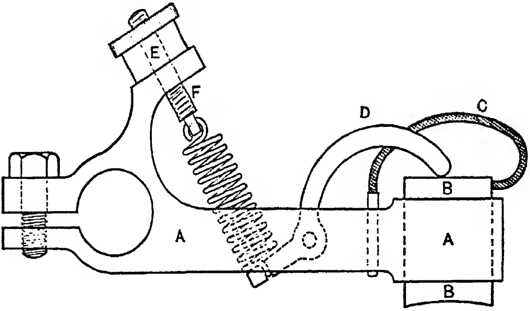


Fig. 10.33.—BOX TYPE OF BRUSH-HOLDER.

A A, Arm clamped to brush spindle.
B B, Carbon brush.
C, Flexible copper connector.
D, Pressure arm.
E, Adjusting nut.
F, Screw with flat.

bottom, in which the brush can slide. The brush is pressed on to the commutator by an arm which is free to rotate, and a spring with a screw and nut for adjusting the pressure. The brush is connected to the fixed arm by a flexible conductor, so as to avoid sending the current through sliding contacts unnecessarily. The flexible connector may be attached to the brush by a screw, or soldered to a clip which can be slid on to the dovetailed end of the brush (see Fig. 10.34). The upper part of the brush is coppered to improve the connexion.

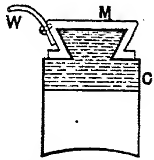


Fig. 10.34.—CLIP FOR CARBON BRUSH.

C, Coppered end of carbon brush.
M, Metal clip.
W, Flexible connecting wire.

An alternative is to make in the brush a hole, wider at the bottom than at the top. The flexible is placed in this, and the whole filled with powdered metal; by compressing this a good connexion is secured.

The advantage of the box type is that the moving parts are light, and consequently the brush can follow irregularities of the commutator even at high speeds. On the other hand there is a tendency for the brush to tilt and so work stiffly in the box.

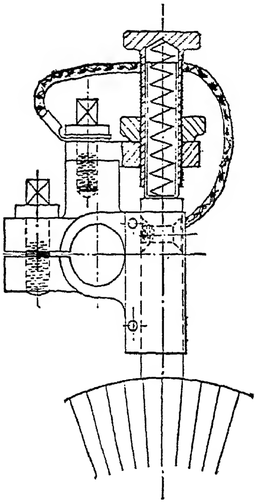


Fig. 10.35.—BOX TYPE BRUSH-HOLDER WITH RADIAL SPRING.

In turbo-generators carbon brushes are sometimes used with pads supplied with air under pressure instead of springs. It was usual to employ copper or brass wire brushes, but carbon brushes have become more general. Another box type brush-holder, made by the Westminster Engineering Co., is shown in Fig. 10.35. Its advantages are that it occupies very little circumferential space and that the spring is entirely enclosed and presses directly on the brush. A modified form with stronger parts is suitable for turbo-generators.

25. Brush Rocker

Each brush spindle carries several brushes, except in the smallest machines, because a number of small brushes make better contact than a single large one. Moreover, if one requires attention it can be removed from the commutator temporarily without interfering with the running of the machine.

All the spindles are attached to the same brush rocker, or brush

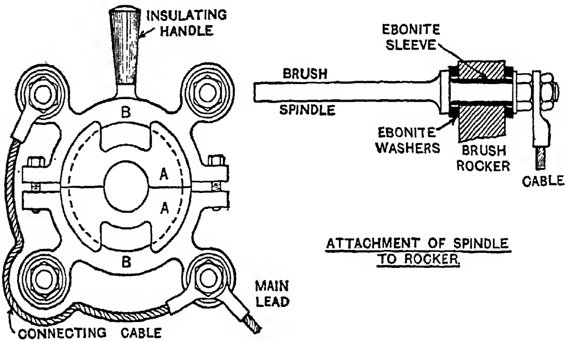


Fig. 10.36.—BRUSH ROCKER.

A A, Supporting pieces clamped to bearing. B B, Halves of brush ring.

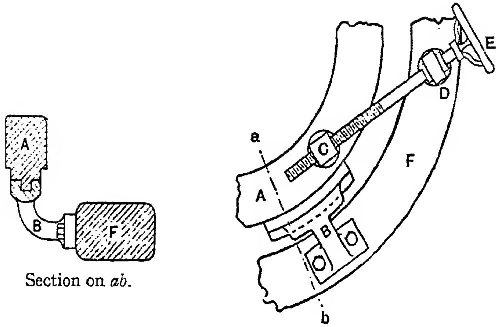


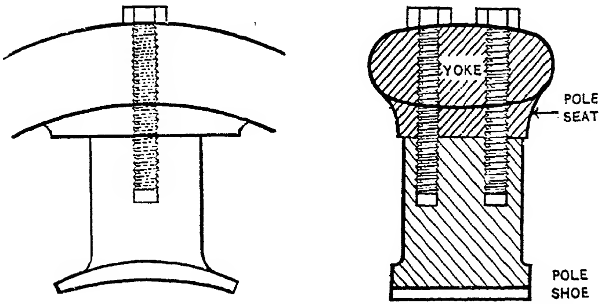
Fig. 10.37.—PORTION OF LARGE BRUSH ROCKER.

A, Rocker ring. B, Bracket attached to yoke. C, Nut pivoted to rocker. D, Collar pivoted to yoke. E, Hand wheel. F, Magnet yoke.

ring, but are insulated from it by a sleeve and washers of ebonite or mica (see Fig. 10.36).

In small machines the rocker is supported on a casting which is either in one piece with the bearing cover, or is attached to it. Alternate spindles are connected together by insulated cables, and the main leads are each attached to one spindle. Only the positive connexions are shown in the figure. A handle is provided for shifting the brushes.

In larger machines the rocker is supported by a number of brackets attached to the yoke. The connexions of alternate spindles are made by two rings of copper strip, with short flexible



(*a*) Side view. (*b*) Sectional end view.

Fig. 10.38.—ATTACHMENT OF POLE TO YOKE.

leads to the spindles. The rocker is shifted by a hand wheel and screw (see Fig. 10.37).

26. Poles

The simplest method of attaching the poles to the yoke is by means of a couple of screws (see Fig. 10.38). When the yoke is of cast-iron the pole seat consists of a wrought-iron plate so as to avoid high reluctance in this part. With cast-steel yokes the pole seat is cast with the yoke, and sometimes the pole-cores are cast with it also.

Laminated pole-cores may be attached by screws holding a bar. This bar is either dove-tailed or circular, and passes through corresponding holes in the pole-core laminations [see Fig. 10.39 (a), (b)]. An alternative method is to cast the poles into the yoke. They are provided with grooves to ensure firm attachment [see Fig. 10.39 (c), (d)].

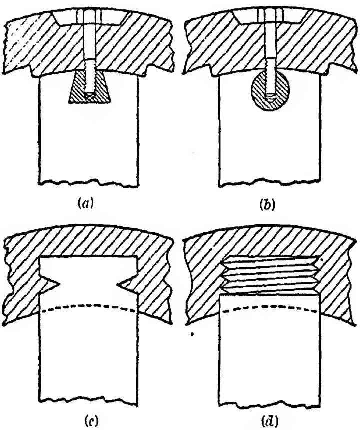


Fig. 10.39.—METHODS OF ATTACHING POLES TO YOKE.

If the poles are "cast-in," or are in one piece with the yoke, the pole-shoes must be separate from them to enable the field coils to be placed on the pole-cores. In this case the pole-shoes are usually attached to the cores by screws.

With poles attached to the yoke by screws the pole-shoes may be in one piece with the cores, but they are sometimes separate in this case also.

Except in small machines the yokes are divided across a horizontal diameter to facilitate removal of the armature, etc.

Laminated pole-shoes are required when the air-gap is narrow and wide, open slots are used. Otherwise eddy currents are produced in the shoes by the swaying to and fro of the lines as the armature teeth pass under the poles; some hysteresis loss is produced by the same cause. The shoes are set in vibration by the varying magnetic pull, and therefore emit a characteristic hum.

QUESTIONS ON CHAPTER X

1. Draw external characteristics for shunt and series generators respectively, and explain their shapes.

2. Draw to scale characteristics of a series-wound and of a shunt-wound generator. From these obtain curves showing how the current varies as the external resistance is altered in each case.

3. For what purposes are shunt-wound, series-wound, and compound-wound generators respectively used? Give reasons.

4. What are the external, total, and magnetisation characteristics of a machine? Draw the first for series- and shunt-wound generators, and show how to obtain the " total characteristic " from the external in a shunt-wound generator, giving a numerical example.

5. Describe an experiment to determine the external characteristic of a shunt-wound generator.
What further measurements are required to obtain the total characteristic?

6. A certain armature generates 115 volts at 600 r.p.m. and can give out 90 amperes.
State the approximate values of the E.M.F. and of the watts or:—

(*a*) The same armature at 1 200 r.p.m.
At 600 r.p.m.: (*b*) An armature twice as long.

(*c*) An armature of double the diameter with conductors of the same size.

(*d*) An armature of double the diameter but with the original number of conductors instead of original size of conductor.

State the assumptions made, and give the general relation for all cases.

7. State approximately the way in which the output of a generator depends on the speed and the dimensions of the armature. What two causes limit the permissible speed of a given armature?

If a 150-kilowatt generator driven at 420 r.p.m. has an armature 41 in. diameter × 9 in. long, find suitable dimensions for the armature of a 400-kilowatt generator driven at 100 r.p.m.

8. Why are multipolar generators usually employed?

9. When must (*a*) the armature core, (*b*) the pole-pieces, and (*c*) the conductors of a dynamo be laminated, and in what direction should the laminations go?

10. Describe and sketch the method of building up an armature core of moderate size. Give reasons for the method and materials used.

11. What are the advantages and disadvantages of fitting a fan to a generator? Why are the advantages less marked (*a*) at very low speeds; (*b*) at very high speeds; than at moderate speeds?

12. A direct current series generator is driven by a motor and connected to a resistance of 50 ohms. It fails to give an appreciable current. State two possible causes for this failure and how you would overcome the difficulty. [C. & G., II.

13. A "long shunt" compound-wound generator driven at constant speed gives the following readings:—

External current: 0; 10; 20; 30; 40; 50 amperes.
Terminal P.D.: 228; 231; 234; 236; 234; 228 volts.
Resistance of armature = 0·18 ohm.
Resistance of series winding = 0·06 ohm.
Resistance of shunt winding = 145 ohms.

Find graphically the total characteristic.

If it were connected "short-shunt" what E.M.F. would be needed to give 228 volts at the terminals at full load (50A)?

14. Show by diagrams all the possible ways of altering the connexions of a compound-wound interpole generator so as to make it suitable for driving in the reverse direction.

15. A shunt wound D.C. generator driven at 500 r.p.m. has the open-circuit curve given by the following table:—

Field amperes	0·5	1·5	2·5	3·5	4·5	5·5	6·5
Open-circuit volts	64	170	242	284	308	320	326

Find the open-circuit voltage of the machine when the resistance in the shunt circuit equals 64, 80, and 90 ohms respectively. What will be the open-circuit voltage at 600 r.p.m. with shunt circuit resistance 80 ohms?

16. Two shunt generating sets are adjusted to share a load of 100 kW. equally at a terminal P.D of 230 V. Assuming that their volt-ampere characteristics are straight lines rising to 240 V. and 245 V. respectively on no load; find how the load is divided between them when the total current is reduced to half its original value.

What is the new terminal voltage?

CHAPTER XI

DIRECT CURRENT MOTORS

1. Dynamo Used as a Motor

Every dynamo can be used as a motor, *i.e.* when supplied with electrical energy it can convert some of this into mechanical energy, and so may be used to drive machinery of any sort. This is the reverse of its action as a generator, when it receives mechanical energy, *e.g.* from a steam engine, and delivers electrical energy.

The motor action depends on the fact that a current-carrying conductor lying across a magnetic field is acted on by a force, perpendicular to itself and to the direction of the field. The direction of the force is given by the Left Hand Rule which is: Place the thumb, fore-finger, and middle finger of the left hand at right angles. Point the fore-finger in the direction of the field, and the middle finger in the direction of the current. Then the thumb points in the direction of the force exerted, and therefore of the motion due to this force (cf. the Right Hand Rule, Chapter VIII., Art. 5). When the field magnets are excited and the armature is supplied with current from an external source, all the conductors under one pole carry currents in the same direction, all those under the next pole (of opposite polarity) carry currents in the opposite direction, and so on round the armature (cf. Chapter VIII., Art. 12).

All the forces therefore tend to rotate the armature in the same direction, and this distribution of currents is maintained by the commutator in spite of the rotation of the armature. Thus a continuous driving torque (*i.e.* twisting moment) is maintained, which keeps the armature rotating unless the resisting torque becomes greater than that which the armature can exert.

In the case of a toothed armature the pull comes mainly on the teeth, but the torque has the same value as for a smooth armature with the same total flux, and the same number of conductors with the same current in them.

2. Back E.M.F. and Torque Calculations

When the armature of a motor rotates its conductors have an E.M.F. developed in them. By Lenz's law (see Appendix A) this E.M.F. opposes the current. This agrees with the result obtained

by applying the Left Hand Rule and the Right Hand Rule to the case of a single conductor.

This E.M.F. is therefore usually called the back E.M.F.

Let E = P.D. applied to the brushes in volts,

E_b = back E.M.F. of the armature in volts,

I_a = armature current in amperes,

R_a = ,, resistance in ohms.

Then $I_a = \frac{E - E_b}{}$, or $E = E_b + I_a R_a$.

The electrical power supplied to the armature is EI_a watts, which is equal to $I_a(E_b + I_a R_a) = E_b I_a + I_a^2 R_a$ watts. The second term represents the power wasted in heating the armature. Therefore $E_b I_a$ gives the remainder of the power which is converted into mechanical power; or in words:—

Mechanical power developed by the armature

= (back E.M.F.) × (armature current).

Let T = torque in lb.-ft.

n = r.p.m. of armature.

Then, since 1 H.P. is 33 000 ft.-lb. per minute, the power of the

armature $= \frac{}{33000}$ H.P. $= \frac{}{33000} \times 746$ watts;

33000

$\frac{33000}{2\pi n \times 746}$]

Now $E_b = \frac{}{60 \times}$ VIII.,

$\frac{}{60 \times 10^8}$.

Moreover, the current in any one armature conductor is $\frac{I_a}{2a}$, so

$\frac{NI_a}{2a}$ is the ampere-conductors on the armature;

∴ the torque in lb.-ft. $= \frac{\cdot 1173}{10^8} \times$ (armature ampere-conductors)

× (flux per pole) × (number of poles).

The above torque is the total torque developed, not the useful torque, *i.e.* it includes that necessary to overcome mechanical friction and the resistance due to the hysteresis and eddy currents in the armature core.

The same formula can be obtained in the following alternative way:—

The force on a conductor carrying a current I in a field of strength **B** is $\frac{\mathbf{B}l\mathrm{I}}{10}$ dynes (see further Example 3),

where l = length in cm. of conductor measured perpendicularly to the direction of the field (*i.e.* if the conductor is not itself $\perp$ to field or is not straight, its projected length on a plane $\perp$ to field is taken).

Let d (cm.) be the diameter of the armature.

Then the average field strength is $\Phi \div \left(\frac{\pi d l}{2p}\right)$, since the expression in brackets is the area over which the Φ lines are spread;

$\therefore$ the average force on a conductor is $\frac{2p\Phi}{\pi d l} \cdot \frac{l\mathrm{I}}{10}$ dynes;

$\therefore$ the average turning moment of one conductor is

$$\frac{\quad}{\pi d . 10} \times \frac{d}{2} \text{ dyne-cm.};$$

$\therefore$ the total torque in dyne-cm. is $\frac{2p\Phi\mathrm{I}}{20\pi}.\mathrm{N}$,

$$= \frac{(\text{flux per pole}) \times (\text{no. of poles}) \times (\text{armature ampere-conductors})}{\quad}.$$

To convert this into lb.-ft. it must be divided by

$$(981 \times 453{\cdot}6 \times 2{\cdot}54 \times 12),$$

which gives the same result as on p. 324.

Note that for a given motor the torque varies directly as the product of armature current and flux.

Example 1. *Two parallel wires carry currents of* 150 *amperes and* 250 *amperes respectively, in the same directions. Find the force between them per foot of length if their centres are* 2 *in. apart.*

The field produced by the current of 150 amperes at the centre of the other wire is of strength (see Chap. IV., Art. 3).

$$= \frac{2 \times \mathrm{I}}{10 \times r} = \frac{2 \times 150}{10 \times 2 \times 2{\cdot}54} = \frac{15}{2{\cdot}54}.$$

The direction of this field is as shown in Fig. 11.01.

The force due to this field on 1 ft. length of the second wire is

$$\frac{\mathrm{BI}'l}{10}\text{ dynes} = \frac{15}{2{\cdot}54} \times \frac{250}{10} \times 12 \times 2{\cdot}54,$$

$$\frac{4500}{981 \times 453{\cdot}6}\text{ lb.} = 0{\cdot}0101\text{ lb.}$$

By applying the Left Hand Rule it will be found that the force is one of attraction. The force of the second wire on the first is equal and opposite, *i.e.* it too is an attraction.

Example 2. *In a 4-pole direct current motor the number of conductors on the armature is* 180, *and there are* 2·9 *megalines per pole. What torque in kilogram-metre units will the motor exert when a current of* 50 *amperes flows in each conductor?* (C. & G., El. Eng., II.)

The average field strength is $2{\cdot}9 \times 10^6 \div (\frac{1}{4}\pi dl)$ lines/sq. cm.

where d = diameter of armature in cm.

l = length " "

$$\therefore \text{ average force per conductor} = \frac{2{\cdot}9 \times 10^6 \times 4}{\pi dl} \times \frac{50}{10} \times l \text{ dynes.}$$

But torque = force × radius × number of conductors

$$= \frac{2{\cdot}9 \times 10^6 \times 4}{\pi d} \times 5 \times \frac{d}{2} \times 180 \text{ dyne-centimetres}$$

$$= \frac{2{\cdot}9 \times 10^6 \times 4 \times 5 \times 180}{2\pi \times 10^2} \text{ dyne-metres}$$

$$= \frac{2{\cdot}9 \times 10^4 \times 4 \times 5 \times 180}{2\pi \times 981 \times 10^3} \text{ kilogram-metres}$$

$$= 16{\cdot}95 \text{ kilogram-metres.}$$

Example 3. *Prove the expression for the force on a conductor carrying a steady current in a magnetic field. Hence find the torque on an armature carrying a total of* 50 000 *ampere-conductors, the diameter of the core being* 36 *in., the length* 12 *in., the pole arc being* 70 *per cent., and flux-density in the gap* 4 500. (Lond. Univ., El. Tech.)

From Chapter VIII., Art. 1: $\mathrm{E} = \mathrm{B}lv \times 10^{-8}$ volts, for a conductor moving in a uniform field.

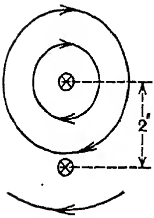


Fig. 11.01.

⊕ Conductors in section.

→ Lines of force.

Let this conductor carry a current I amperes flowing against the E.M.F., and let the force on it be F dynes.

Then the mechanical work done by the conductor in t sec. = Fvt dyne-cm. (or ergs).

And the electrical energy supplied to it in the same time, apart from that which is transformed into heat = EIt watt-sec. (or joules).

These must be equal, and 1 joule = 10^7 ergs;

$$\therefore \mathrm{F}vt = \mathrm{EI}t \times 10^7;$$

$$\mathrm{F}v = \mathrm{EI} \times 10^7 = \frac{\mathrm{B}lv}{10^8}. \qquad 10^7 = \frac{\mathrm{B}lv\mathrm{I}}{10};$$

Q.E.D.

Neglecting fringing, the number of ampere-conductors in the field is $\frac{70}{100}$ of 50000 = 35000.

The length of each conductor is $12 \times 2{\cdot}54 = 30{\cdot}5$ cm.

The flux density is 4500 lines per sq. cm.;

$$\therefore \text{ sum of forces on the conductors} = \frac{4500 \times 30{\cdot}5 \times 35000}{10} \text{ dynes}$$

$$= \frac{4500 \times 30{\cdot}5 \times 35000}{10 \times 981 \times 453{\cdot}6} \text{ lb. weight}$$

$$= 1080 \text{ lb.};$$

$$\therefore \text{ the total torque} = 1080 \times \frac{18}{12} \text{ lb.-ft.}$$

$$= 1620 \text{ lb.-ft.}$$

This result can be checked by the formula of Art. 2 as follows:—

$$\text{Total area of pole faces} = \frac{70}{100} \times \pi \times 36 \times 12 \text{ sq. in.}$$

$$= 950 \text{ sq. in.};$$

$$\therefore \text{ (flux per pole)} \times \text{(number of poles)} = 950 \times 4500 \times 6{\cdot}45 \text{ lines}$$

$$= 27{\cdot}6 \times 10^6 \text{ lines};$$

$$\therefore \text{ torque} = \frac{{\cdot}1173}{10^8} \times 50{,}000 \times 27{\cdot}6 \times 10^6$$

$$= 1620 \text{ lb.-ft.}$$

3. Armature Reaction in a Motor

Since the current in a motor flows against the E.M.F. instead of with it as in a generator, the magnetising effect of the armature ampere-turns is reversed. The field is therefore distorted against the direction of rotation. Consequently, unless interpoles or compensating windings are used, the brushes have to be given a negative lead (or a lag) to obtain satisfactory commutation.

If the brushes could be given a lead the weakening effect of the back ampere-turns of a generator (see Chapter IX.) would become a strengthening effect. But since they must have a lag this is again reversed, and a weakening of the field is caused just as in a generator. The amounts of the two effects can be found by dividing the armature reaction into cross ampere-turns and back ampere-turns in the same way as for a generator.

4. Motor Characteristics

Motors may have permanent magnets, or shunt-, series-, or compound-wound electromagnets, much as for generators. The behaviour of the various types similarly may be compared by

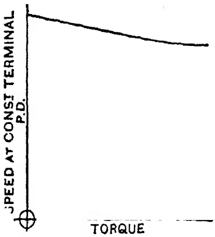


Fig. 11.02. — MECHANICAL CHARACTERISTIC OF SHUNT-WOUND MOTOR.

means of characteristics, but for motors the mechanical characteristic is the one required. This is a curve connecting speed at constant voltage, and torque (cf. Chapter X.).

The magnetic characteristic of a dynamo is the same whether it is used as a generator or as a motor.

In considering the forms of mechanical characteristics it will be of assistance to remember that the back E.M.F. is normally only a few per cent. below the brush P.D. (see further Chapter XII.) and so is approximately constant.

Now $$= \frac{NnZ}{60 \times 10^8} \times \frac{p}{a} \text{ volts.}$$

Therefore for a given motor supplied at constant voltage nZ is nearly constant, or the speed (n) varies approximately inversely as the flux.

5. Characteristic of Shunt-Wound Motor

A shunt-wound motor supplied at constant voltage has a mechanical characteristic of the type shown in Fig. 11.02. Its field current is constant, after the field winding has warmed up to a steady temperature, since the terminal P.D. is constant. The drop in speed with increase of load is due to the diminution of back E.M.F. needed to allow an increased current to flow through the armature (see Art. 2).

Armature reaction weakens the field somewhat as the load increases, and this diminishes the drop of speed (see Art. 4).

It is in fact possible to design a shunt-wound motor with the same (or even a higher) speed at full load as at no load. Usually other considerations make it advisable to keep the armature reaction below the value which would produce this effect.

The heating up of the field windings, and the resulting increase of their resistance, weakens the field, and so increases the speed of a shunt-wound motor after it has been in use for some time.

Shunt-wound motors are suitable for driving machine tools or other machinery requiring an approximately constant speed.

Magneto motors have similar characteristics but are suitable only in very small sizes, chiefly toys.

6. Characteristic of Series-wound Motor

The characteristic of a series-wound motor is shown in Fig. 11.03. The reason for its shape is that an increase of torque, requiring an increase of armature current which is also the field current, causes a strengthening of the field, and therefore a drop of speed (see Art. 4).

At first the field strength increases nearly in proportion to the current and so the drop of speed with increasing torque is rapid. But with larger currents the field strength increases much less than in proportion to the current, owing to the diminishing permeability of the iron in the magnetic circuit of the machine. Consequently the speed diminishes much less rapidly.

Series motors are very suitable for traction work: they exert their largest torque at low speeds, *i.e.* when starting; as the load decreases they automatically raise the speed. They are also suitable for crane work: the size of motor used depends on the speed at which the maximum load is to be lifted; if it is shunt-wound it will raise all loads at practically the same speed, but if series-wound lighter loads will be raised at higher speeds. Fans are generally driven by series motors because the load at normal speed is constant, and therefore the speed keeps constant whatever type is used. The series type has the advantage in starting, and is also a little cheaper, especially in small sizes, because fewer turns are required for the field windings.

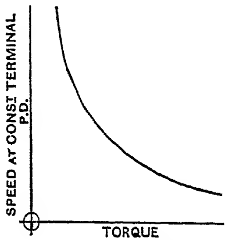


Fig. 11.03.—CHARACTERISTIC OF SERIES-WOUND MOTOR.

Series motors should never be used where the load may be completely removed, because the speed then becomes very high and there is a probability of damage by centrifugal force.

7. Compound- and Differential-Wound Motors

A motor with both series and shunt field windings may have them connected in either of two ways. If their magnetising effects are in the same direction it is called a cumulative compound-wound (or simply a compound-wound) motor. If the series turns oppose the effect of the shunt turns it is called a *differential-wound motor.* In the latter the series turns may be made to weaken the field sufficiently to make the speed at some particular load the same as at no load. The speed at other loads will, however, be slightly

different, as shown in Fig. 11.04. This type is scarcely ever used in practice, as the simpler shunt winding can be made at least as satisfactory (see Art. 9).

The mechanical characteristic of a cumulative compound-wound motor is intermediate between those of a shunt-wound and of a series-wound motor, as shown in Fig. 11.05.

This type is employed where a series characteristic is desired, but the load may be almost entirely removed, *e.g.* in some types of coal-cutting machines. The shunt winding then prevents the speed from rising above a safe value. It is also useful in conjunction with a fly-wheel when there are sudden temporary overloads, *e.g.* in rolling mills. In this latter case when the overload comes on the fly-wheel supplies part of the power as its speed diminishes, so decreasing the maximum power which has to be supplied.

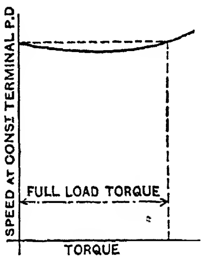


Fig. 11.04.—CHARACTERISTIC OF DIFFERENTIAL-WOUND MOTOR.

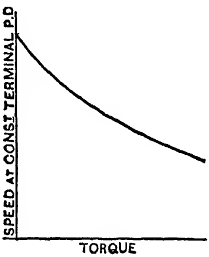


Fig. 11.05.—CHARACTERISTIC OF COMPOUND-WOUND MOTOR.

8. Motor Startors

Motor-starting switches or *motor startors* are necessary because there is no back E.M.F. in the armature when it is at rest. Consequently if the motor were switched directly onto the mains an enormous current would flow through it and melt the fuses. For instance a 5 H.P. 220-volt motor has a normal full load current of about 20 amperes, and an armature resistance of 0·5 ohm. If this were connected directly to the mains a current of $\frac{220}{0 \cdot 5} = 440$ amperes would flow through it. To avoid this a resistance is connected in series with the armature, and gradually cut out as the speed (and therefore the back E.M.F.) increases. The way in which the resistance is divided is explained in Art. 17.

For a series-wound motor the resistance is in series with the armature and the field winding as shown in Fig. 11.06. It is cut out step by step by moving the startor arm over the contact studs.

For a shunt-wound motor the resistance is in series with the armature only. The field winding is connected to the *first* contact stud, C (see Fig. 11.07). It is necessary to have the field circuit, CADE, always closed through the armature circuit, EFGC, because of the former's high inductance. If it were opened when a current was flowing in it a momentary high E.M.F. would be induced, which would cause bad sparking and damage the insulation of the field coils (see also Volume II.). The field must not be connected to the last stud, G, as this would result in the P.D. across the field winding at starting being cut down to the same extent as that across the armature: the field would therefore be very weak, and the motor would start slowly or not at all.

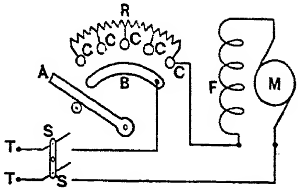


Fig. 11.06.—STARTOR FOR SERIES-WOUND MOTOR.

A, Movable arm. B, Contact strip. CC, Contact studs. F, Field winding of motor. R, Starting resistance. S S, Double pole switch. T T, Main terminals.

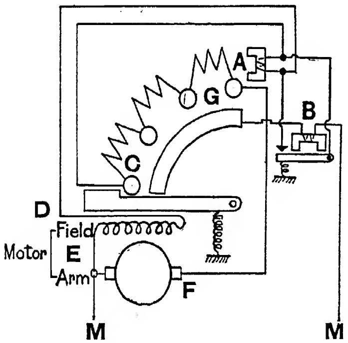


Fig. 11.07.—CONNEXIONS OF STARTOR FOR SHUNT-WOUND MOTOR.

A, No-voltage release. B, Overload release. MM, Main leads.

When the starting resistance is all cut out of the armature circuit the field current has to traverse the whole of it. This weakens the field, but not seriously, since this resistance is small compared with that of the field winding. The weakening can be prevented entirely by providing an additional stud, close to G, connected to the first stud (see Fig. 11.08).

A "no-voltage release" is fitted to startors nearly always. This consists of a spring to bring the startor arm to the off position

and a small "hold-on magnet." The winding of this magnet is usually connected in series with the field, as shown in Fig. 11.07, but is sometimes connected directly across the mains. Its action is, in case of a failure or disconnexion of the supply or a break in the field circuit, to release the arm and allow the spring to bring it to the off-position. This prevents the fuses blowing as they would if the supply were restored with the arm in the full-on position.

An "overload release" is sometimes fitted in addition. This consists of a magnet with an armature (or keeper) pivoted at one end. The magnet winding is connected in one of the supply mains. If the current exceeds a desirable value the armature is raised and connects two studs, which are connected as shown in Fig. 11.07. The "no-voltage" magnet is thus short-circuited and the arm released. The current at which this occurs can be adjusted by altering the distance of the armature below the poles of the overload magnet by means of a screw.

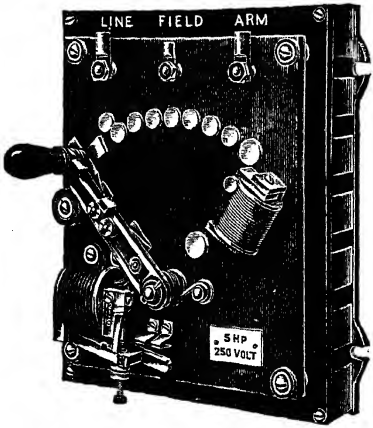


Fig. 11.08.—STARTOR WITH "NO-VOLTAGE" AND "OVERLOAD" RELEASES.

A compound-wound motor may be started by a similar startor, the series winding and armature being connected as the armature is in the shunt-wound motor.

A liquid startor is illustrated in Fig. 11.09. In this the current is passed through a solution of washing soda (or of caustic potash) contained in an iron tank. The current is led out by iron plates which can be lowered into the solution by a screw and hand-wheel. The resistance is thus gradually reduced, and finally short-circuited by two contacts on the plates and tank respectively.

9. Speed Regulation of a Shunt-Wound Motor

As has been shown in Art. 4, the speed of a motor supplied at constant voltage varies inversely as the field strength. Conse-

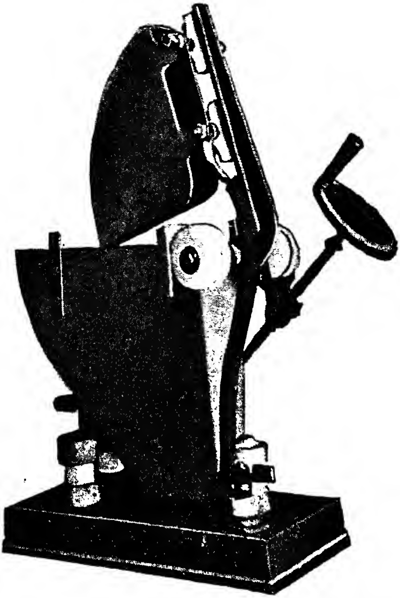


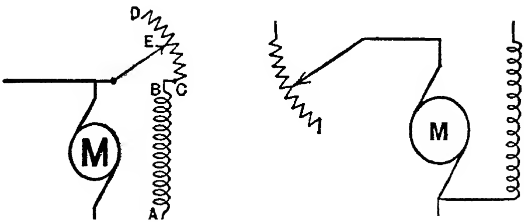
Fig. 11.09.—LIQUID STARTOR FOR 100 H.P. MOTOR.

quently the speed of a shunt-wound motor can be raised by placing a resistance in series with the field winding [Fig. 11.10 (*a*)]. As this resistance is increased the speed will increase. *E.g.* to double the speed the field must be halved, which requires the field current to be reduced to less than half its normal value. The extent to which the field current must be reduced depends on the magnetic circuit of the motor (see Fig. 11.11, Example 5).

On the other hand the speed can be lowered by putting resistance in the armature circuit and keeping the field current constant. The speed then varies approximately as the brush P.D., more exactly as the back E.M.F.

The latter method has the following disadvantages:—

(*a*) The variation of speed with load becomes much greater,



(*a*) (*b*)

Fig. 11.10.—SPEED REGULATION OF SHUNT-WOUND MOTOR.
(*a*) By field resistance. (*b*) By armature resistance.

because as the armature current increases the P.D. across the rheostat increases, thus diminishing the brush P.D. (see Example 4).

(*b*) The maximum horse-power is diminished in the same ratio as the speed.

(*c*) The efficiency is diminished in the same ratio as the speed, for the power supplied remains the same in spite of the diminution of horse-power. The wasted power is expended in heating the rheostat.

This method is therefore employed only when low speeds are required occasionally, *e.g.* in printing machines when "making-up."

None of these disadvantages apply to the field rheostat method.

If, however, the motor is required to exert the same torque at all speeds (not the same H.P.) a smaller motor can be used if the armature rheostat method of speed regulation is employed. For with a field rheostat the field is weakened as the speed increases, and so a greater current is required to produce a given torque. The armature must be capable of carrying this current without overheating, *i.e.* a larger motor is necessary. It may therefore be worth while to effect a saving in first cost by using armature rheostat control, at the expense of efficiency, etc.

Example 4. *A shunt-wound motor whose armature has a resistance of* 0·65 *ohm is running at* 600 *r.p.m. and taking* 20 *A. at* 220 *volts in addition to its field current. Find the resistance necessary to reduce the speed to* 400 *r.p.m. with no change in the armature current. In what ratio is its H.P. reduced?*

If the current then decreases to 10 *A. what happens to the speed?*

$$\text{The back E.M.F. is } 220 - 20 \times 0{\cdot}65 = 220 - 13 = 207 \text{ volts.}$$

$$\text{At 400 r.p.m. the back E.M.F. becomes } \frac{400}{600} \text{ of } 207 = 138 \text{ volts;}$$

$$\therefore \text{ P.D. at armature terminals} = 138 + 13 = 151 \text{ volts;}$$

$$\therefore \text{ P.D. across series resistance} = 220 - 151 = 69 \text{ volts;}$$

$$\therefore \text{ Series resistance} = \frac{69}{20} = 3{\cdot}45 \text{ ohms.}$$

$$\text{The H.P. is reduced to } \left(\frac{400}{600} =\right)\frac{2}{3} \text{ of its former value.}$$

When the current drops to 10 A. (owing to reduction of load) the P.D. across the series resistance drops to $10 \times 3{\cdot}45 = 34{\cdot}5$ volts;

$$\therefore \text{ P.D. across armature} = 220 - 34{\cdot}5 = 185{\cdot}5 \text{ volts;}$$

$$\therefore \text{ Back E.M.F.} = 185{\cdot}5 - 10 \times 0{\cdot}65 = 179 \text{ volts.}$$

[N.B.—This can be obtained in one step by subtracting $10 \times 4{\cdot}1$ (*i.e.* armature current × resistance of armature circuit) from the applied P.D.]

$$\therefore \text{ Speed} = \frac{179}{138} \text{ of } 400 = 519 \text{ r.p.m., } \textit{i.e.} \text{ an increase of 30 per cent.}$$

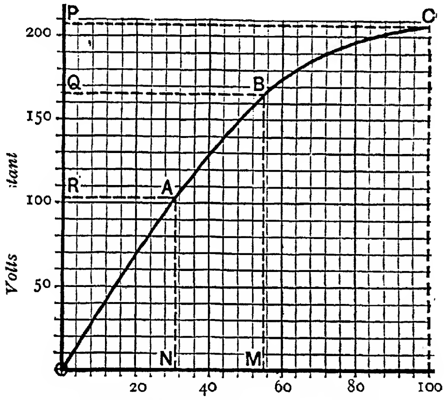
With no series resistance and 10 A. through the armature—

$$\text{Back E.M.F.} = 220 - 6{\cdot}5 = 213{\cdot}5 \text{ volts;}$$

$$\therefore \text{ Speed} = \frac{213{\cdot}5}{207} \text{ of } 600 = 619 \text{ r.p.m., } i.e. \text{ an increase of } 3{\cdot}3 \text{ per cent.}$$

Example 5. *If the above motor has a shunt of* 200 *ohms resistance, what additional resistance would raise the speed to* 750 *r.p.m.?*

The field strength must be reduced in the ratio $\frac{600}{750}$, *i.e.* to 80 per cent. of its normal value. The reduction of field current to effect this will be greater, and to determine its extent the magnetisation characteristic of the machine must be known. Suppose it to be as shown in Fig. 11.11. Then to reduce the field strength to 80 per cent. of normal the field current must be reduced



Field current (per cent. of normal value).

Fig. 11.11.—MAGNETISATION CHARACTERISTIC OF SHUNT-WOUND MOTOR.

OL=Normal field current. OM=Current for 80 per cent. field. ON=Current for half field.

to 55 per cent. of its normal value (note that OQ is equal to 166 volts, which is 80 per cent. of 207, the back E.M.F. of the motor, see Example 4).

$$\therefore \text{ Resistance of shunt circuit must be increased to } \frac{200}{0{\cdot}55} = 364 \text{ ohms,}$$

i.e. a resistance of 164 ohms must be inserted.

10. Speed Regulation of a Series-Wound Motor

A resistance in series with the armature (and the field) of a series-wound motor reduces the speed, just as for a shunt-wound one. The same disadvantages apply, but the variability of speed is not important since a series-wound motor's speed is always variable with changes of load.

To increase the speed of a series-wound motor a resistance in parallel with the field winding may be used (see Fig. 11.12): this is sometimes called a divertor. Its effect is to shunt some of the main current from the field windings, thus weakening the field and increasing the speed. Its action resembles that of a field rheostat for a shunt-wound motor, and in neither case is the efficiency impaired.

In a compound-wound motor a resistance in the armature circuit will as before reduce the speed. An increase can be effected either by a resistance in series with the shunt turns, or one in parallel with the series turns. The method adopted depends on whether the shunt or the series ampere-turns are normally the greater.

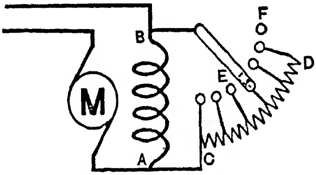


Fig. 11.12. SPEED REGULATION BY DIVERTOR.

A B, Field winding. C D, Divertor. E, Regulating handle. F, Position for *lowest* speed.

11. Reversal of Rotation

To reverse the direction of running of a motor, either the field or the armature current, but not both, must be reversed. The truth of this can be tested by means of the Left Hand Rule (Art. 1). If the positive and negative mains are interchanged the motor, whatever the type of winding, will run in the same direction as before, because the field will be reversed as well as the armature current. The only exceptions are motors with permanent magnets, or with their excitation supplied from some independent source.

In order to produce reversal the connexions of the field windings to the brushes must be changed. These changes are the same as those necessary when a generator is driven in the reverse direction. [See Chapter X., Art. 11, and Fig. 11.15 (*b*) and (*c*).] If the motor has interpoles, care must be taken not to reverse their connexions when reversing those of the field windings. This will occur if the brushes are shifted a pole-pitch, and no other change is made.

Motors which require frequent reversing are provided with combined starting and reversing switches. The connexions of such a switch are shown in Fig. 11.13. It can be seen that the field current flows in the same direction to whichever side the switch is moved, but the armature current flows in opposite directions according to the position of the switch.

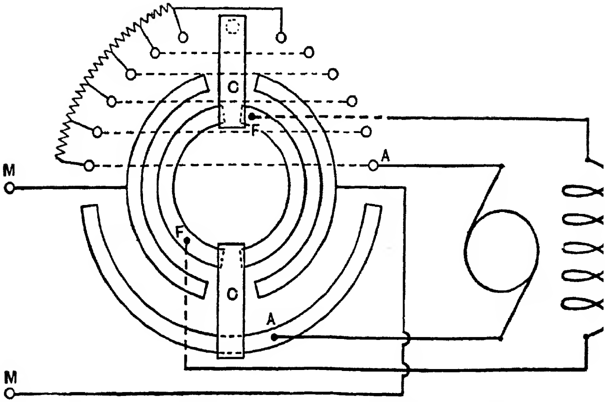


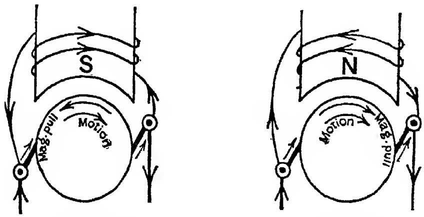
Fig. 11.13. REVERSING SWITCH FOR SHUNT-WOUND MOTOR.

A A, "Armature" terminals. C C, Double moving contact. F F, "Field" terminals. M M, Supply mains.

12. Generator Run as Motor

When a *generator is used as a motor* the direction of running can be obtained by remembering that the E.M.F. of a motor armature opposes the current. Thus in a shunt-wound dynamo, if the field current is in the same direction in the motor as when it was used as a generator the armature current will be reversed. Therefore the E.M.F. is the same as before, and the motor runs in the same direction as it was driven when a generator. If the direction of the armature current is unchanged the same result follows (see Fig. 11.14).

On the other hand in a series-wound motor, if the field current is in the same direction as when it was a generator the armature

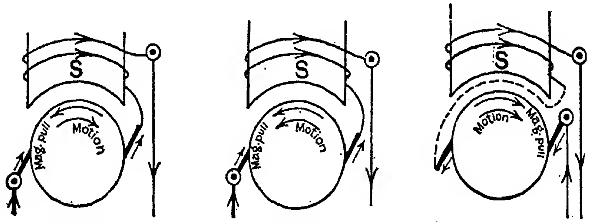


(a) As generator. (b) As motor.

Fig. 11.14—SHUNT-WOUND DYNAMO'S ROTATION.

current also will be in the same direction. Therefore the E.M.F. is reversed, so it will run in the reverse direction (see Fig. 11.15).

A compound-wound generator used as a motor will run in the same direction as before, but if no change is made in the connexions it will be a differentially-wound motor. To make it act as a com-

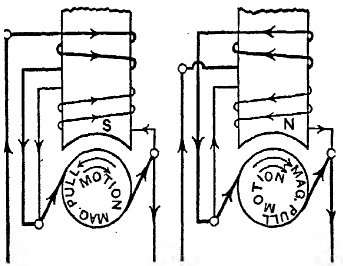


(a) As generator. (b) As motor. (c) As motor with connexions changed.

Fig. 11.15.—SERIES-WOUND DYNAMO'S ROTATION.

pound-wound motor the series winding must have its connexions reversed (see Fig. 11.16).

No change is required in the connexions of interpoles or compensating windings. If the dynamo is shunt-wound the polarity of the interpoles is reversed when it is used as a motor,* and this is necessary (see Art. 3, and Chapter IX., Art. 25). In a series-wound motor the polarity is unchanged (if that of the main poles remains the same), but the direction of rotation is reversed (see above). Consequently the interpoles have the same polarity as the "preceding" main poles, instead of that of the "next" main poles as they had when the machine was a generator.



(a) As generator. (b) As motor.

Fig 11.16.—COMPOUND-WOUND DYNAMO'S ROTATION.

Similarly in a compound-wound generator the connexions of the interpole winding must not be changed, even if the main series winding is reversed so as to make it a compound-wound motor (see Fig. 11.17).

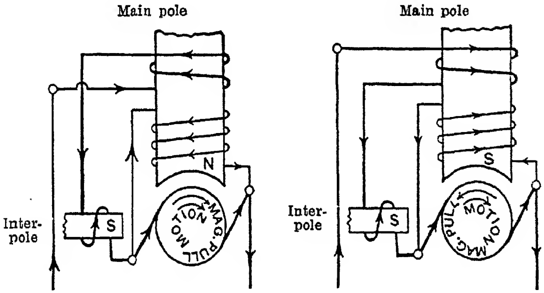
* On the assumption of unchanged polarity of main poles.

13. Subdivision of Startors

The amount of variation of the current taken during starting depends on the number of sections into which the rheostat is divided. With a continuously variable rheostat the current could be kept at a constant value till all the resistance was cut out. With a given number of resistance sections the variation of the current, and the most suitable subdivision of the resistance can be obtained by making certain assumptions, as follows:—

The armature current (I amperes) flowing through a shunt motor of armature resistance (R_a ohms) with a resistance (R ohms) in series with it is given by—

$$I = \frac{V - E}{R_a + R}, \quad \text{where} \quad \begin{cases} V = \text{applied P.D. in volts,} \\ E = \text{back E.M.F. in volts.} \end{cases}$$



(a) As generator. (b) As motor.

Fig. 11.17.—CONNEXIONS OF COMPOUND-WOUND INTERPOLE DYNAMO.

If the maximum current I_1 is given, the maximum total resistance is obtained from the above by putting $E = 0$, the value when the motor is at rest. Denote this resistance by R_1. Let the current fall to I_2 owing to the increase of E, and then be brought up again to I_1 by reducing the total resistance to R_2. Then, assuming that the back E.M.F. does not change while altering the resistance,

$$\frac{R_2}{R_1} = \frac{I_2}{I_1}.$$

If this resistance (R_2) is left in until the current again falls to I_2, and a further reduction to R_3 restores the current to its original value, and this process is repeated, then $\frac{R_3}{R_2} = \frac{I_2}{I_1} = \frac{R_4}{R_3}$, etc.

The final step is to cut out all the external resistance, leaving only that of the armature (R_a). Therefore if there are n positions of the starter, *i.e.* (n – 1) sections in the resistance,

$$\frac{R_2}{R_1} = \frac{R_3}{R_2} = \ldots = \frac{R_a}{R_{n-1}} = k;$$

$$\therefore \frac{R_a}{R_1} = k^{n-1};$$

$$\therefore k = \sqrt[n-1]{\frac{R_a}{R_1}}.$$

This enables the values of R_2, R_3, etc., to be calculated if R_1, R_a, and n are known.

To obtain the values of the external resistances, R_a must be subtracted from the values of the total resistance (see Example 6).

The assumption of no change in the back E.M.F. during alteration of resistance is accurate in this case, because the field current is independent of the armature current. In the series-wound motor a reduction of resistance will cause less change of current then the amount calculated on the above assumption, since the field strength and back E.M.F. increase when the main current increases.

Example 6. *Calculate the sections of the starting resistance for a 5 H.P. 220-volt shunt-wound motor. Starting current not to exceed $1\frac{1}{2}$ times full load current.*

Assume— 6 sections in starting resistance.
Motor efficiency 85 per cent.
Half of total losses in armature copper.

$$\text{Full load current} = \frac{5 \times 746}{0{\cdot}85 \times 220} = 20 \text{ A.}$$

Strictly the shunt current should be subtracted from this before proceeding, but this amount of accuracy is unnecessary in practice.

$$\therefore \text{Starting current} = 1\tfrac{1}{2} \times 20 = 30 \text{ A.};$$

$$\therefore \text{Total resistance of armature circuit at start} = \frac{220}{30} = 7{\cdot}33 \text{ ohms.}$$

$$\text{Total losses} = \frac{15}{100} \times 20 \times 220 = 660 \text{ watts};$$

$$\therefore \text{Armature } I^2R \text{ loss} = \frac{1}{2} \times 660 = 330 \text{ watts};$$

$$\therefore \text{Armature resistance} = \frac{330}{(20)^2} = 0{\cdot}83 \text{ ohm};$$

$$\therefore \text{Starting resistance} = 7{\cdot}33 - 0{\cdot}83 = 6{\cdot}50 \text{ ohms.}$$

To obtain the sections:—

$$\sqrt[6]{\frac{0{\cdot}83}{7{\cdot}33}} = 0{\cdot}696 \text{ and } \begin{array}{l} 7{\cdot}33 \times 0{\cdot}696 = 5{\cdot}10, \\ 7{\cdot}33 \times (0{\cdot}696)^2 = 3{\cdot}55, \text{ etc.} \end{array}$$

Hence the following results:—

Stud number ..	1,	2,	3,	4,	5,	6,	7.
Total resistance ..	7·33,	5·10,	3·55,	2·47,	1·72,	1·20,	0·83 ohms.
Inserted resistance	6·50,	4·27,	2·72,	1·64,	0·89,	0·37,	0 ohms.
∴ sections are ..		2·23,	1·55,	1·08,	0·75,	0·52,	0·37 ohms.

The following is an alternative way of obtaining the above figures. Draw a line (see Fig. 11.18) whose length is the distance between 8·3 and 73·3 on the scale of a slide-rule.

Divide this line into six equal parts.

Read off the positions of the dividing points on the slide-rule scale. These give the required sections of the total resistance.

In the above example the value to which the current should fall before a section is switched out is—

$$30 \times 0{\cdot}696 = 20{\cdot}9 \text{ amperes,}$$

i.e. a little over full load current. Thus the motor will be able to start against full load torque.

Figures read on scale of slide-rule.

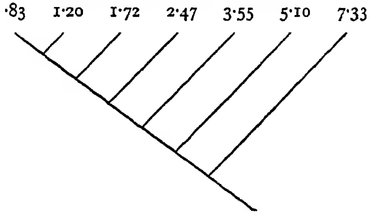


Fig. 11.18.—SEMI-GRAPHICAL SUBDIVISION OF RHEOSTAT.

If the number of sections is not given it can be obtained as follows if the above condition is to be satisfied:—

$$\frac{\text{Minimum starting current}}{\text{Maximum starting current}} \quad \frac{20}{30} = {\cdot}667.$$

Let $({\cdot}667)^n = \dfrac{0{\cdot}83}{7{\cdot}33}$. Then n is over 5;

∴ Minimum number of sections is six.

The remainder of the calculation is then as above.

14. Rating of Motors and Generators

The rating of a motor or of a generator is its maximum safe output. Three ratings are recognised by the B.S.I.*—

(A) Continuous maximum rating.
(B) Rating permitting overloads.
(C) Short-time rating.

* See British Standard Specifications, Nos. 168, 169, and 226.

(A) is tested by a full load run continued until the final steady temperature is ascertained; (B) is tested in the same way, but the temperatures permitted are lower; so that the machines are capable of sustaining a 25 per cent. overload for 2 hours after having reached the maximum temperature due to their rated load. (C) is tested by a full load run for either one hour or half an hour, the period being specified in stating the rating.

Machines are classified into—

(i) Open. (ii) Protected.
(iii) Enclosed ventilated. (iv) Totally enclosed.
(v) Duct-ventilated, with self-ventilation, or forced or induced draught.
(vi) Flame-proof. (vii) Enclosed self-cooled.

Protected means a machine in which the live parts are protected mechanically from accidental contact, but so that there is free access to the interior.

Enclosed ventilated means a machine in which ventilation is provided, but access to live parts can be obtained only by opening covers having perforations under $\frac{1}{2}$ sq. in. and over $\frac{1}{50}$ sq. in. in area.

Enclosed self-cooling means an enclosed machine with a special device (forming part of the machine) for cooling the enclosed air, *e.g.* a fan for circulating the air through cooling chambers.

15. Heating and Sparking Limits

The output of a dynamo is limited either by heating or by sparking.

The heating limit depends on whether the working is to be continuous or intermittent, and on the permissible rise of temperature. The latter should depend on the nature of the insulation, but, since this is similar in all makes and since the durability of insulation can be tested only by prolonged working, certain standard figures have been adopted. In the case of the British S.I. these figures are 60° C. (108° F.) for field windings, whose temperature rise if shunt or separately excited must be measured by resistance (see Chapter III., Art. 6); and 55° C. (99° F.), measured by thermometer or thermo-couple, for the armature, commutator, brushes, etc. If asbestos or mica is used for insulating, these temperature rises may be increased by 20° C. If the air temperature of the room in which the machine is to work may exceed 40° C. (94° F.) the above temperature rises must be diminished by the amount of this excess.

Evidently a given motor must receive a much lower rating if it is totally enclosed than if it is open or merely protected. The diminution is about 15 per cent. for semi-enclosed motors and 50 per cent. for totally enclosed ones. The latter figure can be lessened by supplying ventilation through special ducts. Such motors are termed **duct-ventilated,** and have the advantages of total enclosure together with a higher rating.

Sparking depends on reactance voltage (see Chapter IX., Art. 19) and therefore becomes more troublesome as the speed increases.

As far as the heating limit is concerned the higher the speed the higher the rating: for the armature current and therefore the torque can be kept constant, and so the horse-power increased in proportion to the speed. In fact, owing to the high speed improving the ventilation, the rating can be increased slightly more than in proportion to the speed. When the reactance voltage has reached its maximum safe value the armature current must be diminished inversely as the speed to prevent any further rise in the reactance voltage, *i.e.* the H.P. remains constant (Fig. 11.19).

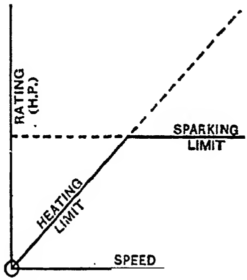


Fig. 11.19.—HEATING AND SPARKING LIMITS.

By the use of interpoles the sparking limit can be raised above the heating limit, so that they become useful only when the former is reached. This is usually the case in motors whose speed is varied by a field rheostat. Moreover, the heating limit can be raised by improving the ventilation and thus the output of a given size of motor increased.

In cases where the sparking limits the rating of the open type, enclosure will cause no lowering of the rating unless it brings the heating limit below the sparking limit.

16. The Temperature Time Constant

The *temperature time constant* of a resistance or a machine is required to determine its rating under intermittent loads, or the length of time for which it will stand a given overload. When a machine is run on a steady load its temperature rises rapidly at first and then more and more slowly, and according to its size and design it may require three, six, or more hours to approximate to the *final steady* temperature corresponding to the load.

The Temperature Time Constant may be defined as *the time in which the temperature would rise to the final steady value if no heat were given out to the surrounding atmosphere.* It can be seen that the same constant (in seconds) is obtained by dividing the total heat (in joules or watt-seconds) stored at the final steady temperature by the rate at which heat is produced (in watts) in the machine.

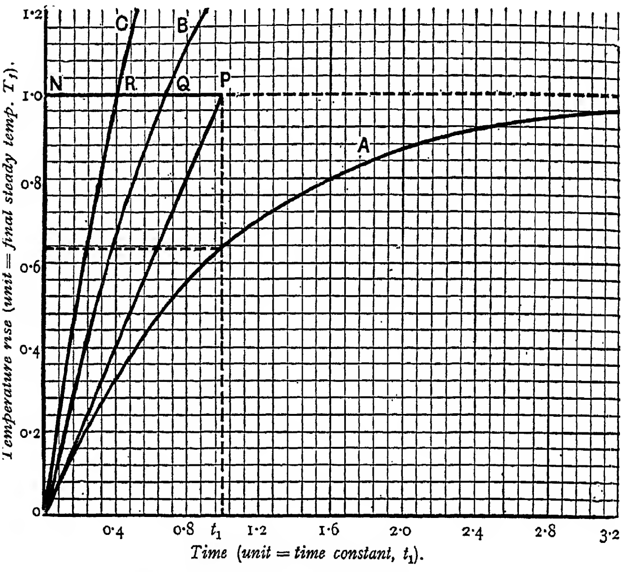


Fig. 11.20.—CURVES OF TEMPERATURE RISE.

A, With normal load. B, With double normal load. C, With treble normal load. O P, Tangent to curve A at O.

If temperature rise is plotted against time (see Fig. 11.20) the curve obtained is always approximately exponential. In other words, it can be expressed by the equation

$$T = T_f\left(1 - e^{-\frac{t}{t_1}}\right),$$

where e = base of Napierian logarithms = 2·718,

T = temperature rise after loading for time t,

T_f = final steady temperature rise,

and t_1 = temperature time constant in same unit as t.

Assuming the exponential law, it follows that the temperature time constant is the time in which the temperature rise reaches 0·632 of its final amount. Or by determining T_f and the time required to reach any intermediate temperature the value of t_1 can be found.

Example 7. *The final temperature rise of a motor is 50° C. After 1 hour from the start the temperature has risen 25° C. Find its temperature time constant.*

Substituting these values in the above equation—

$$25 = 50\left(1 - e^{-1/t_1}\right);$$

$$\therefore e^{-1/t_1} = 1 - \frac{25}{50} = 0{\cdot}5;$$

$$\therefore e^{1/t_1} = \frac{1}{0{\cdot}5} = 2{\cdot}0.$$

$$\frac{1}{t_1} = \log_e 2{\cdot}0 = 0{\cdot}693;$$

$$\therefore t_1 = \frac{1}{0{\cdot}693} = 1{\cdot}44 \text{ hours.}$$

If a tangent OP is drawn to the curve at the origin (see Fig. 11.20) it will intercept on the horizontal line of T_f a distance NP equal to the temperature time constant. Because this tangent gives the temperature time curve which would be obtained if no heat were given out (see definition). If the load is increased so that heat is developed twice as fast, the slope of this tangent is doubled (see Fig. 11.20, B). But the final temperature is very approximately doubled too, so that the time temperature constant remains almost exactly at its former value. Thus the equation for the new time temperature curve is

$$T = 2T_f\left(1 - e^{-\frac{t}{1}}\right).$$

Or in the general case when the rate of production of heat is increased to m times its original value—

$$T = mT_f\left(1 - e^{-\frac{t}{t_1}}\right).$$

17. Application to Overloads

Suppose T_f in the above equations is the maximum permissible temperature. Then the original load is the maximum which can be carried continuously, *i.e.* it is the continuous rating of the machine. If a greater load is placed on the machine when it is cold, the permissible temperature T_f will be reached after a time

which is shorter the greater the overload. It can be shown that if the rate of heat production is increased m times

$$t_m = t_1 \log_e \frac{m}{m-1} = 2{\cdot}303 t_1 \log_{10} \frac{m}{m-1},$$

where t_m = time to reach T_f under the increased load,

and t_1 = temperature time constant.

Thus in Fig. 11.20 NQ $= 0{\cdot}69 t_1$, *i.e.* this is the time to reach T_f with doubled rate of heat development. And NR $= 0{\cdot}41 t_1$.

Example 8. *The temperature time constant of a motor is 2 hours. How much may the heating rate be increased if the load is applied for only 1 hour?*

$$1 = 2 \log_e \frac{m}{m-1} = 4{\cdot}606 \log_{10} \frac{m}{m-1};$$

$$\therefore \frac{m}{m-1} = \text{antilog } 0{\cdot}217 = 1{\cdot}65;$$

$$\therefore m = \frac{1{\cdot}65}{0{\cdot}65} = 2{\cdot}54,$$

i.e. heat may be developed for one hour at 2·54 times the normal rate.

The total heat developed has to be taken into account, *i.e.* in the case of a machine that due to copper, iron, and friction losses. The armature copper loss (and the field copper loss in a series-wound machine) increases as the square of the current; while the others increase less rapidly, or remain constant. If the first loss alone is considered, an increase of rate of heat production in the ratio m corresponds to a current $\sqrt{m}$ times the normal. Thus in example, since $\sqrt{2{\cdot}54} = 1{\cdot}59$, the "one hour rating" is 59 per cent. more than the continuous rating. The effect of the other losses is to increase the permissible load beyond this, the amount of the increase depending on the relative amounts of the different losses. If, however, the overload is a heavy one, and so of short duration, there is not time for the temperature to become equal throughout the machine, and each part must be considered separately.

18. Short-Run Tests

To determine the final steady temperature rise the load must be maintained for a long time, especially in large machines. This time could be shortened by running the machine under overload until the temperature approached the expected final value, and then reducing the load to normal. This can be done easily as far as the armature is concerned, but it is not easy to arrange this for the field windings if the machine is shunt-wound.

If the exponential curve can be trusted any two points on it will suffice to determine the two constants, T_f and t_1. The calculation is simplified if two points are taken at times from the start, of which one is double the other. For if T_1 is the temperature rise after time t', and T_2 is the temperature rise after time $2t'$,

$$T_1 = T_f\left(1 - e^{-\frac{t'}{t_1}}\right),$$

and

$$T_2 = T_f\left(1 - e^{-\frac{2t'}{t_1}}\right).$$

By division

$$T_2/T_1 = 1 + e^{-\frac{t'}{t_1}};$$

$$\therefore\ 1 - e^{-\frac{t'}{t_1}} = 2 - \frac{T_2}{T_1} = \frac{2T_1 - T_2}{T_1};$$

$$\therefore\ T_f = T_1 \div (2T_1 - T_2)/T_1 = T_1^2/(2T_1 - T_2).$$

It is advisable to make t' reasonably large, otherwise $2T_1$ is little greater than T_2 and small errors in the readings will cause a large error in the calculated value of T_f (see Question No. 24).

Having found T_f the value of t_1 can be found by substitution in either of the first two equations.

19. Intermittent Loads

To find the rating under intermittent loads the cooling time constant must be known. For if the load is re-applied before the machine has cooled down to the temperature of the air a higher temperature will be reached in the same time, though the temperature *rise* will be less. This cooling time constant is usually larger than the heating constant, and if the machine is stopped the time constant is further increased because the ventilation is less effective.

The following equation represents what happens during cooling:—

$$T = T_1 . e^{-\frac{t}{t_2}},$$

where T_1 = temperature rise at the start of cooling,

T = temperature rise after cooling has continued during a time t,

and t_2 = cooling time constant.

The following are approximate values for the heating time constants of various D.C. machines:—

	Open type.	Enclosed type.
H.P.	$2\frac{1}{2}$; 10 ; 100	$1\frac{1}{2}$; $2\frac{1}{2}$; 6; 15.
Time constant (hours)	1·0; 1·5; 2·0	1·0; 2·0; 4; 5.

Example 9. *A motor's final temperature rise under normal full load is 50° C., its heating time constant is 2 hours, and its cooling time constant 7 hours. A load which causes heat to be developed at double the normal rate is applied for 1 hour. The motor is then allowed to cool for 3 hours, and the same load is again applied for 1 hour. Find the temperature rise at the end of this.*

Substitution in the equation $T = 2T_f\left(1 - e^{-\frac{t}{t_1}}\right)$ (see Art. 16), or a reference to curve B, Fig. 11.20, shows that at the end of the first hour (= 0·5 of time constant) the temperature rise is 0·79 × 50° C. = 39·5° C.

The equation during cooling is therefore

$$T = 39{\cdot}5\, e^{-\frac{t}{7}};$$

$\therefore$ after 3 hours' cooling $T = 39{\cdot}5\, e^{-\frac{3}{7}} = 39{\cdot}5 \times 0{\cdot}65 = 25{\cdot}7°$ C.

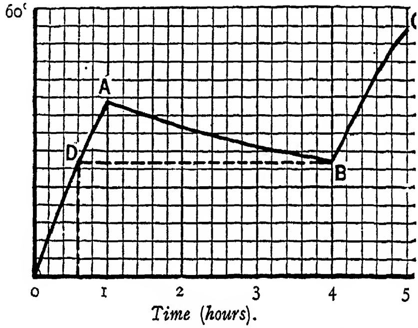


Fig. 11.21.—HEATING AND COOLING OF A MOTOR.

25·7° = 0·51 of the normal final temperature rise of 50° C., and curve B (Fig. 11.20) shows that this is reached under double load in a time 0·3 t_1 (= 0·6 hour) from the first start. Thus when the load has been again applied for 1 hour the temperature rise is as if the load remained on for 1·6 hour (= 0·8t_1) from the first start.

Referring again to curve B, this is seen to cause a temperature rise of 1·1 T_f = 55° C. in this case.

Note that the rise of temperature during the last hour is under 30° C. compared with 39·5° C. in the first hour.

The changes of temperature are shown in Fig. 11.21. OA is the heating curve during the first hour, AB the cooling curve, and BC the heating curve during the second application of the load. The temperature at B is the same as that at D, viz. 0·6 hour from the start. OA and BC are both portions of curve B of Fig. 11.20.

If the same cycle of 3 hours' cooling and 1 hour's heating were applied again, the fall of temperature would be increased, and the rise of temperature would be diminished. These changes would go on until the fall during cooling became equal to the rise during heating.

QUESTIONS ON CHAPTER XI

1. What is the fundamental fact on which the action of electric motors depends? From this derive an expression for the torque in pound-feet: apply this to the case of a two-pole motor with 480 conductors, $2\cdot5 \times 10^6$ lines through the armature, and 25 amperes supplied.

2. The flux in each pole of a 4-pole motor is 2×10^6 lines. What total number of ampere-conductors must be carried by the armature to produce a torque of 400 lb.-ft.? Prove any formulae you use for the calculation. [C. & G., II.

3. Find an expression for the force on a conductor carrying a steady current in a magnetic field. Hence find the torque on an armature carrying a total of 25,000 ampere conductors, the diameter of the core being 36 in., the length 12 in., the polar arc 70 per cent., and the flux-density in the gap 6 000. [Lond. Univ., El. Tech.

4. Explain why the brushes of a motor are given a lag, instead of a lead as in a generator. Does this strengthen the field?

5. Show how the speeds of shunt-wound and of series-wound motors respectively vary with load. Give reasons for the variation, and state for what purpose each type of motor is used.

6. What is the purpose of a "no-voltage" release attached to a direct current motor starting switch? Make a sketch of such a device and its connexions, and explain briefly how it operates. [C. & G., II.

7. Give a diagram of a starting switch for a shunt-wound motor, including "no-voltage" and "overload" releases.

8. Why is one end of the field winding connected to the first stud of a motor startor? Give full reasons.

9. What is the relation between the speed and mechanical load on a shunt motor supplied at constant voltage? Illustrate your answer by a torque speed characteristic, and give reasons why the speed alters when the load is changed. [C. & G., II.

10. If the armature resistance of a shunt-wound motor is 0·42 ohm, and it makes 600 r.p.m. at no load, taking 4 amperes at 220 volts; what will the speed be when the current is raised by increasing load to 20, 30, and 40 amperes respectively, assuming constant field. How is this modified in an actual motor?

11. How does resistance in (*a*) the field circuit, (*b*) the armature circuit of a shunt-wound motor affect its speed? Which is the preferable method and why?

12. If a shunt-wound motor takes an armature current of 20 amperes at 220 volts and 800 r.p.m., find the resistance necessary to reduce the speed to 520 r.p.m. with the same current. Armature resistance = 0·9 ohm.
If the load is reduced till the armature current falls to 12 amperes, what change of speed results (*a*) without the resistance in circuit, (*b*) with it?

13. Why is the speed of a shunt-wound motor for a given current higher after several hours' running than it is soon after it has been put to work?

14. If a motor on being started blows its fuse, and, when this is replaced by a larger one, runs at an excessive speed, what is the probable cause? Give reasons.

15. What is the effect on the speed of a shunt-wound motor of placing a resistance in series with the motor as a whole (*i.e.* not in the armature or field circuits separately)?

16. A six-pole lap-wound motor has poles 20 cm. square, and a constant flux-density in the gap of 5 000. The armature is wound with 500 wires having a total length of wire of 24 000 cm. and ·07 sq. cm. area. Find the speed of the motor with 100 volts on the terminals and 120 amperes in the line. [Lond. Univ., El. Tech.

17. How can the speed at which a series motor runs with a given current from the mains be (*a*) lowered, (*b*) raised?

How are the torque and the H.P. affected in each case?

18. Describe the various methods which have been used in practice for obtaining a wide range of speed in a direct current electric motor. Discuss the advantages and disadvantages of each method from the point of view of (*a*) efficiency, and (*b*) convenience. [Lond. Univ., El. Eng.

19. A shunt-wound motor is required to produce the same torque over a speed range of 2 to 1. Compare the sizes of the motors required if the speed regulation is effected (*a*) by a field rheostat, (*b*) by a rheostat in the armature circuit.

What other considerations would determine the method adopted?

20. Explain how the torque and speed characteristic of a series-wound motor operating on a constant voltage may be determined from the magnetisation curve of the machine when running as a generator at a constant speed. [C. & G., II.

21. A direct current motor is supplied at constant voltage. What fixes the speed at which it will run, and upon what does the current it will take depend? [C. & G., II.

22. A compound-wound generator to be used as a motor with the direction of rotation unchanged. What alterations, if any, will be required in the connexions? Give full reasons and, if need be, diagrams to explain your answer. [C. & G., II.

Sketch the mechanical characteristics of the motor. State its advantages or disadvantages compared with shunt-wound and series-wound motors.

23. A motor armature has a final temperature rise of 50° C. when fully loaded. Find the temperature rise after $1\frac{1}{2}$ hr. at full load if the time constant is 80 min.

If the motor is then run at no load for 2 hr. and then at full load for a further period of $1\frac{1}{2}$ hr., find the temperature rise at the end of this (time constant for cooling 3 hr.):—

(*a*) neglecting heat produced at no load;

(*b*) assuming final temperature rise at no load to be 10° C.

24. A motor run at steady load gave the following test results:—

Time	1·0;	1·5;	2·0;	3·0 hr.
Temperature rise	15·8;	21·0;	25·5;	31·0 ° C.

What will be the final temperature rise at this load; and what is the heating time-constant of the motor?

CHAPTER XII

DYNAMO EFFICIENCIES

1. Dynamo Losses

The waste of power in any dynamo is due to the following seven causes:—

(*a*) The resistance of the armature winding.
(*b*) The power required by the field windings.

These two together form the " copper losses."

(*c*) Eddy currents.
(*d*) Hysteresis.

These occur mainly in the armature core, and when added are the " iron losses."

(*e*) Brush friction and resistance (" brush losses ").
(*f*) Bearing friction.
(*g*) Windage.

The loss due to brush resistance (mainly contact resistance) is sometimes included in the armature resistance loss (*a*), to simplify the measurements and calculations.

The " friction losses " comprise those due to brush and bearing friction (*f*) and windage (*g*).

These losses can be calculated for a given design from certain data; and they can be measured in an actual generator or motor. The rest of this chapter gives details of both these processes.

All the wasted energy is converted into heat; and has to be got rid of by radiation, convection, and conduction, except what remains in the machine when its temperature rises (cf. Chapter XI., Art. 16).

2. The Copper Losses

These can be calculated if the resistances of the copper windings and the currents in them are known, *e.g.* the armature copper loss is $I_a^2R_a$ watts, where I_a is the armature current in amperes and R_a the armature resistance in ohms. The resistance to be used in such calculations are the *hot resistances*, *i.e.* the resistance after the windings have reached their maximum temperatures by a run on load lasting for six hours, or more if necessary.

The resistances can be calculated from the dimensions of a design, or measured for an actual dynamo.

The brush resistance may be included in that of the armature, in which case part of the brush loss is included in the armature copper loss, as mentioned in Art. 1. This is done sometimes to simplify matters, but it is more accurate to keep them separate (see Art. 4).

3. Iron Losses

Iron losses in the armature core can be calculated only roughly, owing to the variation of the flux-density from point to point.

The teeth are treated separately from the main part of the core.

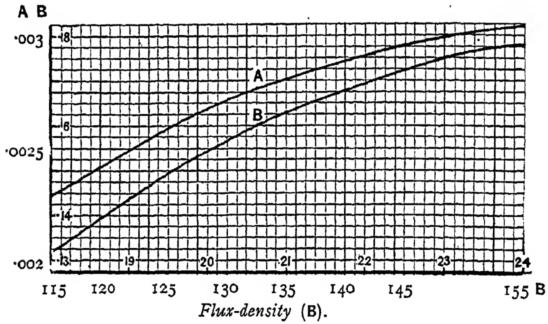


Fig. 12.01.—HYSTERESIS LOSS IN TEETH OF ARMATURE CORE.

A, Loss in watt-secs. per c.c. per cycle: flux-density in kilolines per sq. cm.
B, Loss in watt-secs. per lb. per cycle: flux-density in kilolines per sq. in.

For the latter Steinmetz's Law (see Chapter IV., Art. 22) can be used. Thus the hysteresis loss may be written as $h\mathbf{B}^{1.6}m$ watts per c.cm.,

where $\mathbf{B}$ = average number of lines per sq. cm.,
m = number of cycles of magnetisation per second
= revolutions per second $\times$ number of pairs of poles,
h = "hysteretic constant" of the iron used.

The value of h varies between 3×10^{-10} and 4×10^{-10} for the quality of iron employed for armature cores. Good average iron $3{\cdot}3 \times 10^{-10}$. If British units are employed the formula becomes

$$\text{hysteresis loss} = h_1\mathbf{B}_1^{1.6}m \text{ watts per lb.,}$$

where $\mathbf{B}_1$ = lines per sq. inch,

and h_1 varies between $1{\cdot}9 \times 10^{-8}$ and $2{\cdot}5 \times 10^{-8}$ (good average 2×10^{-8}).

At flux-densities as high as those usual in the teeth Steinmetz's Law does not hold.

The hysteresis loss in this case may be obtained from the curves in Fig. 12.01.

The mean flux-density along the teeth or the flux-density half-way down the teeth is to be used in obtaining the hysteresis loss from these curves.

The eddy current loss varies as m^2 (m as above). For the voltage producing the eddy currents varies as the speed, and therefore the magnitude of these currents also varies as m. But the watts lost vary as the square of the current (since watts $= I^2R$), and therefore as m^2.

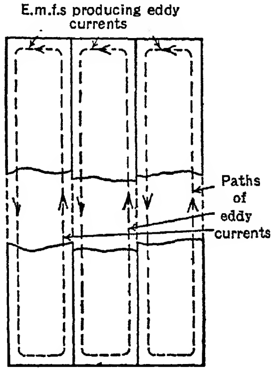


Fig. 12.02.—EDDY CURRENTS IN LAMINATIONS.

Again this loss varies as $\mathbf{B}^2$ from similar reasoning.

And finally the loss varies as b^2, where b is the thickness of the core discs. For the E.M.F. producing the eddy current in one disc is proportional to b (see Fig. 12.02). And the resistance in the path of the eddy current is *inversely* proportional to b, for the length of the path is very nearly constant, while the width of it varies as b and $R \propto \frac{l}{A}$. Thus the eddy current loss in a single disc equals I^2R watts or $\frac{E^2}{R}$ watts, and therefore varies as $\frac{b^2}{1}$ or as b^3. But the number of discs necessary for a given thickness varies *inversely* as b. So the total loss varies as b^2 as stated.

Combining the above three results the following formulae are obtained:—

$$\text{Eddy current loss} = k\,(m\mathbf{B}b)^2 \text{ watts per c.c.}$$

watts per lb.,

where m = magnetic cycles per sec.,
$\mathbf{B}$ = kilolines per sq. cm.,
$\mathbf{B}_1$ = " " sq. inch.,
b = plate thickness in mm.,
b_1 = " " " mils.

The value of k is about $1 \cdot 5 \times 10^{-7}$ for ordinary iron
and " $0 \cdot 8 \times 10^{-7}$ " alloyed " .
" " " k_1 is " $1 \cdot 4 \times 10^{-10}$ " ordinary "
and " $0 \cdot 7 \times 10^{-10}$ " alloyed " .

A variant of the above which is sometimes useful is (for ordinary iron)

$$\text{eddy current loss} = \left(\frac{m\mathbf{B}b_1}{10^5}\right) \text{ watts per c.cm.}$$

Note that the flux-density is in kilolines per sq. cm., in the last formula, while the disc thickness is in mils.

Alloyed iron is iron containing about $3\frac{1}{2}$ per cent. of silicon. Its permeability is much the same as good quality pure iron. Its hysteresis loss is, however, only about two-thirds as great. Moreover its resistivity is nearly twice that of pure iron, and so its eddy current loss is about half the normal. Consequently thicker plates can be used without increasing this loss beyond its usual value. The advantages of using thicker plates are a reduction in the labour of building the cores, and in the space wasted in plate insulation.

4. Brush Losses

The resistance portion of this loss is due chiefly to the contact resistance. But this contact resistance varies approximately inversely as the current, so that the contact drop is constant and has a value of about 2 volts (see Chapter IX., Art. 16). Therefore the contact resistance loss in watts is equal to the armature current multiplied by this drop, *i.e.* it varies approximately as the armature current, while the armature copper loss varies as the square of this current. Hence the advisability of treating the two separately.

The friction loss depends on the pressure of the brushes which is about $1\frac{1}{2}$* lb. per sq. inch, and on their coefficient of friction which may be taken as 0·25 to 0·3 if not more accurately known. The tangential frictional force is therefore about 0·4* lb. per sq. inch.

Let A = sum of areas of contact of all the brushes (+ve and −ve),
v = peripheral velocity of commutator in ft. per min.

* For motors these figures may be increased by about 50 per cent.

Then power absorbed by brush friction

$$\times\ 746 \text{ watts} = \qquad \text{watts.}$$

The *bearing friction* loss is due chiefly to fluid friction, and may be calculated for plain bearings by the formula

$$\text{watts lost in bearing friction} = 0{\cdot}8' \ \overline{\ \ 10^3}$$

where d = diam. of bearing in inches,

l = length ,, ,,

v = peripheral velocity of shaft in ft. per min.

The use of ball or roller bearings reduces this loss greatly.

The *windage* loss can be estimated only roughly. It increases rapidly with the speed, varying from about 10 per cent. of the bearing friction loss in rather low-speed dynamos, to as much as 120 per cent. in turbo-generators. It may be assumed that roughly

$$\text{watts lost in windage} = 4 \times (\text{barrel surface in sq. in.}) \times \frac{\ }{10^8}.$$

But the index of the speed may have values from 1·5 to 3 in different cases, which makes the formula of little use unless experience enables a value to be settled for the particular machine considered.†

5. Temperature Rise

This depends on the rate at which heat is produced, on the area of the surface from which it is dissipated, and on the speed with which this surface moves.

In the case of stationary field coils the mean temperature rise (t° C.) is given approximately by Lister's formula for open type machines

$$t = \frac{200\ddagger\, W}{A},$$

where W = watts lost in the coil,

A = total cooling surface (outer, inner, and ends)§ in sq. in.

* For motors this figure may be increased by about 50 per cent.

† See further "Change of Energy Loss with Speed in C.C. Machines," Thornton, *Journal Inst. E. E.*, Vol. 50, p. 492.

‡ 190-230, higher values for large machines. Reduce by one-third for machine ventilated by fan.

§ With divided field coils 0·8 of surfaces facing one another may be taken.

For the armature the results are more liable to error owing to great variations in ventilation. A formula for well-ventilated armatures is

$$t_a = \frac{}{(1 + \cdot 0005v)A},$$

where t_a = temperature rise (° C.) of armature as measured by thermometer,

W = watts lost in armature winding *and* core,

A = exposed surface of armature in sq. in., including one side of end connexions and of ventilating ducts,

v = peripheral velocity of armature in ft. per min.,

a = from 50 for large armatures to 90 for small ones.

Similarly for the commutator $t_c = \dfrac{25W}{(1 + \cdot 0006v)A}$,

where t_c = temperature rise (° C.) of commutator,

W = watts lost in brush resistance and friction,

A = cylindrical surface of commutator in sq. in.

In very long commutators t_c will be greater, and in very short ones less, than the formula given, owing to the effect of the end surface.

If considerable sparking occurs there will be additional losses in the commutator, with a corresponding increase in its temperature rise.

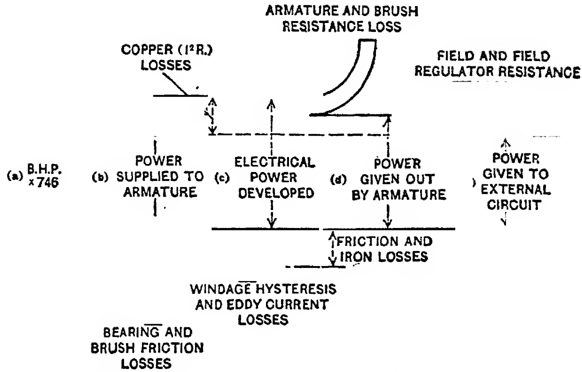
In totally enclosed machines the temperatures of the various parts are largely equalised by the circulation of air inside the case. The mean temperature rise in ° C. is approximately equal to $\dfrac{120W.}{A}$, where W = total watts lost, A = area of case in sq. in.

If a ribbed case is used only half the additional area due to the ribs should be included.

6. Efficiency

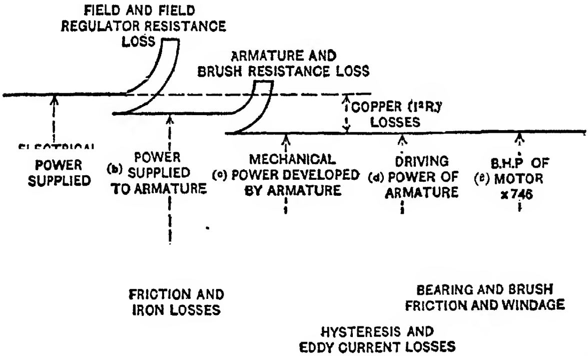
The *efficiency* of any machine is the ratio $\dfrac{\text{output}}{\text{input}}$, both these being measured in the same units. It must be less than unity.

Thus— the efficiency of a generator $= \dfrac{VI}{\text{B.H.P.} \times 746}$



Commercial efficiency $=\frac{(e)}{(a)}$. Electrical efficiency = (Mechanical efficiency = (c)

Fig. 12.03.—LOSSES AND EFFICIENCIES OF A GENERATOR.



Commercial efficiency $=\frac{(e)}{(a)}$. Electrical efficiency . Mechanical efficiency = (e)

Fig. 12.04.—LOSSES AND EFFICIENCIES OF A MOTOR.

where V = the terminal P.D. of the generator in volts,
I = the amperes in the external circuit,
B.H.P. = the brake horse-power, *i.e.* the horse-power actually supplied to the generator.

And— the efficiency of a motor $= \dfrac{\text{B.H.P.} \times 746}{\text{VI}}$,

where V = the supply P.D. in volts,

I = the total amperes supplied,

B.H.P. = the *useful* horse-power of the motor.

The above are sometimes called the **commercial efficiencies**, to distinguish them from two others. These are the **electrical efficiency**, which is the efficiency when only the I^2R (or "copper") losses are considered; and the **mechanical efficiency**, which is that obtained when only the remaining losses are considered, viz. those due to friction and iron.

It follows that—

$$\text{the electrical efficiency of a generator} = \frac{\text{VI}}{\text{VI} + I^2R \text{ losses}},$$

$$\text{,, ,, ,, ,, motor} = \frac{\text{VI} - I^2R \text{ losses}}{\text{VI}}.$$

The reason for the difference in these and the corresponding formulae for the mechanical efficiencies can be seen by an inspection of Figs. 12.03 and 12.04, which are diagrammatic and not to scale.

It should be noted that the product of the electrical and mechanical efficiencies gives the commercial efficiency for both generators and motors.

The **combined efficiency** of a generating set is the ratio $\dfrac{\text{output of generator in watts}}{\text{indicated H.P. of steam engine} \times 746}$. This can be measured easily and is equal to (commercial efficiency of generator) $\times$ (mechanical efficiency of steam engine).

7. Calculation of Efficiencies

When the losses have been calculated, the efficiencies can be obtained by using the formulae of the preceding Art. There are, however, one or two further points of interest, mainly in connexion with the calculation of the copper losses. In a series-wound generator or motor, the field and armature resistances may be added together and treated as one in calculation, whether the brush resistance is kept separate or not.

In a shunt-wound generator the armature current is the sum of the field and external currents, but in a shunt-wound motor the external current is the sum of the armature and field currents. In

any shunt-wound dynamo the field current $I_{sh} = \frac{V}{R_{sh}}$, where V is the brush P.D. and R_{sh} is the resistance of the shunt winding. The loss in the shunt may be calculated as $I_{sh}{}^2R_{sh}$ or VI_{sh} or $\frac{V^2}{R_{sh}}$, whichever is most convenient; but in the case of the armature copper loss, if either of the latter two is used V must be the armature "drop" and not the voltage across it.

In compound-wound dynamos the current in the series turns will, or will not, be the same as that in the armature according as the shunt is connected across the terminals, or across the brushes.

Example 1. *Find the electrical efficiency of a short-shunt compound-wound motor supplied with 190 amp. at 440 volts.*

Resistance of armature	= 0·065 ohm.
,, ,, shunt	= 105 ohms.
,, ,, series winding	= 0·013 ohm.

See Fig. 8·01 (*c*) for diagram of connexions.

$$\text{Copper loss in series winding} = (190)^2 \times \cdot 013 = 470 \text{ watts.}$$

$$\text{Drop in series winding} = 190 \times \cdot 013$$
$$= 2\cdot 5 \text{ volts;}$$

$$\text{Current in shunt} = \frac{440 - 2\cdot 5}{105} = 4\cdot 17 \text{ amperes;}$$

$$\therefore \text{ Copper loss in shunt} = (4\cdot 17)^2 \times 105 = \frac{(}{105} \quad 437\cdot 5 \times 4\cdot 17$$
$$= 1820 \text{ watts.}$$

$$\text{Armature current} = 190 - 4\cdot 17 = 185\cdot 8 \text{ amperes;}$$

$$\therefore \text{ Armature copper loss} = (185\cdot 8)^2 \times \cdot 065 = 2240 \text{ watts;}$$

$$\therefore \text{ Total copper loss} = 470 + 1820 + 2240 = 4530 \text{ watts;}$$

$$\text{Electrical efficiency} = \frac{190 \times 440 - 4530}{190 \times 440}$$

$$\frac{4530}{190 \times 440} \quad \cdot 946 = 94\cdot 6 \text{ per cent.}$$

Example 2. *If the motor in Example 1 is of 100 B.H.P., what is its commercial efficiency, and what are the total iron and friction losses?*

$$\text{Commercial efficiency} = \frac{}{190} \times 100 \text{ per cent.} = 89\cdot 2 \text{ per cent.}$$

$$\text{Total of losses} = (190 \times 440) - (100 \times 746) = 9000 \text{ watts;}$$

$$\therefore \text{ Total of iron and friction losses} = 9000 - 4530$$
$$= 4470 \text{ watts.}$$

TESTING SECTION

8. Brakes

In testing small motors the output is absorbed and measured by means of a brake.

The simplest type is the *rope brake* (see Fig. 12.05). A double rope passes round the motor pulley and supports a hanger on which weights can be placed. The other end of the rope loop is fastened to a spring-balance supported above the pulley. The turns of the rope may be kept apart by wooden distance pieces. Often the pulley is water-cooled.

Let W = weight of hanger and weights in lb.,

w = pull on spring-balance in lb.,

r (ft.) = radius of pulley + $\frac{1}{2}$ diameter of rope.

Then torque = $(W - w)r$ lb.-ft., and

$$\text{H.P.} = \frac{2\pi r\,(W - w)n}{33000},$$

when n = r.p.m. of motor.

Another type is the *eddy current brake*. This consists of discs of copper or aluminium driven by the motor. The discs rotate between electromagnets which induce eddy currents in the discs, producing a torque opposing their rotation. The discs produce an equal but opposite torque on the magnets, and this can be measured by means of a lever and weights, the magnets being otherwise free to rotate through a small angle. The amount of the torque is adjusted by altering the current in the magnet windings. This form is convenient but expensive.

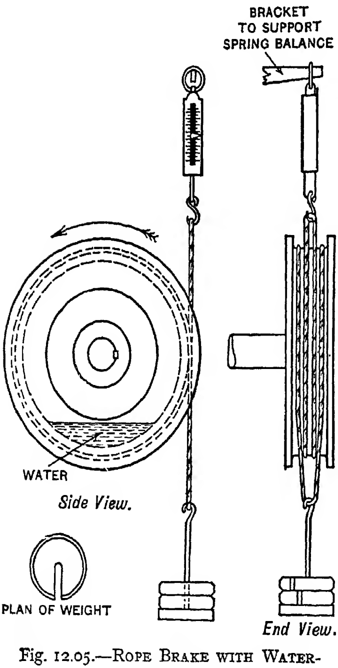


Fig. 12.05.—ROPE BRAKE WITH WATER-COOLED PULLEY.

A similar method is to make the motor drive a calibrated generator, *i.e.* one whose efficiency is known at all loads.

The output of the generator is measured by means of an ammeter and voltmeter, and the output of the motor

= (generator output) ÷ (generator efficiency),

both of which are known.

9. Hopkinson Test

The efficiency of a generator can be measured by driving it by a calibrated motor and measuring the input of the motor and the output of the generator by voltmeters and ammeters. The input of the generator = (input of motor) × (motor efficiency). This is the converse of the above test.

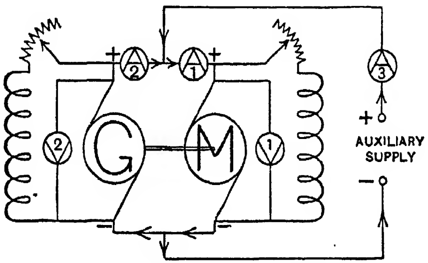


Fig. 12.06.—HOPKINSON TEST.

$V_1 \times A_1$ = Motor input.
$V_2 \times A_2$ = Generator output.
$V_3 \times A_3$ = Auxiliary power supplied.

A method which saves power and gives more accurate results is that usually known as the Hopkinson test. Two exactly similar machines are required for this, and either a third (auxiliary) generator or some other source of electrical power. In the original Hopkinson test the wasted power was supplied by a steam engine. The modified test is still known by Hopkinson's name in Great Britain, though the modification is due to G. Kapp.

The two similar dynamos are connected in parallel and are mechanically coupled. They are started as unloaded motors, through a resistance, or by gradually raising the voltage of the auxiliary generator (if one is used) to the normal value for the machines under test.

Then, by strengthening the field of one and weakening the field of the other, the former can be made to act as a generator and the

latter as a motor. The electrical power given out by the generator together with that from the auxiliary supply is taken in by the motor and is mostly given out as mechanical power, the rest going in the various motor losses. This mechanical power is given to the generator, and is given out again as electrical power except that which is wasted in the generator. Thus the power taken from the auxiliary supply is the sum of the generator and motor losses, and this can be measured directly by a voltmeter and an ammeter as shown. The currents in the test machines and their speed can be adjusted by altering the strengths of the two fields, and in this way the total loss measured at various loads.

$$\text{Then combined efficiency} = \frac{\text{Generator output}}{\text{Motor input}}$$

$$= \frac{\text{Generator output}}{\text{Generator output} + \text{auxiliary supply}}$$

$$= 1 - \frac{\text{Auxiliary supply}}{\text{Motor input}}.$$

Assuming that the efficiencies of the two machines are the same, the efficiency of each $= \sqrt{\text{combined efficiency}}$, *e.g.* if the combined efficiency is 81 per cent. the efficiency of each machine is $\sqrt{\cdot 81} = \cdot 90$ or 90 per cent.

The advantages of this test are that the power supplied is only the total loss instead of the motor input, and that it is more accurate to measure the loss directly instead of obtaining it as the difference of the measured input and output. Further, all the measurements are electrical, which are simpler and more accurate than mechanical measurements.

The disadvantages are that two similar dynamos are required and that the currents in the two machines differ, consequently the assumption of equal efficiencies is not correct. The results are nevertheless fairly accurate if taken as corresponding to the mean of the two loads.

It is unnecessary to measure both generator and motor currents, but this is advisable as a check on the readings, since motor current = generator current + auxiliary current.

10. The Series Hopkinson (or Potier) Test

In this test the two test machines have their armatures connected in series with each other and with the auxiliary supply, which must be capable of giving the full current of the test machines at a fairly

low voltage. This may be done by means of a booster, or by accumulators of large size. The best method of excitation is to connect the field windings in parallel to an independent source of supply at the normal voltage of the two machines. The speed can be adjusted by altering the excitations. The loads are altered by changing the voltage of the auxiliary supply, which assists the generator to send a current through the motor armature against the latter's E.M.F.

In this case combined efficiency

$$= \frac{\text{Generator armature output}}{\text{Generator armature output} + \text{auxiliary supply} + \text{excitation supply}},$$

and, as before,

$$\text{Efficiency of each machine} = \sqrt{\text{combined efficiency}}.$$

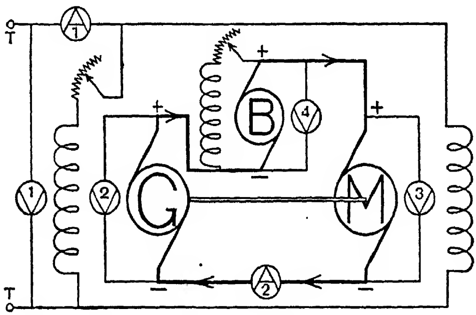


Fig. 12.07—SERIES HOPKINSON TEST.

B, Booster (or accumulators). G, Generator. M Motor.
T T, Terminals of excitation supply.
$V_1 \times A_1$ = Power supplied for excitation
$V_2 \times A_2$ = Generator armature output.
$V_3 \times A_2$ = Motor armature input.
$V_4 \times A_2$ = Auxiliary power supplied.

One ammeter measures the armature current of the two machines which is also the auxiliary current. The auxiliary voltage and either the motor or generator voltage must be measured, and it is advisable to measure both the latter so as to check the voltages by the relation

$$\text{Motor volts} = \text{generator volts} + \text{auxiliary volts}.$$

The power supplied for excitation is measured by a separate ammeter and voltmeter.

The advantage of this method is that the two armature currents are equal. The disadvantages are that two separate auxiliary

supplies are required, and that a supply at full current and low voltage is not usually obtainable so readily as one at full voltage and low current.

11. Hopkinson Test for Series-Wound Dynamos

This is a modification of that for shunt-wound machines.

If they are fitted with field "divertors" (see Chapter X., Art. 8) either the parallel (Art. 9) or the series (Art. 10) Hopkinson test for shunt-wound motors may be applied to them. The fields are then regulated by the "divertors" instead of by the rheostats used in

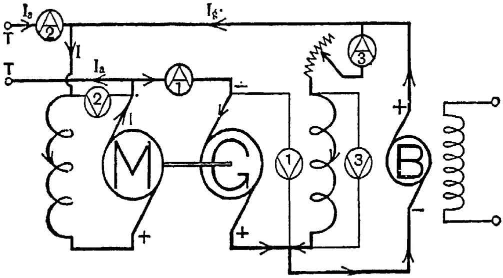


Fig. 12.08.—HOPKINSON TEST FOR SERIES-WOUND DYNAMOS.

$V_1 \times A_1$ = Generator output.
$V_2 \times (A_1 + A_2)$ = Motor input.
$V_3 \times A_3$ = Power supplied to field of generator.
T T, Terminals of auxiliary supply.

the shunt-wound machines. The current flowing through the machines is otherwise adjusted just as in the previous methods.

An alternative method is to use a booster (see Vol. II.). This has its armature connected in series with the machine which is to act as a generator, and thus enables it to supply power to the mains (see Fig. 12.08).

The booster is driven by a small motor, and is either separately excited or shunt-wound. Its excitation and the resistance in the generator field circuit are adjusted till the generator field current is the same as that in the field (and in the armature) of the motor.

Voltmeters and ammeters connected as shown measure:—

the terminal P.D. of the motor, V volts,
the armature P.D. of the generator, V_g volts,
the current in the generator armature, I_g amperes,
the auxiliary current, I_a amperes.

The current in the motor (I) $= (I_g + I_a)$ amp.
The power lost = motor input − generator output

$$= VI - V_g I_g \text{ watts.}$$

To obtain the total loss add the watts supplied to the generator field, which can be measured by a third ammeter and voltmeter.

Assuming half the total loss to occur in the motor its efficiency can now be calculated from—

$$\text{Efficiency} = \left(1 - \frac{\text{losses}}{\text{input}}\right) \times 100 \text{ per cent.}$$

12. Field's Test for Series Motors

This requires two exactly similar machines. When applied to

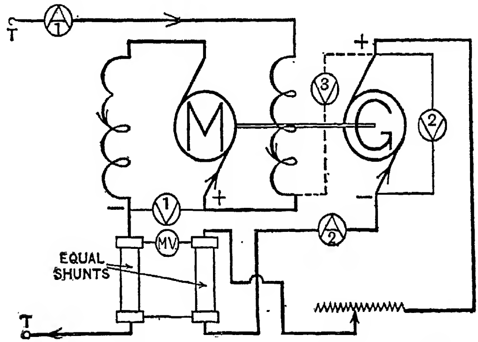


Fig. 12.09.—FIELD'S TEST FOR SERIES-WOUND DYNAMOS.

tramway motors they are connected by one of the gear wheels actually used on the axles of the car, so as to obtain the combined efficiency of the motors and gearing. The general principle is to measure the motor input (W_1) and the generator output (W_2), whence the total loss ($W_1 - W_2$) is obtained. This is divided between the two machines, and the efficiency of the motor obtained from—

$$\text{Efficiency} = \frac{\text{input} - \text{losses}}{\text{input}} \times 100 \text{ per cent.}$$

The connexions are as shown in Fig. 12.09. The field winding of the generator is connected *in series with the motor*, consequently the field strengths are equal except for armature reaction.

The speeds of the two armatures also are necessarily equal, and so the iron and friction losses are the same in the two machines.

The motor input (V_1I_1) less the generator armature output (V_2I_2) gives the motor losses and the generator armature loss. Subtracting the I^2R losses in the motor and in the generator armature (including brush contact resistance losses) the total iron and friction losses are obtained, and half this gives these losses for each machine (say W_2 watts). Then—

$$\text{The efficiency of the motor} = \frac{V_1I_1 - W_2 - I^2R \text{ loss in motor}}{V_1I_1}.$$

By using two ammeter shunts of equal resistance and a millivoltmeter connected as shown, the difference (I_3) between the motor and generator currents can be measured directly, for the P.D. across the millivoltmeter is the *difference* between the drops over the two shunts. Similarly, a voltmeter connected as shown by the dotted lines gives the difference (V_3) between the motor P.D. and the generator armature P.D. Then instead of using ($V_1I_1 - V_2I_2$) in calculating the losses, the expression $V_1I_1 - (V_1 - V_3)(I_1 - I_3)$ is used. This is more accurate than measuring V_2 and I_2 directly, and the instruments for this purpose are then used only as checks. This modification is due to E. Wilson. The efficiency thus calculated is for the motor at the actual input at which the measurements are taken, and the assumption of equal efficiencies made in the Hopkinson test is not made in the Field test.

13. Swinburne's Test

Swinburne's test can be applied to a single generator. The hot resistances of the armature and field coils are obtained first (see Art. 2). From these the resistance losses can be calculated.

The iron and friction losses are then obtained by running the dynamo as a motor. The P.D. applied to the brushes is adjusted to be equal to the E.M.F. of the machine when generating the current at which the efficiency is desired.

I.e. Applied P.D. $= V + IR_a$ volts,

where V = terminal voltage of generator, at load I,

I = *armature* amperes at desired loads,

R_a = armature resistance in ohms.

The speed is then adjusted to its normal value by the field regulator. The armature current (I_o) is measured under these conditions. Then the watts supplied to the armature (P.D. $\times I_o$) are all wasted in friction and iron losses, except for a negligible I^2R loss.

Since the speed is normal and the total flux is the same as under load (because the E.M.F. is the same in the two cases) it is assumed that the (friction + iron) loss thus measured remains constant. The calculated resistance losses are then added on to obtain the total loss.

$$\text{Then efficiency of generator} = \frac{\text{output}}{\text{output} + \text{total loss}} \times 100 \text{ per cent.}$$

This test is a very convenient one, but is not quite accurate since the iron losses under an actual load are greater than those measured. This is due mainly to armature reaction distorting the field, any departure from uniform distribution increasing the iron losses on the whole.

14. Separation of Losses (1)

By repeating the second part of Swinburne's test (Art. 13) at constant excitation but various speeds, the eddy current loss can be separated from the hysteresis and friction losses. The connexions for this test are shown in Fig. 12.10. The field current is kept constant at its normal value, and the speed varied by altering the resistance in the armature circuit.

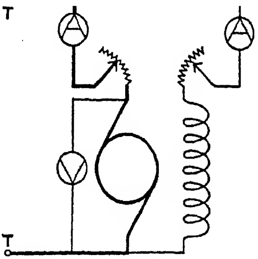


Fig. 12.10.—CONNEXIONS FOR SEPARATION OF LOSSES.

Then, as in Art. 13, VI_o (less I^2R loss in armature, which is usually negligible) = losses by eddy currents, hysteresis, and friction. But the eddy current loss varies as (speed)^2 (see Art. 2), while the hysteresis and friction losses vary as the speed;

$$\therefore\ VI_o - I_o^2R = a.n^2 + b.n \quad \ldots\ldots\ldots\ldots(1)$$

where n = r.p.m.,

a is eddy current loss constant,

b is hysteresis and friction losses constant;

$$VI_o - I_o^2R_a \qquad \cdot(2)$$

A number of observations are taken and $\frac{VI_o - I_o^2R_a}{n}$ is plotted against n. This should give a straight line, and if this is produced

to meet the vertical axis the intercept gives b (see Fig. 12.11). The slope of the line gives a.

The eddy current loss and the sum of the hysteresis and friction losses can then be calculated for any speed. Alternatively they can be obtained graphically as shown by the shaded areas in Fig. 12.11.

When the armature I^2R loss is negligible V is proportional to n, since the field is constant. The left-hand side of equation (2) then reduces to $I_0 \times$ a constant. Therefore n need not be observed and I_0 can be plotted against V. The remaining procedure is unaltered.

15. Separation of Losses (2)

The friction loss can be separated from the others by an experiment with similar connexions to those of Art. 14.

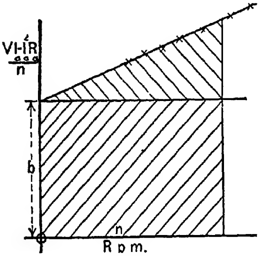


Fig. 12.11.—GRAPHICAL SEPARATION OF EDDY CURRENT LOSS.

Half eddy current loss.
Friction and hysteresis losses.

The armature voltage is reduced step by step as before, but the speed is kept constant by weakening the field as the armature voltage is reduced. The watts supplied to the armature (less its I^2R loss) are plotted against the armature E.M.F. As the armature E.M.F. and therefore the excitation also, are reduced the iron losses diminish [eddy currents loss $\propto \mathbf{B}^2$, hysteresis loss $\propto \mathbf{B}^{1 \cdot 6}$ (Art. 3)]. Therefore by continuing the curve down to zero E.M.F. the friction loss at the constant speed is obtained (see Fig. 12.12).

Since the curve cannot be obtained experimentally for very low voltages (since the motor will not run) there is some doubt as to the exact shape of the dotted part of the curve. If this appears likely to cause considerable error the friction loss may be calculated by taking two points on the curve and using the formula

$$\text{Friction loss} = \frac{W_1 - W_2\left(\frac{E_1}{E_2}\right)}{1 - \left(\frac{E_1}{E_2}\right)^k}$$

where W_1 is the watts supplied when armature voltage is E_1,

W_2 is the watts supplied when armature voltage is E_2,

and k has a value between 1·6 and 2, dependent on the relative amounts of the hysteresis and eddy current losses.

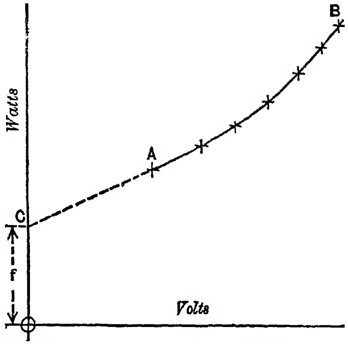


Fig. 12.12.—SEPARATION OF FRICTION LOSS.

f = Friction loss.

16. Retardation Method

If the machine is run as a motor at a high speed with no load, on cutting off the power supply it will slow down gradually and finally stop. During this process the driving power is supplied by the loss of kinetic energy of the armature.

Now Kinetic energy $= \frac{1}{2}K\omega^2$,

where $K =$ moment of inertia of the armature,

$\omega =$ angular velocity in radians per sec.

$= \frac{2\pi n}{60}$, where $n =$ r.p.m.;

Driving power = loss of kinetic energy per sec.

$$= \tfrac{1}{2}K \times (\text{diminution of } \omega^2 \text{ per sec.}) \quad \ldots\ldots (1)$$

$$= \tfrac{1}{2}K \times \left(-\frac{d\omega^2}{dt}\right)$$

$$= -K\omega\frac{d\omega}{dt} \quad \ldots \quad (2)$$

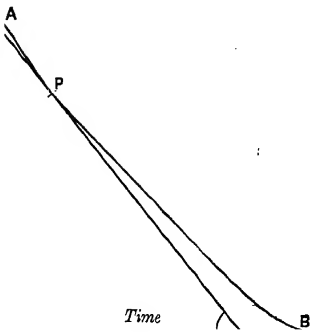


Fig. 12.13.—RETARDATION METHOD (a).

Then if K is known and ω (or n) is observed during the running down of the motor the driving power can be determined in either of the following ways:—

(a) Plot ω^2 (or n^2) against time. At the point P on this curve corresponding to normal speed draw a tangent, PT, to

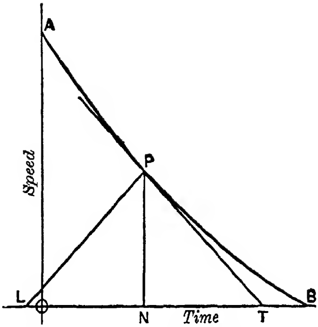


Fig. 12.14.—RETARDATION METHOD

the curve. Then the slope of the curve at P, *i.e.* the tangent of $\angle$ OTP, gives the change of ω^2 per sec. This, multiplied by $\frac{1}{2}$K, gives the driving power [equation (1)].

(*b*) Plot ω (or n) against time. At the point P on this curve corresponding to normal speed draw the normal PL (*i.e.* the line perpendicular to the tangent PT) and the ordinate PN. (See Fig. 12.14.)

The subnormal $\text{LN} = -\,\omega\dfrac{d\omega}{dt}$;

$\therefore$ LN $\times$ K gives the driving power.

Since it is difficult to determine K directly or by calculation, it is eliminated by a second experiment. In the latter, either the necessary driving power is increased by a known amount by loading the motor with a brake; or alternatively K is increased by a known amount K′ by means of a fly-wheel or rotating weights.

Denoting the driving power in the first experiment by P, and the power taken by the brake by p,

$$\text{P} = \tfrac{1}{2}\text{K} \times \text{diminution of } \omega^2 \text{ per sec. in the first experiment,}$$

$$\text{P} + p = \tfrac{1}{2}\text{K} \times \text{diminution of } \omega^2 \text{ per sec. in the second experiment.}$$

Thus P (and K, too, if desired) can be determined from the two experiments.

By the alternative (fly-wheel) method—

$$\text{P} = \tfrac{1}{2}\text{K} \times \text{diminution of } \omega^2 \text{ per sec. in the 1st experiment,}$$

and

$$\text{P} = \tfrac{1}{2}(\text{K} + \text{K}') \times \text{diminution of } \omega^2 \text{ per sec. in the 2nd experiment.}$$

Whence, as before, both P and K can be determined.

The retardation method can be applied to determine the total friction and iron losses, by opening the armature circuit but keeping the excitation constant. The driving power during running down must then supply the iron and friction losses.

By opening the field circuit as well, the necessary driving power is reduced to that required for the friction losses. The latter can therefore be separated by a second retardation test. Since K has been determined by the two experiments of the first test only one experiment is needed in the second test. Note that the field circuit must be disconnected from the armature (*i.e.* an ordinary motor startor cannot be used), otherwise the latter sends a current through the field windings during the running down. A field-breaking switch (see Vol. II.) must therefore be used. An alternative is to drive the motor by another connected to it by a belt, knocking off the belt before taking observations.

The main difficulty in connexion with the retardation method is the accurate determination of the speed, which is continually changing.

17. Combined Method

The preceding methods may be combined in the following way as an alternative to either separately:—

(*a*) Separate the eddy current loss from the other two by the method of Art. 14.

(*b*) Carry out a retardation test with constant field, but do not determine or eliminate K (Art. 16).

(*c*) Repeat (*b*) with the excitation cut off.

From the results of (*b*) and (*c*) the ratio of the friction losses to the sum of the friction and iron losses can be determined at any desired speed. But this sum is known from the observation in experiment (*a*), being the total power supplied to the armature less the I^2R losses (cf. Art. 14). Hence the friction loss is determined. Finally, by subtracting the friction and eddy current losses from the above sum the hysteresis loss alone is left.

This method has the advantage over that of Arts. 14, 15 of avoiding extrapolation in the determination of the friction loss. At the same time it does away with the necessity of determining K (Art. 16), which is somewhat difficult to do accurately.

QUESTIONS ON CHAPTER XII

1. Define the "electrical" and "commercial" efficiencies of a generator. To what causes is the difference between these due?

2. The I.H.P. of a steam engine driving a generator is 500 when the generator is delivering 580 amperes at 525 volts.

Calculate (*a*) "combined" efficiency, (*b*) "commercial" efficiency of generator, if mechanical efficiency of engine is 88 per cent., (*c*) "electrical"

efficiency of generator if the copper losses amount to 10·7 kilowatts, (*d*) sum of friction and iron losses.

3. A shunt-wound generator is supplying 250 amperes at 440 volts terminal P.D. The resistances of the windings are: armature 0·058 ohm, shunt 81·7 ohms. Determine the armature current, the generated E.M.F., and the "electrical" efficiency.

Find the "commercial" efficiency if the friction and iron losses together are 3 per cent. of the output.

4. A "short-shunt" compound-wound generator supplies 50 amperes at 230 volts.

Armature resistance = ·0985 ohm.
Shunt winding res. = 106·9 ohm.
Series ,, ,, = ·041 ohm.

Determine the armature current, the E.M.F., and the electrical efficiency.

5. If the friction and iron losses together equal $3\frac{1}{2}$ per cent. of output, determine the commercial efficiency of the above generator.

6. Find the "electrical" efficiency of a compound-wound motor supplied with 100 amperes at 440 volts:

resistance of armature = 0·135 ohm.
,, shunt = 205 ohms.
,, series turns = 0·026 ohm.

If the B.H.P. of the motor is 50, what is the "commercial" efficiency, and what are the iron and friction losses together?

7. Compare two generators having the same armatures and magnet cores, with regard to voltage, output, losses of various sorts, efficiency, size of wire for shunt and resistance of shunt, and commutating qualities, if one runs at twice the speed of the other.

8. Describe the general characteristics of the Hopkinson, or double conversion, method of testing electrical plant, and criticise its usefulness and convenience (*a*) for a long run at full load, (*b*) as an overall test of efficiency. [Lond. Univ., El. Tech.

9. Describe in detail all the adjustments and tests you would make in determining the efficiency of a direct current shunt motor by the loss method at a given output. Deduce the formula for the current corresponding with a load of W watts supplied at V volts, assuming the loss tests have furnished the requisite data. [Lond. Univ., El. Tech.

10. Describe in detail the retardation method of testing the losses in machines, and compare its reliability with other standard methods. How can the moment of inertia of the rotating parts be found by this method? [Lond. Univ., El. Mach.

11. A shunt motor running light at 480 volts takes a current of 2·5 amperes. The resistance of its field winding is 800 ohms and of its armature 0·6 ohm. Determine the efficiency of the motor when loaded so that the current is 40 amperes, the terminal voltage being maintained at 480 volts. [Lond. Univ. (Mining), El. Tech.

CHAPTER XIII

ACCUMULATORS

1. Accumulators

Accumulators are used for storing electrical energy and giving it out again when required. They are called also *storage cells* or *secondary cells*. The electrical energy is converted into chemical energy, and retained in that form until electrical energy is required again. The chemical energy is then reconverted into the electrical form, apart from the portion turned into heat.

A primary cell gives out electrical energy which is derived from the chemical actions which occur in it, nearly always by the solution of zinc in an acid or a salt solution. The zinc forms as it were the fuel whose consumption produces energy when required. In some cells the effect of sending a current against the E.M.F. of the cell is to reverse the chemical actions, and ultimately to restore the cell to its original condition. For instance, in the Daniell cell (see Chapter II., Art. 8) a current sent through the cell from the copper to the zinc will cause some of the copper to dissolve to copper sulphate, which will crystallise out from the solution, and in the other portion of the cell zinc will be deposited from the zinc sulphate. Cells of this sort are called **reversible**. (N.B.—Sometimes this name is restricted to those cells in which in addition the E.M.F. is the same for both directions of the current.) Thus electrical energy may be used to re-deposit the zinc, which on being dissolved once more again gives out electrical energy.

A secondary cell must be reversible. The most important of the other properties it should possess are:—

(*a*) low resistance, without which it cannot have—

(*b*) high efficiency, for which low resistance is only one essential;

(*c*) large storage capacity;

(*d*) fairly constant E.M.F.;

(*e*) durability;

(*f*) cheapness.

The only cell which possesses these in a sufficient degree to be used largely is the lead, sulphuric acid, lead peroxide cell, though Edison's iron-nickel cell has certain advantages.

2. Chemical Changes

The chemical changes are actually very complex, but their general character is as follows. When the cell is charged the active material of the positive plates is lead peroxide (PbO_2), and that of the negative ones is metallic lead in a spongy state.

When the cell discharges the sulphuric acid (H_2SO_4) is dissociated into H_2 and SO_4 ions. The former move towards the positive plate and there reduce the peroxide to monoxide (PbO), which then combines with the sulphuric acid to form lead sulphate. These reactions may be written as follows:—

$$PbO_2 + H_2 = PbO + H_2O,$$
$$PbO + H_2SO_4 = PbSO_4 + H_2O,$$

or in one step: $PbO_2 + H_2 + H_2SO_4 = PbSO_4 + 2H_2O$.

At the negative plate the SO_4 ions combine with the lead to form lead sulphate; or in symbols—

$$Pb + SO_4 = PbSO_4.$$

The lead sulphate formed is not actually ordinary lead sulphate ($PbSO_4$), which is insoluble, but some more complex compound, probably containing a larger proportion of lead.* Since its exact nature is uncertain the simple formula $PbSO_4$ has been used in the above equations, which therefore must be taken only as representing the general nature of the changes.

On recharging the cell the H_2 ions move to the *negative* plate and the SO_4 ions to the positive. The above chemical reactions are therefore reversed, and the changes may be represented as follows:—

At the negative plate: $PbSO_4 + H_2 = Pb + H_2SO_4$

and at the positive plate:

$$PbSO_4 + SO_4 + 2H_2O = PbO_2 + 2H_2SO_4.$$

Thus the two plates of lead sulphate are converted into lead peroxide and lead respectively. For every molecule of lead peroxide formed on the positive, one atom of lead is produced from sulphate on the negative, and two new molecules of sulphuric acid are formed.

When the cell is nearly fully charged the amount of lead sulphate still present in the plates is insufficient to combine with all the ions reaching the plates. Consequently hydrogen gas is given off at the negative plates. At the same time oxygen gas is given off at the positive plates from the SO_4 ions, the SO_3 recombining with water

* Féry states it to be Pb_2SO_4; see *Sci. Abstr.*, Vol. 22, No. 351.

to form sulphuric acid. This action is known as the gassing of the cell. Gassing occurs to some extent during the whole of charging, but extensive gassing from both sets of plates is a sign that the cell is charged practically completely.

The insoluble lead sulphate ($PbSO_4$) sometimes appears in the plates, which are then said to be **sulphated**. It has a bad effect on a cell, increasing its resistance and diminishing its capacity. It is

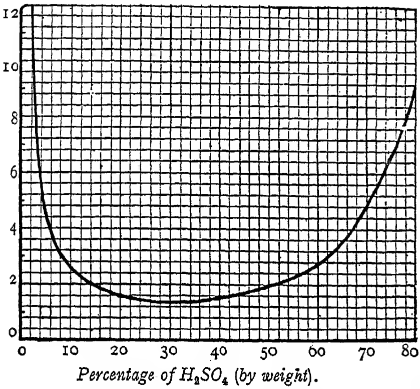


Fig. 13.01.—Specific Resistance of Sulphuric Acid.

Fig. 13.02.—Hydrometer for Accumulators.

usually the result of leaving the cell incompletely charged for a long time, or of continuing the discharge too long (see Arts. 4 and 9).

3. Changes of Specific Gravity of Acid

It can be seen from the equations of the chemical actions that the amount of sulphuric acid present increases during charge, and decreases during discharge. Since pure sulphuric acid has a specific gravity of 1·84 this results in an increase during charge of the specific gravity of the dilute acid used. The values of the specific gravity usually range from 1·21 when the cell is fully charged to 1·18 when it is discharged, but these values vary slightly in different makes of cell. The less acid there is for a given weight of active material the greater the change of specific gravity during a charge or a discharge. The above values correspond with $28\frac{1}{2}$ per cent. and $24\frac{1}{2}$ per cent. respectively of pure sulphuric acid

by weight; or by volume to $17\frac{1}{2}$ per cent. and 15 per cent. of acid respectively. The last percentage corresponds nearly with 6 volumes of water to 1 of acid.

The specific resistance of dilute sulphuric acid decreases as the percentage of acid increases up to about 30 per cent. by weight (see Fig. 13.01). The use of stronger acid up to this value therefore decreases the internal resistance of a cell, but increases the risk of damage to the active material. If the acid is too weak the grid is liable to attack.

In mixing, the acid should be added to the water and not vice versa. The diluted acid should be allowed to cool before its specific gravity is measured, as the value when hot is lower than that at normal temperature.

The specific gravity can be measured by a flat bulb hydrometer. This consists of a glass bulb (see Fig. 13.02) with a long stem showing a scale. It is loaded with shot, so that it floats upright in the acid. The thickness is under $\frac{3}{16}$ in., so that it can go between the plates of a cell if necessary. The specific gravity is read on the scale at the level of the acid.

4. Change of Voltage during Charge and Discharge

The E.M.F. of a charged cell depends on the specific gravity of the acid, varying from 2 volts with a specific gravity of 1·15 to 2·1 volts with a specific gravity of 1·28. When a cell has been recently charged its E.M.F. is higher than these values (about 2·3 volts), but it gradually loses this extra voltage, even if left on open circuit.

On commencing discharge the P.D. at the terminals immediately falls owing to internal resistance. In addition to this the E.M.F. changes, owing to polarisation and to changes of specific gravity in the acid in the pores of the plates. If the circuit is broken the E.M.F. immediately commences to rise, due to the stronger acid diffusing into the plate pores, so that it is very difficult to obtain its true value during discharge. But for this difficulty the resistance of the cell at any time could be obtained readily by measuring the terminal P.D. on closed and on open circuit.

For then:—Internal resistance of cell $= \dfrac{E - V}{I}$,

where E = terminal voltage on open circuit (E.M.F.),

V = ,, ,, on closed circuit,

I = amperes flowing through closed circuit (see further Art. 15).

If the discharge is continued at a constant current the terminal P.D. falls rapidly for a short time, then slowly for some time, and again more rapidly towards the end of discharge (see Fig. 13.03). When the P.D. has fallen to a value dependent on the rate of discharge (about 1·85 volts) the discharge should be stopped. If it is continued the cell will deliver only a comparatively small amount of energy owing to the rapid fall of P.D. At the same time there is a possibility of damage to the cell if treated in this way.

The chief cause of the final rapid fall of voltage is probably an increase of internal resistance owing to dilution of the acid in the pores of the plates (cf. Fig. 13.01). It is not due to exhaustion of the active materials.

On charging the cell the P.D. rises as shown in Fig. 13.03. The charge curve resembles the discharge curve reversed, but is everywhere higher owing to the effects of resistance and polarisation.

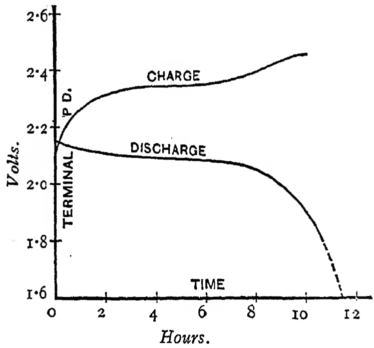


Fig. 13.03.—CHARGE AND DISCHARGE CURVES OF LEAD ACCUMULATOR.

5. Formed Plates

In the original cell of Planté the plates were both pure lead at the start. On passing a charging current through these plates in dilute sulphuric acid the positive becomes covered with a thin layer of transparent brown lead peroxide. This does not increase in thickness if the current is continued. If, however, the cell is left on open circuit the peroxide is reduced to oxide by the lead underneath it. On again passing the current the double layer is peroxidised. By repeating this the thickness of the layer can be gradually increased, but the intervals of rest required grow longer. These can be reduced by discharging the cell instead of leaving it on open circuit. This opens up the layer of peroxide and allows the acid to penetrate it more readily. Even then the time of rest increases, till it becomes so long that further increase of capacity requires more time than it is worth.

This process is known as **formation**, and plates made by this method are called **Planté plates**, or **formed plates**. Negative plates are formed by the same process, and finally turned from positive to negative plates by reversing the current until the whole of the peroxide has been reduced to spongy lead.

A quicker method of formation is obtained by using a forming bath of dilute sulphuric acid (about 1·1 specific gravity) to which a "forming agent" has been added. The forming agent is a substance which on electrolysis produces a *soluble* lead salt, *e.g.* the nitrate, the acetate, or the perchlorate. The action appears to be as follows:—A layer of the soluble salt is formed and is at once converted into lead sulphate, which in turn becomes peroxide. These latter actions release ions which form more of the soluble salt, and so the action continues to a considerable depth of the lead.

The difficulty is to obtain the best proportion of forming agent to acid. If too little is used the plate becomes covered with a protective layer preventing further formation; and if too much the peroxide flakes off.

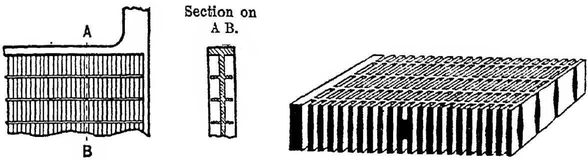
The final stage of formation should be carried out in dilute sulphuric acid alone, otherwise the plates are liable to deteriorate rapidly in use, due to continued action by the forming agent.

The disadvantage of this and other similar quick methods is that very careful adjustment of all the conditions is necessary to obtain satisfactory results. The labour costs are higher, and partly neutralise the saving effected in electric energy compared with the Planté process.

6. Pasted Plates

Faure introduced the use of a paste of red lead or minium, which usually has a composition represented by Pb_3O_4, but varies between Pb_6O_7 and Pb_2O_3. Later litharge (PbO) was used. Whichever is used it is mixed into a paste with dilute sulphuric acid (specific gravity about 1·15) to which other substances (*e.g.* acetic acid) are sometimes added. The paste is applied while moist to a supporting grid (see Art. 7) of lead, and in many cases subjected to considerable pressure to consolidate it. On the other hand when porous plates are desired other substances are mixed with the paste and afterwards dissolved out.

Litharge is always used for negative plates nowadays because less reduction is required to turn it into spongy lead than for red lead. For positive plates red lead requires less peroxidisation, but is more difficult to make into a satisfactory paste than litharge.



(a) "Tudor." (b) Portion of plate, "D.P. Co."

Fig. 13.04.—PLANTÉ POSITIVES.

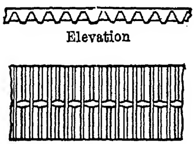
The quantities of electricity required per pound of active material are 109 ampere-hours for the reduction of litharge and the same for its peroxidisation, and 71 ampere-hours for the peroxidisation of red lead if of the composition Pb_3O_4. The formation usually takes from 40 hours to 60 hours, a very much shorter time than that required for Planté plates.

Pasted plates are more liable to disintegration in use than formed (Planté) plates. Hence in the best class of cells, particularly when rapid discharges may occur as in traction work, the positive plates are always of the Planté type. The negative plates are less liable to damage from this cause, consequently pasted negatives are often used.

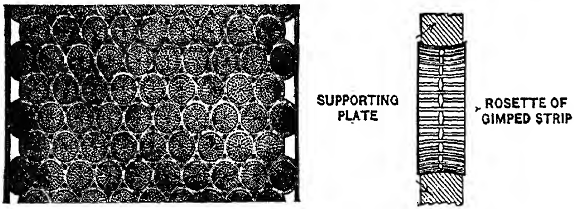
7. Plate Details

Formed positives are made from cast lead plates whose shape is such that they offer a surface up to ten times as great as plain lead plates of equal size (*i.e.* a surface equal to twenty times the area of one side). Two forms are shown in Fig. 13.04.

The D.P. (Dujardin-Planté) Company uses a plate with vertical ribs strengthened at intervals by horizontal ribs. The Tudor plate is somewhat similar, consisting of



(a) "Gimped" strip.

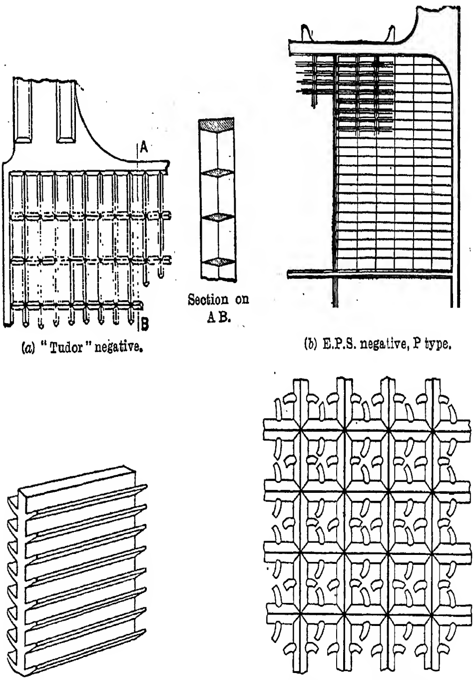


(b) Portion of completed plate. (c) Section of plate.

Fig. 13.05.—"CHLORIDE" POSITIVE.

vertical ribs strengthened by horizontal ribs, the latter not quite reaching the tops of the former, and with a central plate of metal.

The Chloride Company (whose name comes from their former use of lead chloride with 10 per cent. of zinc chloride in the manufacture of their plates) now make plates similar to the above, but



(a) "Tudor" negative. (b) E.P.S. negative, P type.

(c) E.P.S. positive, P type. (d) E.P.S. negative, K type.

Fig. 13.06.—GRIDS FOR PASTED PLATES.

employ also a positive plate of unusual design. Lead strip is passed through rollers which "gimp" it with deep triangular grooves, leaving a central supporting ridge. Lengths of gimped strip are rolled into rosettes. These are placed in round holes in a stout supporting plate. Finally the plate is subjected to hydraulic pressure, which compresses the rosettes and forces them

into contact with the support. As the holes are slightly dovetailed there is no danger of the rosettes falling out after this process.

In pasted plates the active material is held by a lead grid, of which there is a large number of varieties. Four are shown in Fig. 13.06.

In the box type which has been adopted widely in recent years the grid is made in two halves. Each consists of a thin perforated sheet of lead with supporting ribs. After the paste has been applied the halves are riveted together, thus forming a perforated box inside which the active material is held. Projecting pins and corresponding holes are provided on the two halves to ensure the correctness of their relative position.

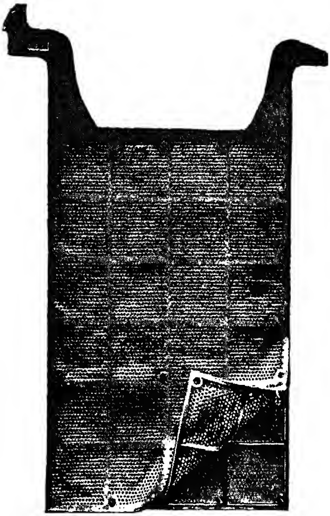


Fig. 13.07.—"BOX" TYPE OF PASTED PLATE.

Supports and grids are made either of pure lead or of lead with from 1 per cent. to 5 per cent. of antimony. This hardens the lead and also renders it less liable to attack by local action. The hardening may be disadvantageous for positive supports owing to the expansion of the active material to which pure lead will yield more readily. Hardened lead is therefore more usual in negative grids, and in positives of special type, *e.g.* the Chloride, in which the support is of hard lead and the rosettes of pure lead.

8. Cell Details

The number of plates in a cell is nearly always odd, an extra negative plate being used so that each positive plate has a negative one on each side of it. If this is not done the end positive plate is very liable to buckle owing to unequal expansion. The plates are prevented from bending so as to touch each other by means of **separators.** These are usually rods or tubes of ebonite or glass,

or thin sheets of specially prepared wood held by slotted wooden rods (see Fig. 13.08). Other separators consist of perforated ebonite, or of ribbed wooden sheets for portable cells.

Stationary cells are contained in glass boxes with open tops. The plates are supported by lugs which rest either on the edge of the box, or on vertical glass slabs inside the box. A sheet of glass rests on the lugs to diminish loss of acid by spraying when gassing occurs. A considerable space is left between the bottom of the box and the plates, so that any loosened active material or other deposit in the cell cannot cause a short-circuit. Large glass boxes are placed on shallow wooden trays, filled with sawdust so as to

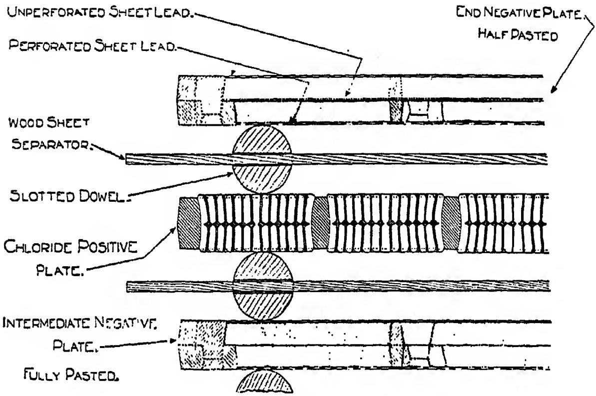
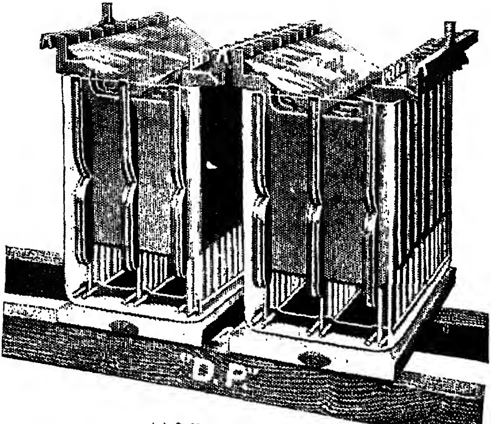


Fig. 13.08.—SECTION OF CELL WITH WOODEN SEPARATORS.

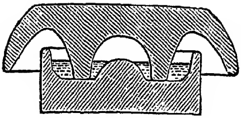
distribute the pressure evenly. The trays are each supported on several glass insulators (see Fig. 13.09) placed on wooden stands made without any metal bolts or nails. These insulators are necessary to prevent loss of charge by leakage.

In modern practice it is usual to omit the trays, and place the glass boxes directly on the insulators. For very large cells lead-lined wood boxes are used in place of glass.

Cells are best connected by burning the end connexions together. In small installations this is too troublesome a process. Brass bolts may be used for these, good brass being much less attacked by acid fumes than iron is. They must, however, be protected by anti-sulphuric enamel, paraffin wax, or vaselin. For the same reason battery connexions are best made by means of lead strip.



(a) Cells, supports, etc.



(b) Section of glass insulator.

Fig. 13.09.—INSULATING SUPPORTS FOR CELLS.

Bolts and nuts with a protective covering of lead or an alloy of lead are often provided (see Fig. 13.10). The bolt and nut may be of brass or iron; the shank of the bolt is not covered, as it is protected when in position by the two lead lugs which it clamps together.

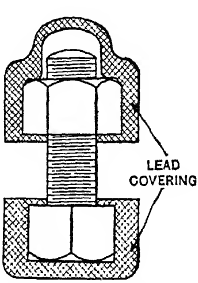


Fig. 13.10.—LEAD-COVERED BOLT AND NUT.

Portable cells of small size are contained in ebonite, celluloid or glass boxes with covers. Apertures must be provided for the escape of gases during charging. The containers are then placed in crates of wood for protection. Larger sizes have lead-lined wooden containers. The plates are supported by glass, celluloid, or other suitable strips at the bottom of the containers so as to leave a free space for deposits.

9. The Capacity of a Cell

The capacity of a cell is the quantity of electricity which it can give out during a single discharge. This is usually taken as the discharge down to some definite voltage in the neighbourhood of 1·8 volts (see Art. 4). The rate of discharge will affect the capacity as thus defined,* for the heavier the current the greater is the voltage used in overcoming the internal resistance of the cell. In addition to this the weakening of the acid in the pores of the plates below the strength of the main body of the acid is necessarily greater with rapid discharges. The effect of rate of discharge on capacity is shown in Fig. 13.11.

The relation of capacity to weight depends upon the ratio of active material to the supporting metal which is not acted on. A high capacity per lb., therefore, means a less durable plate, other things being equal. For stationary batteries a capacity of two to three ampere-hours per lb. is usual, the former figure applying to batteries used for traction supply. Electric automobile cells are made much lighter, because their weight adds to the power they must expend in driving the car. The limit is about 5 ampere-hours per lb. per cell, higher figures increasing excessively the cost of cell renewals. With plates of ordinary thickness the capacity is roughly $\frac{1}{2}$ ampere-hour per sq. in. of plate area (only one side is taken in reckoning the area). The positives are usually thicker, especially if they are formed and the negatives are pasted. Nevertheless the capacity is generally limited by the positives and not by the negatives.

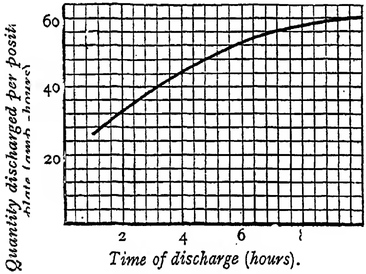


Fig. 13.11.—Relation of Capacity and Discharge Rate.

Temperature has a very great effect on capacity, especially at high rates of discharge. The P.D. is raised slightly and the capacity

* Output is a better name for the quantity discharged, retaining capacity for the total ampere-hours stored in the cell, but the general custom is to give the name capacity to the former quantity, and this practice has been adopted in what follows.

greatly by an increased temperature. Compared with the capacity down to 1·7 volts at 15° C.; the 10-hour rate capacity is increased by $\frac{3}{4}$ per cent. per degree C. rise, that at the 3-hour rate by 1 per cent. per degree C., and at the 1-hour rate by $1\frac{1}{2}$ per cent. per degree C.

These increases are due mainly to the higher temperatures permitting more rapid diffusion of the acid, and increasing the velocities of the ions taking part in the chemical changes.

10. Efficiency

Two different sorts of efficiency are used in connexion with accumulators: (*a*) The quantity or ampere-hour efficiency, (*b*) the energy or watt-hour efficiency. In each case the efficiency is considered for a complete charge and a complete discharge down to the state in which the cell was at the commencement. Under this condition—

$$\text{The quantity efficiency} = \frac{\text{ampere-hours discharged}}{\text{ampere-hours of charge}}.$$

$$\text{The energy efficiency} = \frac{\text{watt-hours discharged}}{\text{watt-hours of charge}}.$$

The latter is always the smaller of the two because the average P.D. during discharge is lower than the average P.D. during charge, owing to internal resistance and polarisation.

Both depend on the rate of discharge, but the quantity efficiency varies only slightly with different rates. The energy efficiency is reduced in a greater proportion by rapid discharges, since the terminal P.D. is lowered more the greater the current. This refers to complete charges and discharges; the energy efficiency for partial charges and discharges alternating at intervals of a few minutes or less (as in a battery used on a traction load, see Chapter XVI., Arts. 5, 6, and 7) is higher, especially with short intervals. This "working" efficiency is about 80 per cent. to 85 per cent., while the ordinary energy efficiency is about 75 per cent. for average conditions. The quantity efficiency is 90 per cent. to 95 per cent., the latter figure being obtained when discharge follows charge almost immediately.

The method of charging affects both capacity and efficiency. The ordinary method is to charge at constant current, raising the charging voltage as the cell E.M.F. increases. An alternative method is to charge at constant P.D. This results in a very large current at first when the cell E.M.F. is low, and a much smaller current towards the end of charging.

The effect of the constant P.D. method is to diminish the time of charging to one half or less, to increase the capacity by about 20 per cent., and to reduce the efficiency by about 10 per cent. The full effect occurs only if discharge follows charge at once, and is due mainly to great concentration of acid in the pores of the positive plates.

11. Maintenance

The following are some of the important points to which attention should be paid in order to maintain a battery in good condition:—

The discharge should not be continued after the P.D. has fallen, *with the current flowing* at the normal rate, below the permissible minimum value for the particular rate of discharge. With larger currents the P.D. may be allowed to fall lower. *E.g.* at the 20-minute rate it may fall to 1·6 v.

The battery should not be left standing in a discharged state longer than necessary. If it is to be out of use for some time it should receive a charge which is continued till gassing takes place from both positive and negative plates. If possible a short charge should be given to it once a fortnight. If the battery can be conveniently divided into sections, or all the cells separated, it is advisable to do so, as this diminishes leakage.

The probable result of over-discharging the cells or of leaving them for long in a discharged state is that the plates become **sulphated,** *i.e.* the insoluble white sulphate of lead ($PbSO_4$) is formed. This has a high resistance, and therefore diminishes the efficiency and the capacity of the cell. It is very difficult to cure. The usual treatment is to give the cell a succession of overcharges. When one or two cells show signs of sulphation or other trouble requiring extra charging for its cure, they may be cut out of circuit during discharge and thus receive two charges with no intervening discharge. An alternative is to use a small "milking" booster or a couple of "hospital" cells to continue the charge to the faulty cell after the battery as a whole has been fully charged. These latter methods have the advantage of not necessitating the breaking of connexions between the faulty cell and the rest of the battery.

When a series of overcharges fails to cure sulphation the free acid should be emptied out and replaced by pure water. This, mixed with the acid in the plate pores, will give a weak acid of Sp. Gr. 1·1 or less. Prolonged charging should then cure the trouble, though it may be necessary to replace the acid again with water when the Sp. Gr. has risen to 1·17.

The level of the electrolyte should be kept well above the tops of the plates by the addition of distilled water when required. Occasionally acid may have to be added to make up for that thrown out when gassing occurs. If this has to be done some of the acid in the cell should be drawn off. Strong acid is added till its specific gravity is raised to 1·4 or less. It is then run back into the cells. If necessary this may be repeated after plenty of time has been allowed for the stronger acid to diffuse thoroughly throughout the cell. The specific gravity of the acid in each cell should be taken regularly at the end of a charge, as its regularity affords a good indication of whether the battery is keeping in satisfactory condition.

12. The Iron-Nickel Accumulator

The iron-nickel storage cell invented by Edison is the only one that has any prospect of competing with the lead cell. It has a nickel positive, an iron negative, and an alkaline electrolyte.

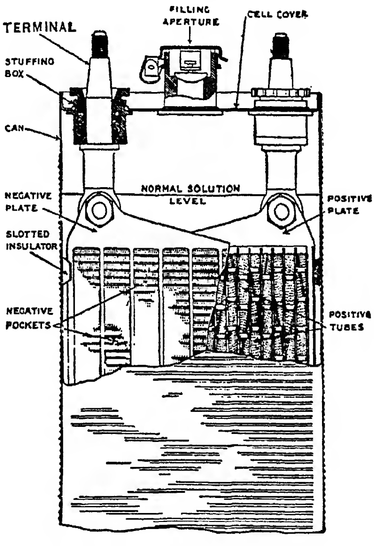


Fig. 13.12.—EDISON IRON-NICKEL ACCUMULATOR.

The positive plate is built up of a number of steel tubes formed from a perforated ribbon with a lapped spiral seam. The active material consists of nickel hydroxide and thin flakes of metallic nickel, which are packed into the tubes in alternate layers. The tubes are then reinforced with seamless steel rings and are flanged at both ends. They are clamped into contact with a frame of cold rolled steel (see Fig. 13.12). Both the tubes and the frame are heavily nickel-plated.

The negative plate is built up of a number of rectangular pockets, stamped from finely perforated nickelled steel ribbon.

These are filled with the active material, which is powdered iron oxide. It is prepared from monosulphide of iron by eliminating the sulphur by alternate oxidisations and reductions in caustic potash. This method of formation is necessary to make the iron susceptible to electrolytic oxidisation and reduction, *i.e.* the actions which occur when the cell is in use. The pockets are inserted in a grid of cold rolled steel, heavily nickel-plated. The plate is subjected to great pressure between dies which corrugate the surfaces of the pockets and force them into intimate contact with the grid (see Fig. 13.12).

The plates are insulated from each other by hard rubber strip separators, and from the container by thin sheets of hard rubber. This container is made of cold rolled sheet steel with welded seams. It is nickel-plated and covered with a flexible insulating compound. The sides of the container are corrugated to increase its stiffness.

The electrolyte consists of a 21 per cent. solution of caustic potash (KOH) in distilled water with a small proportion of lithium hydrate (LiOH). The normal Sp. Gr. is 1·22, and this does not vary much during charge or discharge.

The surface of the electrolyte should be above the tops of the plates. When it falls too low it should be brought up by the addition of distilled water before charging. After nine months or more of use the Sp. Gr. at the end of full charging will have fallen below 1·16. It should then be poured away and replaced with a 25 per cent. solution (Sp. Gr. 1·25). This stronger solution is necessary to compensate for the old weak electrolyte retained by the plates.

This cell has a bigger capacity per lb. than the lead cell, but a lower efficiency. For electromobiles, train-lighting, etc., it has an advantage over the lead cell, but for stationary work the latter is undoubtedly superior.

The ampere-hour efficiency is about 80 per cent., and the watt-hour efficiency 55 per cent. to 60 per cent. A cell with 300 ampere-hours capacity at the 8-hour discharge rate weighs 30 lb. complete. At this rate the average P.D. during discharge is 1·24 volt, varying from about 1·4 volt to 1·1 volt. Allowing for the higher P.D. of the lead cell the weight is equivalent to from six to seven amp.-hr. per lb. per cell (lead), *i.e.* much less than a lead cell weighs (see Art. 9). During charge at the 7-hour rate the P.D. rises from 1·55 to 1·8 volt.

The capacity increases about 10 per cent. with use, and is not diminished by short-circuits nor by long standing in the discharged state. The active material is not thrown down by jolting or over-

charging, so that the life of the cells is very long, and the upkeep is confined almost entirely to occasional renewal of the electrolyte. A "boosting" charge, *i.e.* one of short duration at twice to four times normal rate, may be given at any time during discharge. By doing this during the dinner hour the running distance per day of an electromobile can be increased by 30 per cent. and the efficiency improved. This cell is therefore useful when the conditions of working are very variable or severe.

13. Cyclic State

In testing a secondary cell, especially for efficiency, it is necessary to make sure that the cell has returned to the same chemical state as at the commencement of the test. Otherwise part of the output may have been obtained at the expense of some of the chemical energy originally contained in the cell, and efficiencies of over 100 per cent. are possible.

Even if the P.D. of the cell when delivering a given current, and the specific gravity of the acid have returned to their original values, it is not certain that the chemical state of the plates has also been exactly restored. To ensure this a succession of charges and discharges must be carried out and continued until the voltage-time and specific gravity-time curves are repeated exactly each time. The cell is then said to be in a cyclic state. When this is reached accurate values of efficiency and capacity can be obtained. Since the behaviour of a cell depends on its past history it is necessary to obtain the cyclic state separately for each rate of discharge or charge for which the efficiency or capacity is to be determined.

14. Tests of Capacity and Efficiency

When the cell has reached the cyclic state for the desired conditions of charge and discharge the above tests are quite straightforward. The discharge is nearly always at constant current, and the only other quantity required to obtain the capacity in ampere-hours is the time of a complete discharge.

If the charge is at constant current the ampere-hour efficiency is likewise obtained readily by observing the time of a complete charge, for—

$$\text{Ampere-hour efficiency} = \frac{\text{discharge current} \times \text{time of discharge}}{\text{charge current} \times \text{time of charge}}.$$

When the charging current varies, *e.g.* in charging at constant P.D., it must be observed at frequent intervals. The area of the

current-time curve then gives the total quantity of the charge (see Fig. 13.13).

An alternative method is to place some form of ampere-hour meter in series with the cell and so obtain the ampere-hours of charge directly.

To obtain the watt-hours with a constant current the voltage is observed at intervals, and the total volt-hours of charge or discharge obtained as in Fig. 13.13. Then—

$$\text{Watt-hour efficiency} = \frac{\text{discharge current} \times \text{volt-hours of discharge}}{\text{charge current} \times \text{volt-hours of charge}}.$$

When the current varies, both it and the volts must be observed and the watt-hours obtained by plotting watts against time as above.

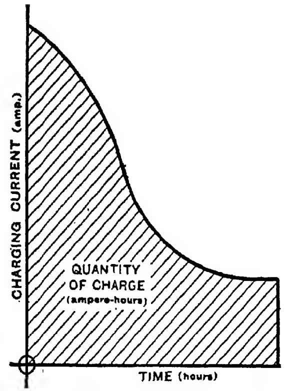


Fig. 13.13.—QUANTITY AND CHARGING CURRENT.

In all cases it is advisable to take the mean values of a series of successive charges and discharges. This reduces the error due to the cyclic state not being quite attained.

15. Tests of Internal Resistance

The internal resistance of a cell at any time during charge or discharge can be obtained by means of an ammeter and voltmeter. The terminal P.D. and the charge or discharge current are noted. The circuit is then opened temporarily and the voltmeter reading taken again as soon as possible, this giving the cell E.M.F.

Then $$\text{Internal resistance} = \frac{\text{E.M.F.} - \text{terminal P.D.}}{\text{discharge current}}$$

or $$\text{,, \quad ,,} = \frac{\text{terminal P.D.} - \text{E.M.F.}}{\text{charge current}}$$

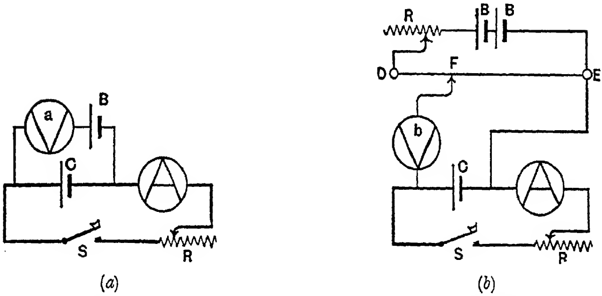
Since the numerator in the above expressions is the difference of two nearly equal quantities, small errors in obtaining their values may make a large error in the value of the internal resistance (see Example). Consequently when accurate values are required other methods must be adopted.

The simplest of these is to use another cell on open circuit to balance the greater part of the voltages measured, and to use a millivoltmeter with a central zero [see Fig. 13.14 (*a*)]. A similar method is to obtain an adjustable balancing voltage by means of two cells and a slide wire [see Fig. 13.14 (*b*)]. The balancing voltage may be adjusted to balance the terminal P.D. on closed circuit exactly, or to have a value intermediate between this and the E.M.F.

Example.—*The terminal P.D. of a cell when discharging at* 12 *amperes is* 1·88 *volts; on opening the circuit the voltage rises to* 2·03 *volts. What is the internal resistance of the cell?*

If there is a possible error of ·02 *volt in the readings* (i.e. *about* 1 *per cent.*) *what are the possible errors in the value of the internal resistance?*

The internal resistance $= \dfrac{2{\cdot}03 - 1{\cdot}[\text{illegible}]}{[\text{illegible}]}$ ohms = 0·0125 ohm.



$R = \dfrac{\text{Change of } V_a}{\text{Current}}$ $\qquad$ $R = \dfrac{\text{Change of } V_b}{\text{Current}}$

Fig. 13.14.—MEASUREMENT OF INTERNAL RESISTANCE OF AN ACCUMULATOR.

With possible errors of ·02 volt,

Maximum possible internal resistance $= \dfrac{2{\cdot}05 - 1{\cdot}86}{12} = 0{\cdot}0158$ ohm.

If this is the true value the error is − ·0033 ohm,

$$\frac{{\cdot}0033}{{\cdot}0158} \times 100 \text{ per cent.} = -\,21 \text{ per cent.}$$

Similarly,

Minimum possible internal resistance $\dfrac{2{\cdot}01 - [\text{illegible}]}{[\text{illegible}]} = 0{\cdot}0092$ ohm.

If this is the true value the error is + ·0033 ohm,

or $$\frac{{\cdot}0033}{{\cdot}0092} \times 100 \text{ per cent.} = 36 \text{ per cent.}$$

In each case the error expressed as a percentage of the calculated value is

$$\frac{{\cdot}0033}{{\cdot}0125} \times 100 \text{ per cent.} = 26 \text{ per cent.}$$

The actual error is not likely to be more than a third of the above values, say about nine per cent. of the calculated value.

QUESTIONS ON CHAPTER XIII

1. In what way does a secondary cell (*i.e.* an accumulator) differ from a primary cell, and what does it accumulate?

State its most important properties.

Define its "capacity," and say upon what it depends.

2. Define the "quantity" and "energy" efficiencies of an accumulator.

Calculate these for an accumulator which is charged in 8 hours by 30 amperes at an average P.D. of 2·2 volts, and is discharged in 9 hours by 24 amperes at an average P.D. of 1·9 volts.

3. State the various ways in which you can tell when an accumulator is fully charged, and when it is discharged down to a safe limit.

4. How many cells would be required for a secondary battery of the ordinary lead type to supply 100 amperes for three hours to a 240-volt system? If you were put in charge of such a battery and told to have it in readiness, how would you find out the state of the charge, and how would you charge it? [C. & G., II.

5. To what point should accumulators be discharged? What is the suitable density of the acid that should be used in them? How would you test a set of accumulators to discover whether they were fully charged? What is the particular harm that results if accumulators are left long in a discharged state? [C. & G., II.

6. Describe the main chemical changes that occur in a lead accumulator during charge and discharge respectively.

What are the relative advantages of "formed" and "pasted" plates?

7. Describe in detail some modern accumulator. Why is there one more negative plate than there are positive ones?

Describe and explain the use of the insulators employed.

8. Describe the actions taking place during charge and discharge of a secondary cell.

What tests would be made on a secondary cell in order to determine its condition? [Lond. Univ., El. Tech.

9. What is the effect of using acid (*a*) stronger than normal, (*b*) weaker than normal?

How would you estimate the maximum current that can safely be taken from a given cell?

10. Describe, with sketches, one important type of lead storage cell suitable for heavy discharge work; and explain the chemical changes taking place in such a cell during "charge," "discharge," and "rest." In what way, if any, are the E.M.F., life, internal resistance, capacity, and efficiency affected by the variations of the rate of charge and discharge, and by the specific gravity of the electrolyte? [Lond. Univ., El. Eng.

11. Describe some form of secondary cell, and explain the electrical and chemical changes that will take place during charge and discharge. How does the ampere-hour capacity of a cell vary with the rate of discharge? What precautions are necessary to keep a secondary cell in good working order? [C. & G., II.

12. Discuss the advantages and disadvantages of iron-nickel cells for electromobiles.

13. A storage cell has 13 plates each 25 cm. by 23 cm. The clearance between neighbouring plates is 12 mm. The resistivity of the acid is 1·6 ohm per cm. cube; find its resistance.

What factors make the internal resistance of the cell higher than this calculated resistance?

14. Describe how to test the resistance of a storage battery (*a*) during charge, (*b*) during discharge?

CHAPTER XIV

LIGHTING AND ILLUMINATION

1. Radiation

Artificial lighting is nearly always done by means of incandescent bodies, *i.e.* bodies raised to a high temperature. An exception is the electric discharge lamp, in which an electric current is sent through a tube containing gas at low or high pressure.

When the temperature of any body is raised above that of its surroundings it radiates energy in all directions. These radiations consist of waves in the ether travelling with a velocity of 3×10^{10} cm. (or 186,000 ml.) per sec. When the temperature of the radiating body is low the waves are long, and are then perceptible as warmth (radiant heat) but not as light. As the temperature rises shorter waves are emitted, and when their wave-length has become under 0·8 micron (1 micron = one millionth of a metre = ·001 mm.) they produce the impression of red light. The longer (heat) waves are still emitted and are of increased intensity. At higher temperatures, still shorter waves are added and the light changes through bright red and yellow to white. Radiation down to a wave-length of 0·36 micron is visible. Shorter waves are known as ultra-violet and produce no sensation of light in the eye, though they can affect a photographic plate.

It can be proved that the best radiator (*i.e* the one that radiates most energy per unit area at a given temperature) is necessarily the best absorber, and vice versa. A body which absorbed all radiations which reached it would therefore be a perfect radiator. Such a body is called a *perfectly black* body. Since all bodies reflect some of the radiation which they receive, a perfectly black body does not exist. An exceedingly close approximation can be obtained for experimental purposes, however, by using an opaque tube closed at both ends, but with a small observation hole left in one end.

2. Efficiency

The *radiant efficiency* of a light-giving body is the ratio

$$\frac{\text{Energy radiated as light}}{\text{Total energy radiated}}.$$

This is sometimes called the *luminous efficiency*, but the latter term is better reserved for the ratio

$$\frac{\text{Energy radiated as light}}{\text{Total energy supplied}}.$$

The denominator in this case includes losses by convection and by conduction, as well as by non-luminous radiations.

As the temperature rises the radiant efficiency increases, since its numerator increases faster than its denominator. This continues to be true up to a temperature of about 6 000° C. Above this the maximum radiation is shifted so far into the ultra-violet region that the radiant efficiency is again diminished. However, since no solid illuminant approaches this temperature, it is true in general that the higher the temperature the greater the radiant efficiency.

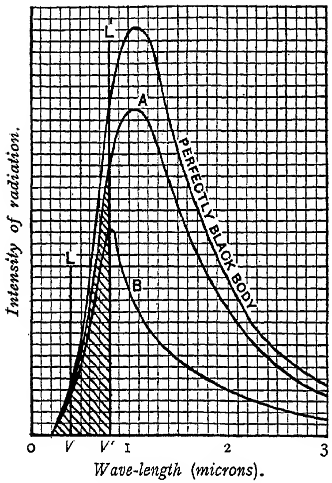


Fig. 14.01.—RADIATION AT DIFFERENT WAVE-LENGTHS.

LV = limit of visibility at violet end of spectrum.
$L'V'$ = limit of visibility at red end.
$0V$ = ultra-violet. V' 1, 2, 3 = infra-red.

3. Selective Emission

As has been mentioned above (Art. 1) all actual bodies radiate less energy than they would if perfectly black. In most cases the amount of energy radiated at each wave-length is diminished in a constant ratio. When this is the case the radiant efficiency at any temperature is the same as that of a perfectly black body, since the numerator and denominator in the expression for efficiency are reduced in the same proportion.

With certain substances, however, the energy radiated at some wave-lengths is reduced by a much smaller proportion than that radiated at others. Such substances are said to be selective radiators, or to possess selective emission. This is illustrated in Fig. 14.01. Curve A represents the connexion between energy and wave-length

of a substance whose radiation is normal. This lies everywhere below the curve for a perfectly black body by a constant percentage. The radiant efficiency is given by the ratio of the area of the curve between the limits of visibility to the area of the whole curve. This efficiency is therefore, as stated above, the same for A as for the perfectly black body.

B is a similar curve for an imaginary substance possessing strong selective emission. Its radiant efficiency is much higher than that of the perfectly black body, because the non-luminous radiations are diminished in a much greater ratio than the luminous ones. Such substances would be very suitable for light sources. Very few solids, however, possess this property to any great degree, and the same applies to liquids. Tantalum and tungsten probably radiate selectively to a small extent, but this has little effect on their radiant efficiency which depends almost entirely on the temperature at which they are run.

With gases, on the other hand, selective emission is the rule, so that high efficiency is possible without a very high temperature. (See Chapter XV., Art. 14.)

4. Candle-Power

Light is a form of energy. It consists of that part of the energy radiated from a body which is capable of affecting human eyes.

The quantity of light emitted per second by any source is known as the *luminous flux* or *flux of light.* Since this is the rate of emitting energy it is of the same nature as power (see Chapter II., Art. 4).

A source may emit light in all directions, but it usually emits it more strongly in some directions than in others. This is expressed by saying that the intensity of the light is different in different directions. The **intensity*** (or **luminous intensity**) in any given direction is measured by taking a small solid angle containing this direction and with the source at its apex, and dividing the luminous flux within it by the angle. This is the most important quantity in *photometry* (*i.e.* the measurement of light), for the intensities of two lights in any given directions can be compared directly. The unit is an arbitrary one, the **candle** (or *candle-power*). (See Arts. 7, 8.)

* This is not a good term, since intensity is generally used for some quantity per unit area. The luminous power would be preferable, but intensity is the term in general use.

The *mean horizontal candle-power* is the mean of the candle-powers in all directions in a horizontal plane through the centre of the source of light.

The *mean spherical candle-power* is the mean of the candle-powers in all directions from the centre of the source. It is therefore equal to the candle-power of a source radiating the same total flux of light as the actual source, but with equal intensity in all directions.

The *mean hemispherical candle-power* lower (or upper) is the mean of the candle-powers in all directions below (or above) the horizontal plane.

One unit of luminous flux is the spherical candle-power, *i.e.* the total flux emitted by a point source giving one candle-power in every direction.

A better unit is the flux emitted in unit solid angle* with one candle-power in every direction from the apex of the angle. This is called the lumen.* Evidently 1 spherical candle $= 4\pi$ lumens, since the surface area of a sphere of radius r is $4\pi r^2$.

The units of quantity of light are the candle-hour and the lumen-hour. The former is the light emitted by a source of one mean spherical candle-power (*i.e.* emitting unit flux) in one hour. Similarly, the lumen-hour is the light emitted when a flux of one lumen continues for one hour.

5. Illumination and Brightness

The *illumination* of a surface is the luminous flux received by it divided by its area. The unit is that produced by a source of one candle-power on a surface at unit distance from the source and perpendicular to the rays of light. In the British system the unit distance is one foot, and the unit of illumination is called the foot-candle. In the international system the unit distance is a metre and the unit of illumination is called the metre-candle or the lux.

The *brightness* (or *intrinsic brightness*) is the candle-power emitted per unit area of a source of light, in a direction normal to its surface. It is applied both to primary sources of light (lamps) and to illuminated surfaces which reflect more or less of the light falling upon them. In the former case a high intrinsic brightness is harmful to the eyes if the lamp can be seen directly. In the case of illuminated surfaces the brightness depends on the illumination, and on the nature of the surface.

* Unit solid angle is one which subtends unit area on a sphere of unit radius: the unit of length employed makes no difference in the resulting unit solid angle. It is sometimes called the "steradian."

6. Laws of Illumination

In the case of light coming from a *point source* the illumination of a surface is inversely proportional to the square of its distance from the source. This law assumes that none of the light is absorbed or refracted (bent aside) in passing over the extra distance.

Further, when the surface is inclined so that the rays are not normal to the surface but make an angle α with the normal, the illumination is reduced in the ratio of $\cos \alpha$ to 1.

These laws are strictly true only for light coming from a point. But as long as the distance of the illuminated surface from the light-giving body is large compared with the size of the body, the laws are sufficiently close approximations.

With a parallel beam of light the cosine law is true, but the illumination is independent of the distance, excepting the diminution due to absorption on the way.

With most light-giving surfaces a similar law holds good as to the light which they give out again. Namely, that the flux of light given out normally by a particular portion of an illuminated surface is greater than the flux in a direction making an angle α with the normal in the ratio of 1 to $\cos \alpha$.

This law does not hold good universally; a polished surface reflecting the light from another source is an evident exception. It is true for most primary sources and for many secondary ones. For most matt surfaces the departures from the exact relationship are about 5 per cent. to 10 per cent.

Often these cosine laws are called Lambert's Law: but strictly the name should be applied only to the first of them.

7. British Standards of Candle-Power

The British standard candle is one of pure spermaceti wax, of size 6 (*i.e.* 6 to the pound) burning 120 grains an hour. It is no longer used as a standard except occasionally for gas photometry. Its place has been taken by the pentane lamp. Standard lamps of two candle-power are used sometimes, but the official standard of the British National Physical Laboratory is the Vernon-Harcourt ten-candle-power pentane lamp.

This is shown in Fig. 14.02. It burns a mixture of air and pentane vapour. Pentane (C_5H_{12}), a volatile liquid, is poured into the tank till it is half full, as can be seen by looking through the windows provided at each end of the tank. The air in contact with the pentane becomes impregnated with its vapour and descends the rubber tube to the burner. This is made of steatite (soapstone) and is of

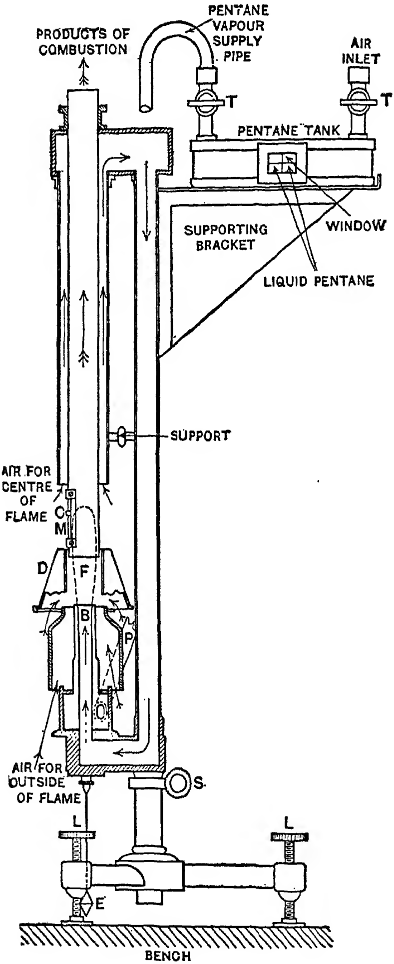


Fig. 14.02.—HARCOURT PENTANE STANDARD LAMP.

B, Burner (perforated ring). C, Cross-bar on mica window. D, Draught shield for flame. E, Plumb bob for adjusting position. F, Flame. L L, Levelling screws. M, Mica window. P, Pentane vapour supply pipe. S, Clamping screw for adjusting height. T T, Regulating taps. → Paths of air supply for centre of flame. →→ Paths of air supply for outside of flame. →→→ Path of products of combustion..

the Argand type. The air supply to the centre of the flame is heated by the products of combustion (see Fig. 14.02). The height of the flame is regulated by the two taps on the tank, until the top of the flame is between the cross-bar and the top of the mica window in the metal chimney. The intensity of the lamp in a horizontal direction is then 10 International candle-power. A blackened metal shield with an opening only in one direction protects the flame from draughts.

The candle-power is affected both by the atmospheric pressure and by the humidity of the air. The following equation has been found to apply within the ranges of variation of these experienced in practice.

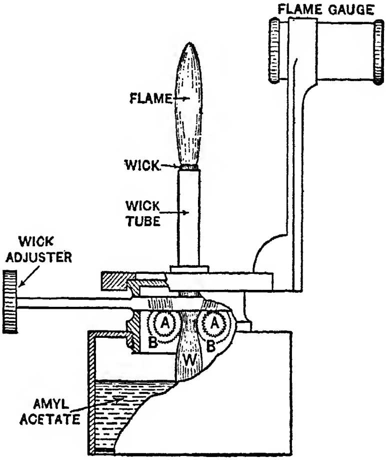


Fig. 14.03.—HEFNER STANDARD LAMP.
A A, Worm wheels gearing with right- and left-handed worms on wick adjuster.
B B, Wheels, on same axles as AA, to grip wick. W, Wick.

$$\text{Candle-power} = 10 + \cdot066\,(10 - h) - \cdot008\,(760 - b),$$

where h = humidity in litres per cubic metre of dry air,

b = height of barometer in mm. of mercury.

The standard conditions are a barometric height of 760 mm. and 10 litres of water per cubic metre of air.

8. Other Standards of Candle-Power

The German standard is the Hefner lamp, which is of very simple construction (see Fig. 14.03). It burns amyl acetate by means of a wick (W) of loosely twisted cotton threads, whose height can be regulated by two toothed wheels (B, B) which are geared together. These are adjusted till the top of the flame is 40 mm. above the top of the gun-metal tube (8 mm. internal and 8·3 mm. external diam.) up which the wick passes. This adjustment is facilitated by

means of a small reading microscope with a cross line, held at the correct height by an arm attached to the lamp. When properly adjusted the intensity in a horizontal direction is taken as 0·9 candle-power.

The Hefner lamp is simpler in construction and in use than the pentane lamp. It has, however, the disadvantages of a low candle-power, and of a colour which is too orange in tone for easy comparison with most electric lamps.

The French standard is the Carcel lamp burning colza oil. It is of 9⅝ candle-power. It is now little used.

The equivalent values adopted internationally in 1911 were:—

	1 pentane candle	= 1	International candle
	1 Hefner	= 0·901	,, ,,
and	1 Carcel candle	= 0·966	,, ,,

or 1 International candle = 1·110 Hefner = 1·035 Carcel candles.

Since 1933 the International candle has been used in all countries.

All these flame standards are liable to errors through variations in atmospheric conditions or in the composition of the fuel used as well as through inaccuracies in construction. The Violle unit was suggested so as to overcome these sources of error. It is the luminous intensity of 1 sq. cm. of pure molten platinum at its solidification point. It is of little practical use owing to the difficulty of working with it. Its value is 20 International candle-power. One-twentieth of its candle-power is called the Bougie Décimale, which is equal to one International candle-power.

9. Secondary Standards

For electric lamp testing it is much more convenient to use an electric lamp as a standard than to employ a flame standard. The lamp thus used has its candle-power in a particular direction determined by careful comparison with the pentane standard; it is therefore a *secondary* standard. This secondary standard is then used to determine the candle-power of one or more working standards. The latter are used as standards during tests, and are re-calibrated at intervals by comparison with the secondary standard. This therefore gets comparatively little use, and so need be checked against the primary standard only at comparatively long intervals.

The usual type of secondary standard is the Fleming large bulb incandescent lamp. This consists of a single-loop carbon filament, mounted in a large cylindrical bulb 4 in. diam. and 8 in. long. The

filament is "aged" by being used in an ordinary bulb until the initial rise of C.P. (see Chapter XV., Art. 8) is over. It is then transferred to the large bulb, in which blackening of the glass takes place much less rapidly than in one of the ordinary size; therefore the candle-power changes very slowly. Since the secondary standard is used only occasionally and for a few minutes at a time, as described above, such a lamp may be used as a reference standard for a long time. The cylindrical shape of the bulb makes errors in setting it to the height of the photometer head cause less change in illumination than would a bulb of ordinary shape.

The International candle is maintained by the National Standardising Laboratories of France, Great Britain, and the United States of America, by means of electric incandescent lamps.

10. Photometry

Photometers are instruments used for the comparison of the candle-powers of two sources in definite directions. If one of these is a standard of known intensity, that of the other can then be expressed in candles.

In their use two similar surfaces are used, each illuminated by one only of the sources, and the illuminations are adjusted till the surfaces are of equal brightness. The rays from the two sources are either normal to their respective surfaces or both make the same angle with the normals. Then, since the surfaces are similar and equally bright, they must be equally illuminated. But, by Art. 6 the illumination of a surface is equal to $\frac{\text{C.P.}}{d^2}\cos\alpha$,

where C.P. = candle-power of the source,

d = distance of source from surface,

α = angle between rays and normal to surface;

$$\cos\alpha = \frac{\text{C.P.}_2}{d_2^2}\cos\alpha,$$

where the suffixes $_1$, $_2$, refer to the two sources respectively;

$$= \frac{d_1^2}{}.$$

or the candle-powers of the sources are *directly* proportional to the squares of their distances from the respective sources.

It is advisable that the two surfaces should be adjacent or should be made to appear so. It is much easier to judge whether they are equally bright in such a case than if they were a distance apart.

It is preferable that the directions from which they are viewed should make the same angles with the normals to the surfaces. Otherwise the accuracy depends on the truth of the cosine law for light-giving surfaces (Art. 6).

Two methods are used for adjusting the illuminations on the surfaces. In one the photometer is stationary, and one or both of the lamps are moved to or from it as required. In the other the lamps are fixed, and the photometer is placed between them: its movement then increases the illumination on one side and decreases it on the other.

The inverse square law of illumination is strictly true only for point sources. As long as the dimensions of the source are small compared with the distances from it, the law remains sufficiently accurate for use as above.

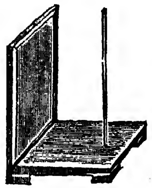


Fig. 14.04.—RUMFORD PHOTOMETER.

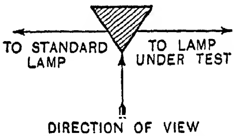


Fig. 14.05.—PLAN OF "WEDGE" PHOTOMETER.

11. Photometers

The *Rumford shadow photometer* consists of an opaque white screen with a vertical rod a short distance in front of it (see Fig. 14.04). The lights under test are arranged so that they throw separate shadows of the rod on the screen. One or both of the lamps is then moved until the shadows appear equally dark. The candle-powers of the lamps are then proportional to the squares of their distances from the screen. This is true because the shadow thrown by each lamp is illuminated by the other lamp only, while the rest of the screen is illuminated by both. Therefore equal darkness of the shadows means that the lamps are producing equal illuminations on the screen.

Good results can be obtained with this photometer after practice, but it requires more practice than some other forms.

The *wedge photometer* consists of a blunt wedge (about 70°) covered with dull white paper or some other substance with a similar surface. It is placed between the lamps to be compared

and is viewed from a direction at right angles to the line joining the lamps (see Fig. 14.05). The wedge is moved until its edge disappears owing to the two sides becoming equally bright. Means are provided for ensuring that the sides of the wedge make equal angles with the direction of view and with the rays from the respective lamps.

Bunsen's grease-spot photometer depends on the fact that a grease spot on paper appears darker than the paper when illuminated from in front, but is brighter than the paper when illuminated from behind. Consequently when equally illuminated on both sides it will vanish, or at least appear equally bright from either side. The photometer consists of a sheet of white paper with a circular or star-shaped grease spot at its centre (or else the whole of the paper is greased except the central spot). Mirrors are provided so that on looking in a direction parallel to the paper (see Fig. 14.06) both sides of it can be seen at once. The photometer or the lamps are then moved till the spot disappears or shows up equally clearly on both sides.

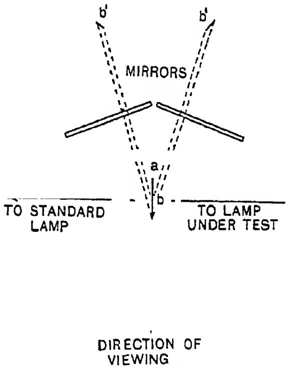


Fig. 14.06—PLAN OF BUNSEN PHOTOMETER.

a a, Paper disc with grease spot.
b^1 b^1, Images of grease spot.

One disadvantage of this photometer is that the two images of the spot are some distance apart. This has been remedied in some of the later forms by means of more complicated arrangements for reflecting the two sides of the paper.

The *Elster photometer* consists of two blocks of paraffin wax separated by a thin opaque sheet. This is moved between the standard and test lamps till the two blocks appear equally bright when viewed in a direction at right angles to the line joining the lamps.

The *Joly photometer* is the same with the opaque sheet omitted, and it is used in the same way.

12. The Lummer-Brodhun Photometer

This is shown in Fig. 14.07. It consists of an opaque white screen, two totally reflecting prisms, and a special double prism.

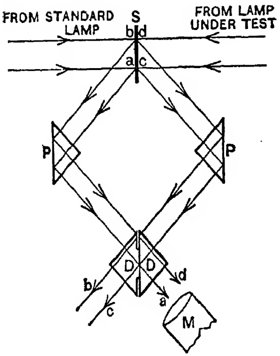


Fig. 14.07.—LUMMER-BRODHUN PHOTOMETER.

D D, Double prism. M, Observing microscope. P P, Right-angled reflecting prisms. S, Screen.

Mirrors are used instead of the reflecting prisms in the cheaper patterns. The double prism consists of two right-angled prisms placed base to base. One of these (the left-hand one in the diagram) has part of its base slightly ground down, leaving the rest in optical contact with the other.

The rays from the left side of the screen are reflected into the double prism. Those which fall on the parts in optical contact pass on into the observing microscope, while those which fall on the ground portions are dispersed and so do not reach the eye. The rays from the right-hand side of the screen are similarly reflected into the double prisms. But in this case those which fall on the parts in optical contact and pass on do not reach the eye; whereas those which fall on the base opposite the ground portions are totally reflected and pass into the observing microscope. Thus images of portions of both sides of the screen are seen absolutely contiguous and with no overlapping. These are the most favourable conditions for comparing the brightnesses of the two.

In the usual pattern the left-hand prism has a central circular portion in optical contact with the right-hand one, and the surrounding portions ground down. The appearance of the images is then as shown in Fig. 14.08 (*a*) and (*b*). By modifying the prisms the " contrast " pattern is obtained. The appearance is then as shown in Fig. 14.08 (*c*) and (*d*). When balance is obtained the trapezoidal patches are slightly darker than the rest of the field.

With the latter pattern more accurate work is possible, especially with differently coloured lights.

This photometer is expensive owing to the

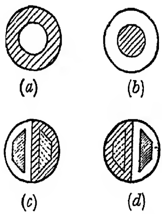


Fig. 14.08.—IMAGES SEEN IN LUMMER-BRODHUN PHOTOMETER.

(*a*) Ordinary pattern, left side brighter.
(*b*) Ordinary pattern, right side brighter.
(*c*) Contrast pattern, left side brighter.
(*d*) Contrast pattern, right side brighter.

accurate manufacture required, but it is very accurate, even in inexperienced hands.

With all the above photometers, except Rumford's, readings should be taken with the instrument reversed, so as to eliminate errors due to differences between the two sides of the screen or between the mirrors or prisms.

It is helpful in getting the point of balance to move the photometer (or lamp) backwards and forwards across this point, instead of attempting to move it to the point of balance without such oscillations.

13. Flicker Photometer

When there is considerable difference between the colours of the lights compared, it is difficult to determine the point of balance with the ordinary types of photometer. Those of the flicker type diminish this difficulty. Their principle is to show the surfaces illuminated

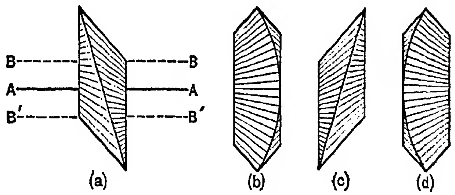


Fig. 14.09.—DISC OF FLICKER PHOTOMETER IN FOUR POSITIONS.

A A, Axis of disc. B B, B′ B′, Axes of cones forming surface.

by the two lamps alternately, instead of simultaneously. As long as they are unequally bright this will produce a flickering effect, which disappears when the brightnesses become equal.

Fig. 14.09 shows the disc of the Simmance-Abady flicker photometer in four positions a quarter of a turn apart. The edges of this disc are portions of the surfaces of two cones with axes parallel to that of the disc, and at equal distances from it on opposite sides. Each curved surface is illuminated by one only of the lamps under comparison. The disc is rotated by clockwork and a portion of its edge is viewed through a microscope. The illuminations are then adjusted in the usual way till the flicker disappears or becomes a minimum.

14. Illumination Photometers

In indoor lighting the effect of reflexion is usually very considerable (see Art. 22). It therefore becomes a difficult, if not impossible,

matter to calculate the illumination on any particular surface. *Illumination photometers* enable this quantity to be measured directly.

Their general principle is to compare the brightness of white matt surface placed in the required position with the brightness of a similar surface with a variable known amount of illumination.

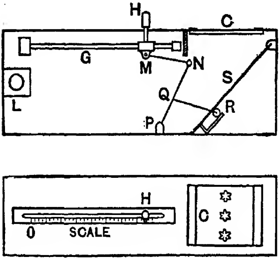


Fig. 14.10.—TROTTER ILLUMINATION PHOTOMETER.

C, Cardboard diaphragm with 3 star-shaped holes in it. G, Guide. H, Handle. L, Lamp. M N, Rod hinged at each end. N P Q R, Rigid rod hinged at P and N. R, Roller. S, Screen.

Fig. 14.10 shows the Trotter illumination photometer.

An adjustable screen is illuminated by reflexion from a low-voltage lamp. The screen is viewed through three star-shaped holes in a sheet of cardboard placed at the top of the instrument. The illumination of the screen is adjusted by altering its inclination until it appears of the same brightness as the cardboard. The illumination of the latter is then read on the scale over which the actuating handle moves. The linkwork is proportioned so as to give a fairly uniform scale. The current for the lamp is provided by a few small accumulators, and a resistance and a voltmeter are used to ensure a fixed voltage across the lamp.

An alternative form utilises a "photo-electric cell." This is usually a very thin semi-transparent layer of gold or platinum on a selenium base. When light falls on one of these, connected to an indicator actuated by small currents, the amount of current produced is nearly proportional to the illumination. The instrument scale can be marked in foot-candles. One pattern gives a

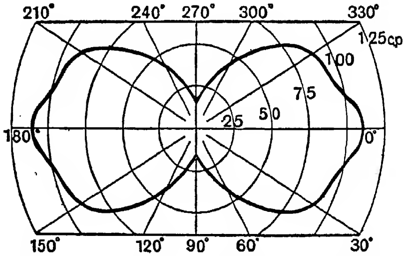


Fig. 14.11.—POLAR HORIZONTAL DISTRIBUTION CURVE OF NERNST FILAMENT.

scale up to 2·5 foot-candles with a current of about 75 micro-amperes. By shunting the indicator (and adding resistance in series) other higher scales can be provided.

15. Determination of Mean Horizontal Candle-Power

The mean horizontal candle-power can be determined by turning the lamp about a vertical axis and measuring the candle-power in a number of directions at equal angular intervals, *e.g.* every 15°. The results are best exhibited on a polar diagram (see Fig. 14.11). Lines are drawn radially from a central point in directions corresponding to those along which the candle-power has been measured. Along each line a distance is marked proportional to the candle-power in that direction. The points thus obtained

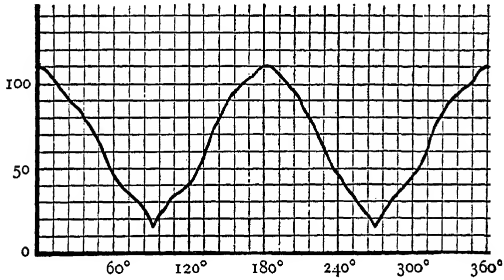


Fig. 14.12.—RECTANGULAR CURVE OF LIGHT DISTRIBUTION.

are joined together and form the horizontal distribution curve of the lamp.

In most cases the candle-power is nearly the same in all horizontal directions. The distribution curve then approximates to a circle, and the mean horizontal candle-power (M.H.C.P.) can be obtained by taking the arithmetic mean of the measured values. When the values vary greatly this method is not accurate enough. A better method is to plot the horizontal distribution with rectangular co-ordinates (see Fig. 14.12) and determine the mean height* of the curve obtained. This mean height is the M.H.C.P.

Mean height · $\frac{\text{Total area}}{\text{Base}}$ The area can be obtained by a planimeter, or by any of the usual approximate methods.

A method by which the M.H.C.P. can be determined by a single measurement is to use a "lamp spinner." This rotates the lamp about a vertical axis at about 200 r.p.m., and, owing to persistence of vision, the apparent illumination of the photometer screen is the same as it would be with a stationary lamp of intensity equal to the M.H.C.P. The cost of the spinner is rather high, as it requires a motor to rotate the lamp, and sliding contacts to supply the lamp with current.

16. Mean Spherical Candle-Power

The determination of the mean spherical candle-power (M.S.C.P.) requires a knowledge of the candle-power in every direction. It is usually sufficient to determine the C.P. in a suitable number of

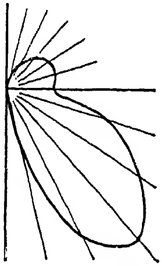


Fig. 14.13.—MEAN VERTICAL DISTRIBUTION CURVE OF ARC.

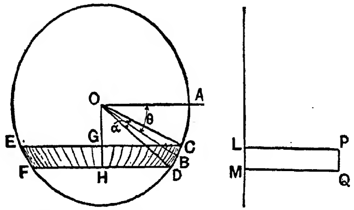


Fig. 14.14 MEAN SPHERICAL CANDLE-POWER.

LP = MQ = c.p. in direction OB.
LM = CD cos θ = height of zone.
∴ LPQM represents the luminous flux onto the zone CDFE.

directions in each of two planes at right angles. The mean of the four values for corresponding directions (*i.e.* those making the same angle with the horizontal plane, two in each vertical plane) is then plotted in this direction, and a polar diagram of mean vertical distribution thus obtained.

It would be quite inaccurate to take the mean of the values in the various directions as the M.S.C.P. For if all the rays of light making angles between $\theta - \frac{\alpha}{2}$ and $\left(\theta + \frac{\alpha}{2}\right)$ below (or above) the horizontal (see Fig. 14.14) are considered they will be seen to illuminate a certain zone on a sphere described with the lamp as centre. The width of this zone is $r\alpha$, where r is the radius of the sphere. But its area is $2\pi r \times$ height of zone $= 2\pi r \times r\alpha \cos\ \theta$

approx. $= 2\pi r^2\alpha \cos \theta$. Thus if a number of zones subtending equal angles (α) at the centre of the sphere are taken, they diminish in area as they get further from the horizontal plane, whether below or above.

In obtaining the mean value, the intensities on the various zones must be given weight according to the areas of the zones, *i.e.* the intensities in directions near the horizontal must be given more weight than those further from it. One method of doing this is by Rousseau's construction.

17. Rousseau's Construction

Let ABCD be the curve of mean vertical distribution of a lamp (a carbon filament one in the example, Fig. 14.15), O being the pole of the diagram. With O as centre describe a semicircle of any convenient radius on a vertical diameter. Draw LM parallel and equal to this vertical diameter. To find the point on the Rousseau curve corresponding to any point (*e.g.* C) on the distribution curve,

Produce OC to meet the semicircle in E.

Draw EN $\perp$ to LM and produce it to P, making NP $=$ OC.

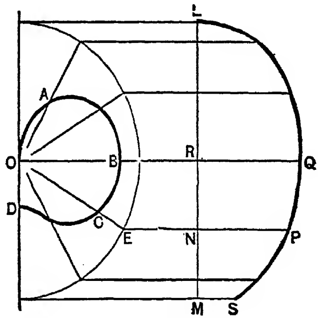


Fig. 14.15—Rousseau's Construction.

Then P is the required point.

This is repeated for a suitable number of points on the distribution curve, usually for each of those obtained by direct measurement. The points are joined to form the Rousseau curve, SPQ. The mean breadth between this curve and LM $\left(= \dfrac{\text{Area of SPQLM}}{\text{Length of LM}}\right)$ is the M.S.C.P. on the same scale as the vertical distribution curve.

It can be seen by referring back to Fig. 14.14 that the portion of the area SPQLM corresponding to any particular narrow zone is a strip whose width is equal to the height of the zone and is therefore proportional to its area, while the height of the strip gives the mean intensity over the zone. Thus each intensity receives its proper weight in obtaining the mean spherical intensity.

The mean hemispherical candle-power (M.H.S.C.P.) can be obtained by carrying out Rousseau's construction for directions below or above the horizontal only, according as the lower or upper hemispherical candle-power is wanted. Thus in Fig. 14.15

18. Other Methods for M.S.C.P.

To avoid the trouble of carrying out Rousseau's construction and the need for a planimeter for measuring the area other methods have been introduced.

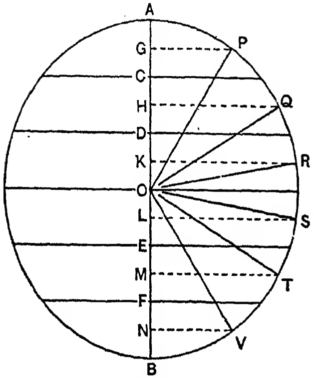


Fig. 14.16.—SPHERE DIVIDED INTO SIX ZONES OF EQUAL AREA.

A B is divided into six equal parts at C, D, O, E and F. These are bisected at G, H, etc. Then O P, O Q, etc., are the directions in which the c.p. is measured.

In one of these, instead of measuring the intensity at equal angular intervals in the vertical plane, the measurements are made at such intervals that the corresponding zones are of equal area.* For instance, if 12 measurements in a vertical plane are to be made the sphere is divided into six zones of equal height and therefore of equal area (see Fig. 14.16).

Instead of measuring the intensity at every 30° it is measured in directions going to the lines which bisect the areas of these zones, *i.e.* which lie midway in height between the top and bottom of the zones. In this case these directions measured from a line vertically upwards are 33·6°, 60°, 80·4°, 99·6°, 120°, and 146·4°, and the same on the other side.

The M.S.C.P. is then obtained simply by taking the mean of the measured candle-powers.

Another construction is that due to Kennelly. Let ABCD be the vertical distribution curve (see Fig. 14.17). In the example measurements are supposed to be taken every 30°. Mark the intensities at the mid-zones, O*r*, O*s*, etc. (*i.e.* 15°, 45°, 75°, and

* A. Russell, *Journal I.E.E.*, Vol. XXXII., p. 631.

$-15°$, $-45°$, $-75°$). With centre O and radius Or describe an arc hra, starting at the horizontal line OB and extending over 30°. Measure ab along aO equal to Os (the intensity at 45°). With centre b radius ba describe an arc ac extending over 30°; i.e. bc is parallel to the 60° radius. Measure cd, equal to Ot along cb. With centre d radius dc describe an arc ce extending over 30°. Project h and e to H and Q on any convenient vertical line. Then HQ represents the M.H.S.C.P. (upper) to the same scale as the original polar curve.

Repeat the construction below the horizontal, obtaining the curve $h'r'a'c'e'$, and Q′ the projection of e'. Then HQ′ represents the M.H.S.C.P. (lower). Thus half QQ′ represents the M.S.C.P.

Moreover, by projecting the intermediate points a, c, etc., QQ′ is divided into lengths proportional to the total flux in the corresponding zones.

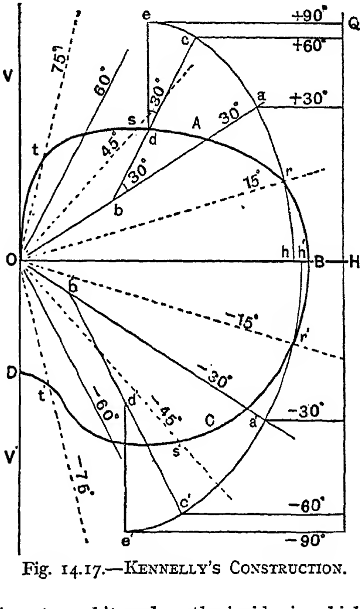


Fig. 14.17.—KENNELLY'S CONSTRUCTION.

19. The Globe Photometer

A third method is the use of the Ulbricht *Globe Photometer*. This consists of a hollow sphere about a metre in diameter, whitened on the inside, in which the lamp is placed. It can be shown that the illumination of every part of this due to reflexion (*i.e.* omitting that due to the direct rays from the lamp) is the same, and therefore depends only on the M.S.C.P. of the lamp and on the reflecting power of the surface.

A milk-glass window 5 cm. diameter in the side of the sphere, shielded from the direct rays of the lamp by an opaque screen, has its brightness measured; this is proportional to the M.S.C.P. of the lamp. The photometer is calibrated by using in it a lamp whose M.S.C.P. has been determined carefully by other methods.

Often for ease of construction the sphere is replaced by a cube with the interior angles rounded off. Suitable sizes for such Cube Photometers are 28 in. for tungsten lamps up to 200 watts, 40 in. up to 500 watts, and 50 in. up to 1 000 watts. For testing complete fittings still larger cubes are required.

20. Reduction Factor

The *reduction factor* (spherical) of a lamp is the factor by which the M.H.C.P. must be multiplied to obtain the M.S.C.P., *i.e.* it is the ratio $\frac{\text{M.S.C.P.}}{\text{M.H.C.P.}}$. In most incandescent lamps this is less than unity (about 0·8), hence the name reduction factor. It is approximately constant for lamps of a given type, *i.e.* with their filaments arranged in a certain way. Therefore in testing a batch of similar lamps it is sufficient to obtain the reduction factor of one or two of them. This is then used to obtain the M.S.C.P. of the rest from measurements of their M.H.C.P.

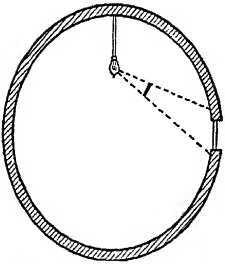


Fig. 14.18.—SECTION OF ULBRICHT GLOBE PHOTOMETER.

Further, if lamps are rated on their M.H.C.P. a knowledge of the respective reduction factors is necessary to obtain a fair comparison of different types.

The increasing use of cube (or globe) photometers has reduced the use of rating on M.H.C.P. The standard British method of rating is to state this in lumens emitted. These are simply $12{\cdot}57 \times$ M.S.C.P. (see Art. 4), but has the advantage of not using "candle-power" for both intensity and total flux.

21. Arc and Projector Lamps Photometry

In obtaining the vertical distribution curve for a filament lamp, the lamp can be placed horizontally (*i.e.* at right angles to its normal position), and then rotated about a vertical axis to obtain the C.P. in the required directions. With arc lamps this cannot be done, because they will not burn properly, if at all, in the horizontal position. The same considerations apply to projection lamps, which are designed for use in a certain position: in testing a complete fitting, the procedure for arcs is more convenient than that for filament lamps.

One method of overcoming this difficulty is to fix the photometer in a position making the required angle with the lamp, which is suspended in its normal position. Balance is then obtained by moving the lamp used as a standard. The photometer screen must be set so as to bisect the angle between the rays from the lamp and from the standard. This equalises the effects of the rays falling obliquely on the screen instead of normally. Different directions can be obtained by raising or lowering the lamp under test, or by moving the photometer horizontally.

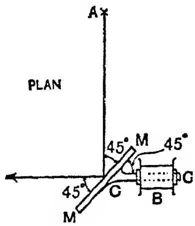


Fig. 14.19.—MIRROR FOR ARC PHOTOMETRY.

B, Bearing (fixed). C C, Shaft carrying the mirror M M.

The disadvantages of this method are that it cannot be applied when the standard lamp is fixed, a very usual arrangement; and that the distance between the arc and the photometer must be measured for each direction of measurement.

A method which overcomes the first of these objections is to use a mirror. This (see Fig. 14.19) can be rotated about a horizontal axis in line with the photometer screen and standard lamp, and making an angle of 45° with the mirror. The arc lamp can be raised or lowered over a point such as A, and the mirror is turned correspondingly so as to reflect the light on to the photometer screen. The distance still requires measuring or calculating for each position. The loss due to the mirror must be determined, but is a constant percentage since the angle between the mirror and the light rays is constant.

By using two or more mirrors the need for measuring the distance each time can be avoided, and at the same time the arc lamp may be kept in a fixed position. The simplest method is shown in Fig. 14.20. The arc is fixed at A in line with the photometer and standard. Two mirrors, L, M, are fixed to a frame, which is rotated about an axis in the same line. The effect is the same as if the arc were a constant distance (AM + ML) to the right of L, but could be tilted in any direction. The loss due to the two mirrors is a constant percentage, as in the previous method.

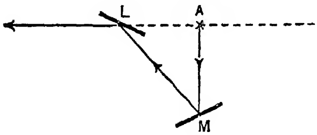


Fig. 14.20.—PAIR OF MIRRORS FOR ARC PHOTOMETRY.

22. Reflexion of Light

The use of reflectors or shades of any sort on a lamp cannot increase the total flux of light, *i.e.* they cannot increase the M.S.C.P. In fact, they must diminish it to some extent, since no shades are perfectly non-absorbent. They can, however, alter the distribution very greatly, and so may increase the illumination to a large extent where it is required particularly. This is especially the case when illumination is wanted chiefly on a comparatively small area below the lamp. An example is shown in Fig. 14.21. The candle-power is increased greatly for all directions near to the vertically downwards, and is decreased by a less amount for directions near to the horizontal. The M.S.C.P. is somewhat diminished, as can be tested by obtaining its values from the two curves.

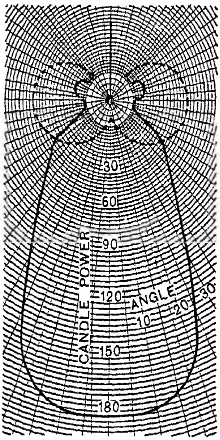


Fig. 14.21.—EFFECT OF SHADE ON DISTRIBUTION OF LIGHT.

---- Without shade.
— With Holophane focusing shade.

In interior lighting the effect of reflexion from ceilings, walls, etc., is to increase the average illumination, often very considerably. The amount of this increase depends on the reflecting powers of the ceiling and walls and, to a less degree, of the various objects in the room. For instance, if on the average half the light coming directly from the lamps is reflected, the *average* illumination will be increased by 50 per cent.

The effect will, as a rule, be least on surfaces receiving the greatest direct illumination, while some which receive none directly will receive some by reflexion; on the whole, therefore, the illumination is made more uniform. Moreover, the reflected light is reflected again and again, so that if the reflecting power of $\frac{1}{2}$ holds throughout the total effect is an increase of 100 per cent. in the average illumination. The candle-power required to light a given room satisfactorily therefore depends largely on the colours of the ceiling and of the walls.

The effects of these can be expressed by means of the **Utilisation Efficiency.** This is the percentage of the light emitted by the lamps which reaches the working plane. Its value depends partly on the

reflectors used and the height and spacing of the lighting units, and partly on the lightness of ceilings, walls, and contents of the room. With good reflectors suitably arranged the value varies between 55 per cent. with very dark walls and ceiling, to 80 per cent. when these are very light. With poor reflectors, bad arrangement, and dark walls and ceiling, the value may fall to 30 per cent.

Example 1. *Calculate the lumens required to give an average illumination of* 6 *foot-candles in a room* 20 *ft. by* 40 *ft. with utilisation efficiency of* 60 *per cent. Supply voltage* 230.

$$\begin{aligned} \text{Lumens required on working plane} &= 6 \times 20 \times 40 \\ &= 4800; \\ \therefore \text{ lumens required from lamps} &= 4800 \times (100/60) \\ &= 8000. \end{aligned}$$

A reference to B.S.S. No. 161 shows that this could be given either by (*a*) two 300-watt lamps, with an excess of 18 per cent. to allow for light depreciation; or by (*b*) eight 100-watt lamps, with an excess of 21 per cent.

Since the room's length is twice its breadth, either of these numbers of fittings will permit uniform spacing.

For outside lighting the effect of reflexion is confined practically to that of the lamps' reflectors. Hence the light thrown in directions above the vertical plane is wasted almost entirely. This is the reason for the frequent use of the M.H.S.C.P. in the case of arc lamps. It forms a fair comparison only when the use of reflectors to concentrate the light in the lower hemisphere is included in obtaining the value of the M.H.S.C.P.

23. The Illumination on a Horizontal Plane

This is required frequently both in street lighting and in the lighting of interiors. The illumination due to a single lamp can be determined from the laws of illumination if its vertical distribution curve is known (see further Example below). The illumination due to a number of lamps is the sum of the illuminations due to each separately.

Example 2. *Draw a curve showing the illumination produced by a lamp giving* 16 *C.P. in every direction, along a horizontal line* 6 *ft. long starting from a point* 8 *ft. vertically below the lamp.*

The illumination vertically below the lamp is

$$\frac{16}{8^2} = 0\cdot25 \text{ foot-candle.}$$

This and the values for other points are plotted in Fig. 14.22, which also shows how the other values are obtained.

L is the position of the lamp, 8 ft. vertically above M. To find the illumination at any point P, level with M.

Join LP and draw PQ vertically.

The illumination at P is $\frac{16}{(LP)^2} \cos \angle$ (Art. 6).

Measure LP on the scale of feet, and calculate the value of $\frac{16}{(LP)^2}$.

Mark off PR along PL, making PR $= \frac{16}{(LP)^2}$ to the foot-candle scale.

Draw RS horizontally, cutting PQ at S.

Then PS $=$ PR $\cos \angle$ QPL $= \frac{16}{(LP)^2} \cos$ QPL.

Thus PS represents the illumination at P.

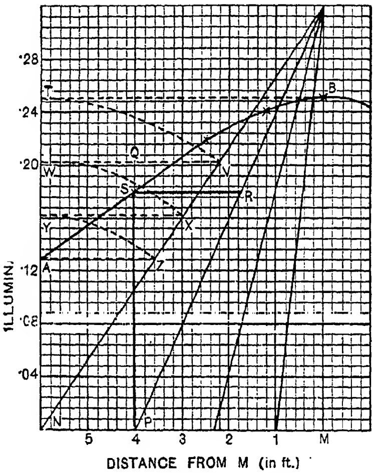


Fig. 14.22.—HORIZONTAL ILLUMINATION DUE TO 16 C.P. LAMP.

LM = 8 ft. on same scale as LP, etc.

By finding a number of points in this way and joining them the required curve of illumination is obtained.

N.B.—The whole of the above can be done graphically in the following way: At any point, such as N (Fig. 14.22), mark off along NL a distance NV $=$ 0·25 foot-candle, *i.e.* the illumination at the point vertically below the lamp. (If the candle-power in the direction LN differs from that in the vertical direction the length of NV must be changed in proportion.)

Let W be the horizontal projection of V on the vertical through N.

Mark off along NL a distance NX $=$ NW.

Let Y be the horizontal projection of X on NW.

Mark off along NL a distance NZ $=$ NY.

Then NZ $= \frac{16}{(LN)^2}$ on ft.-candle scale,* and the construction can be completed as above by projecting Z horizontally to A.

$$* \text{ Because } \frac{NY}{NX} = \frac{NW}{NV} = \frac{LN}{LN'}; \quad \therefore \frac{NY}{NV} = \left(\frac{LM}{LN}\right)^2;$$

$$\therefore \ NZ = NY = NV\left(\frac{LM}{LN}\right)^2 = \frac{16}{(LM)^2} \cdot \left(\frac{LM}{LN}\right)^2 = \frac{16}{LN^2}.$$

QUESTIONS ON CHAPTER XIV

1. Upon what does the illumination at any point of a surface by a single light depend?

2. Draw a curve showing the illumination on a horizontal plane given by a 50 C.P. lamp 5 ft. above it, along a line extending 8 ft. from the point immediately below the lamp.

3. Describe, with a sketch, the pentane standard of light, pointing out its special advantages.

4. Describe some form of flame standard, pointing out its advantages or otherwise.

5. What are the advantages of a large bulb incandescent lamp as a secondary standard for electrical photometry?

6. If the distance between the standard and the test lamps is 4 m., calculate the distances from the centre at which the figures 1·1, 1·2, 1·3, 1·4, 1·5, 1·75, and 2 should be placed on a photometer bar, and show the results in a diagram to scale.

7. Describe a reliable form of photometer, and the method of using it to obtain the candle-power of a lamp.

8. What are meant by the terms "spherical candle-power" and "hemispherical candle-power" of an arc lamp? [C. & G., II.

9. Calculate the mean spherical candle-power and the reduction factor for a lamp from the following readings:—

Angle with upward vertical	0°	15°	30°	45°	60°	75°	90°.
Candle-power	0	6·8	11·6	19·2	23·4	27·4	28·0.

Angle	105°	120°	135°	150°	165°	180°
Candle-power	25·8	21·2	15·6	8·4	6·0	5·4

10. Plot the vertical distribution curve of a mercury vapour lamp and obtain its mean lower hemispherical candle-power from the following readings:

Angle below horizontal	0°	10°	20°	30°	40°	50°	60°	70°	80°	90°.
Candle-power	120	370	700	1 100	1 500	1 800	2 000	1 800	1 700	1 800.

11. Find the mean hemispherical candle-power of an arc lamp from the following readings:—

Angle below horizontal	0°	15°	30°	45°	60°	75°	90°.
Candle-power	2 000	1 550	1 800	2 200	1 200	750	0.

12. Give the theory of the Ulbricht method of determining the mean spherical candle-power of a lamp or group of lamps. Point out the precautions which must be taken to secure a good result, and criticise the accuracy of the method in practice. [Lond. Univ., El. Tech.

13. Explain how the polar curve of light distribution for an arc lamp may be found, and show how the curve of illumination on the surface of the ground below the lamp may be determined. [C. & G., II.

14. Define the units used in connexion with illumination. Describe how you would calculate the number of lamps required to obtain a specified illumination at the table height in a room of given dimensions. [C. & G., II.

15. Define (*a*) illumination, (*b*) luminous flux, and (*c*) the brightness of a luminous surface. State the units in terms of which these quantities are expressed. In what respect does the reflecting action of a diffusing surface differ from that of a mirror?

A large area is illuminated by a number of lamps each placed 12 ft. above the ground on posts erected all over the area so as to stand at the corners of squares of 50 ft. side. The candle-power of each lamp is 300 and may be assumed uniform. Calculate the illumination of the ground (*a*) at the base of each lamp, (*b*) at the centre of each square, and (*c*) as an average for the whole area. [Lond. Univ., El. Eng.

16. A space is illuminated by a lamp which is suspended 10 ft. above the ground, and which gives uniform candle-power in all directions in the lower hemisphere. Find (*a*) the ratio of the ground illumination at a point directly under the lamp to that at a point 20 ft. away, (*b*) the position of a similar lighting unit in order that the minimum illumination on the ground along a line joining the points vertically below the lamps shall not be less than half the maximum value.

Indicate your view of the relative importance in practice of the illumination falling vertically on the ground and that incident on a surface perpendicular to the ground. In case (*b*) above, estimate the relative values of these quantities (i) at the base of a lamp, and (ii) half-way between the lamps. [Lond. Univ. El. P.

17. A road is illuminated by means of lamps suspended 25 ft. above the centre line of the road and spaced 50 ft. apart. The polar curve of each lamp is given below:—

Angle to vertical	0°	10°	20°	30°	40°	50°	60°	70°
candle-power	160	180	190	170	130	100	50	15

Plot a curve showing the illumination along the middle of the road from a point vertically below one lamp to a point on the road midway between two lamps. Neglect the light from a lamp at a greater distance than 50 ft. [C. & G., Final.

CHAPTER XV

INCANDESCENT LAMPS

1. Introductory

In *incandescent lamps* light is obtained by passing through a conductor of high resistance (called the *filament*) a current sufficient to raise its temperature to luminosity. The conductor is usually enclosed in an evacuated glass bulb. The object of the vacuum is two-fold: (*a*) to prevent the conductor burning away, (*b*) to diminish the loss of heat by convection. The first could be effected by filling the bulb with some inert gas, such as nitrogen, but this would not prevent the convection loss. When this loss occurs it requires an additional expenditure of power to maintain the temperature of the filament at the necessary point (see Art. 13).

It is evident from the laws of radiation that the filament of an efficient lamp must almost necessarily be composed of a material capable of withstanding a high temperature. It must also be a conductor of electricity, but preferably not a good conductor.

The only materials which have been found to possess these and the other qualities necessary for a commercial filament are carbon, and the two metals tantalum and tungsten (or wolfram). Tantalum filaments have been superseded by improvements in the manufacture of tungsten filaments (see Art. 7).

Lamps of this type are called filament lamps, to distinguish them from the gas discharge lamps (Art. 15), in which the incandescent body is a gas instead of a solid.

2. Efficiency of Filament Lamp

The efficiency of a filament lamp used to be stated in *watts per candle-power*. This does not correspond with the ordinary meaning of efficiency, because the watts measure the input and the candle-power is proportional to the output. Thus the greater the "efficiency" the *less* efficient is the lamp. Even the *candle-power per watt*, which is greater the more efficient the lamp, is not the efficiency in the sense in which this term is applied to dynamos, etc. For though C.P. per watt gives $\frac{\text{output}}{\text{input}}$, the two are measured in different units. Moreover since "C.P." is not used always in the same sense the meaning is not quite definite (cf. Arts. 4 and 12).

To obtain the real efficiency it is necessary to know the relation between one candle-power and one watt. This is very difficult to determine accurately, but the most recent experiments give a value of about ·06 watt for one candle-power. Thus the real efficiency of a lamp whose "efficiency" (or inefficiency) is 1·5 watts per C.P. is

$$\frac{\cdot 60 \text{ watt}}{1\cdot 5 \text{ watts}} = \cdot 04, \quad \text{or} \quad 4 \text{ per cent.}$$

The present method of stating the efficiency is in lumens per watt. This figure is 4π ($= 12\cdot 57$) times the mean spherical candle-power per watt.

The efficiency depends almost entirely on the temperature, since selective radiation occurs only to a small extent with the materials used for filaments (see Chapter XIV., Art. 3).

3. Relation between Filament Dimensions and Candle-power, etc.

Suppose a number of filaments are to be run at the same temperature and the same efficiency: then the following relations will hold good.

The heat produced per second = the heat got rid of per second (almost entirely by radiation if the vacuum is good).

$$\text{The former} = \text{I}^2\text{R watts} = \text{I}^2\frac{\rho l}{\text{A}}\text{ watts} = \text{I}^2\rho l \Big/ \left(\frac{\pi}{4}d^2\right),$$

where I = current (amperes) in filament,

l = length of filament,

A = cross-area of filament,

d = diameter of filament (if circular),

ρ = resistivity of material of filament *at the working temperature.*

The heat radiated per second varies as the surface and as the emissivity, since the temperature is the same for all. The variations in emissivity are small and will be neglected for the present. Therefore the heat radiated per second is proportional to the surface, or to $l \times d$ for a circular filament. Equating the two heat rates—

$$\text{I}^2\frac{\rho l}{\frac{\pi}{4}d^2} = ld \times \text{a constant, whence } \rho.\text{I}^2 = d^3 \times \text{a constant,}$$

or $$d = \text{constant} \times \sqrt[3]{\rho.\text{I}^2} \quad \text{(see Example 1).}$$

This relation is similar to that of Preece's Rule for fuses (see Chapter III., Art. 24, page 55), but is much more nearly true for vacuum lamps.

Another relation is obtained from the fact that the light (as well as the heat) radiated per second is proportional to the surface, whence candle-power $\propto$ $(d.l)$ for a given material at a fixed efficiency (see Example 1).

It can be readily seen that the effect of an initially high emissivity is to diminish the diameter required for a given current, etc. The effect of an increase of emissivity after the lamp has been in use for a time, *e.g.* by roughening of the surface, is to lower the temperature necessary to radiate the heat produced, and so to lower the efficiency.

Example 1. *Find the relative diameters and lengths of the filaments of two lamps of the same candle-power, efficiency, and material, but* (A) *for* 220 *volts,* (B) *for* 110 *volts.*

Since the candle-powers and efficiencies are equal, so are the watts. But since $I = \frac{\text{watts}}{E}$, I_A is half of I_B. The material being the same for both filaments, the resistivity is constant;

$$\therefore\ d \propto \sqrt[3]{I^2};$$

$$\therefore\ \frac{d_B}{d_A} = \sqrt[3]{\frac{I_B^2}{I_A^2}} = \sqrt[3]{4} = 1{\cdot}59.$$

Since the candle-powers are equal

$$d_A l_A = d_B l_B;$$

$$\therefore \frac{l}{l_A} = \frac{d_A}{d_B} = \frac{1}{1{\cdot}59} = 0{\cdot}63,$$

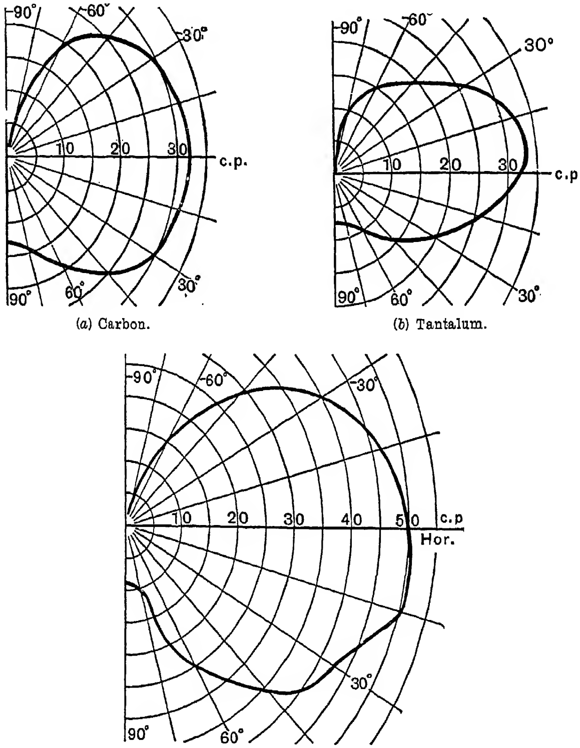
i.e. the low voltage lamp has the shorter and thicker filament.

4. Carbon and Metallic Filaments

Carbon can withstand a very high temperature and has a high resistivity, which means a thick and therefore a short filament (see Art. 3). The working temperature is, however, limited by the fact that in a vacuum particles of carbon are driven off at a much lower point than the normal temperature of evaporation. Consequently the metallic filaments, though their melting points are below the latter temperature,* can be worked at a higher temperature† and therefore a higher efficiency than carbon.‡ The standard efficiencies are from 3·1 to 4·5 watts per mean horizontal candle-power for carbon, *i.e.* 4·1 to 2·8 lumens per watt, and 8·5 to 7·0 lumens per watt for tungsten vacuum lamps.

* 3 600° C. in air. † 2 200° C. ‡ 1 800° C.

The main disadvantages of the metallic filaments are (*a*) their greater cost, and (*b*) the low resistivity of the metals, which necessitates long and thin filaments. They are therefore much more fragile than the corresponding carbon ones in spite of the greater



(*a*) Carbon.

(*b*) Tantalum.

(*c*) Tungsten.

Fig 15.01.—LIGHT DISTRIBUTION CURVES.

strength of the material. Their higher efficiency too increases their length and diminishes their diameter.

In consequence of this the earlier lamps were made only for low voltages. It is now possible, through improvements in manufacture, to obtain tungsten lamps for 200 to 260 volts down to

9 M.S.C.P. (15 W.) at an efficiency of 7·7 to 7·95 lumens per watt. The higher candle-power lamps are less fragile, because the filaments are thicker (see Art. 3) though longer. The tantalum filaments were stronger than the early tungsten ones, and had an efficiency of about 7·4 lumens per watt. They were used where the vibration was too much for the tungsten ones. For trams tungsten filaments with a large number of supports (similar to Fig. 15.06) are used. The lamps take 100 v. to 130 v. each and are connected four or five in series. Their standard efficiency is 6·8 lumens per watt. For train lighting from batteries 15 W. and 20 W. lamps are used, either of 16 v. run at 8·4 lumens per watt or of 24 v. at 8·1 lumens per watt.

The reduction factors are about ·84 for carbon and ·78 for metal filament lamps. For further differences see Arts. 8, 9, and 13.

5. Manufacture of Carbon Filament Lamps

Carbon filaments are now always *squirted*. Cotton-wool is

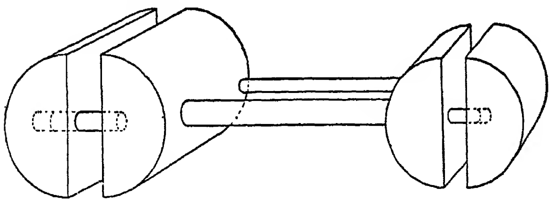


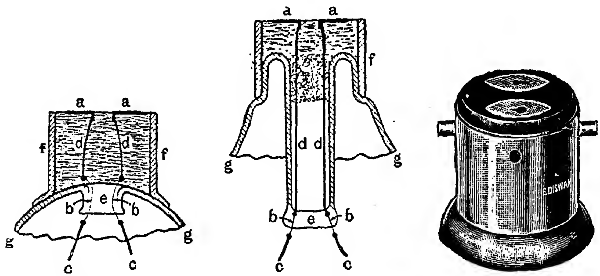
Fig. 15.02.—"FORMER" FOR CARBON FILAMENTS.

dissolved in zinc chloride, and then squirted through a die of the required size into alcohol, where it remains for several days to harden it. The result is a thread of cellulose, which is washed, dried, and cut into suitable lengths. These are wound on carbon *formers* to give them the desired shape. The formers are made in two or more pieces (see Fig. 15.02) to allow for the contraction which occurs during carbonisation, which is the next process.

This is effected by placing the threads and formers in plumbago crucibles, filled with powdered carbon and covered with air-tight lids. These are heated steadily to about 550° C., at which temperature carbonisation is completed, and then are heated more rapidly to the final temperature of nearly 1 700° C. The oxygen and hydrogen in the cellulose are thus driven off, leaving a very hard and pure carbon thread. The filaments are then attached to the *leading-in wires*. Formerly platinum was chosen because it can

stand a high temperature, and because it has very nearly the same coefficient of expansion as the glass into which it is afterwards sealed. Any difference in the expansion of the two when the lamp heats up is liable to destroy the air-tightness of the seal. Satisfactory alloys have now been developed for this purpose, which reduce the cost.

The *flashing* process comes next. This consists of placing the filaments in a hydro-carbon gas or vapour, such as coal gas or benzene, and heating them to incandescence by passing a current through them. This decomposes the vapour and deposits carbon on the filament, rendering it uniform, since the thinner parts get hotter and so receive a greater deposit. At the same time the resistance of the filament can be brought down to any desired value by continuing the flashing till this point is reached.



(*a*) "Pinch" seal. (*b*) "Tube" seal. (*c*) Completed lamp top.

Fig. 15.03.—SEALING OF LEADING-IN WIRES.

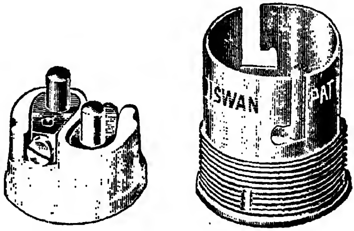
a a, Contact plates. b b, Platinum or nickel-iron wires. c c, Ends of filaments. d d, Copper wires. e, Glass seal. f f, Brass collar. g g, End of glass bulb.

The leading-in wires are then sealed into a glass bulb (see Fig. 15.03) which is exhausted through a tube till the pressure falls to 0·002 mm. of mercury, viz. only $\frac{1}{400000}$ of the air remains in. The bulb is heated by passing a current through the filament during the last stages of evacuation, so as to drive off the air which otherwise would cling to the glass. The tube used to be attached to the bulb opposite to the leading-in wires, and when sealed formed the bulb pip. It is now placed inside the tube seal [Fig. 15.03(*b*)], and when sealed is covered by the lamp top (or "cap").

The final stage of exhaustion is often done chemically. *I.e.* phosphorus dissolved in alcohol is introduced into the tube before pumping starts. When the vacuum has been made as good as the

pump can make it, the tube is sealed a short way from the bulb. The phosphorus is then vaporised and combines with the oxygen, and the tube is then sealed off short. The outer ends of the leading-in wires are connected by copper wires to two brass contact plates, which are mounted in plaster of Paris or a vitreous enamel inside a brass collar with two projecting pins (see Fig. 15.03). Plaster of Paris has the disadvantage of absorbing moisture readily, and so has been superseded.

The type of lamp socket into which such lamps fit is called the bayonet socket, and is shown in Fig. 15.04. The two contact studs are pressed by springs against the contact plates of the lamp, which is kept in position by the pins on its brass collar fitting in the slots in the socket of the lamp-holder.



(*a*) Contact pins and porcelain support. (*b*) Socket.

Fig. 15.04.—BAYONET SOCKET LAMP-HOLDER.

The Edison screw socket is used as one alternative. One contact is in the centre and the screw collar forms the other. Similarly the lamps have one central contact, and a screw collar for the other (see Fig. 15.05).

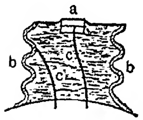


Fig. 15.05.—LAMP END FOR EDISON SCREW SOCKET.

a, Central contact plate. b b, Screw collar of brass (in section). cc, Copper wires to sealed leading-in wires.

6. Manufacture of Tantalum Filaments

Powdered metallic tantalum is obtained by the reduction of potassium-tantalum-fluoride. The powder is fused electrically in a vacuum, thus getting rid of the occluded gases. The metal can then be drawn into a wire of the required fineness.

In order to get the long filament into a bulb of ordinary size it was mounted on four circles (or spiders) of wire hooks, each supported by a glass disc carried on a central glass stem. The hooks are insulated from each other by the glass. In lamps for voltages of 125 and less only two circles of hooks were used (see Fig. 15.06). The resistivity of tantalum at the working temperature is only about 37 microhm-cm., hence the great length of filament needed (see Art. 3).

The remaining details of lamp manufacture are similar to those for carbon filament lamps.

7. Manufacture of Tungsten Filaments

Tungsten is more difficult to work than tantalum, and several processes were in use. These early processes gave only short lengths of filament, and a number of loops had to be joined in series in a lamp of any but very low voltage.

The large number of joints required made these filaments more liable to breakage than a drawn filament, and the filaments themselves were not quite as strong as tantalum filaments.

From 1909 onward processes have been introduced by which tungsten can be drawn as tantalum can. This enables the filaments to be made in one continuous length, and so reduces the risk of breakage, which often occurs near the joints. The drawn filaments are much stronger in themselves at first than the older type, which is a great advantage in the handling and transport of the lamps. During use the strength diminishes, but with some of the processes used the drawn filaments remain stronger during their whole life, in addition to the improvement of fewer joints.

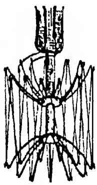


Fig. 15.06. Low Voltage Tantalum Filament.

One process is as follows: A fragile bar is obtained by compressing tungsten powder in a steel mould. This bar is gradually heated in a current of hydrogen and then "vitrified" by heating to 2850° C. in an electric furnace. The metal contracts about 14 per cent. during this process. It thus becomes strong enough to be reduced in diameter by hammering. This is done at about 1300° C. by a special machine which strikes very rapidly. When the diameter has come down to about 30 mils the wire may be drawn cold. It is preferable to use heated diamond dies and restrict cold-drawing to the last stages. The diameter can thus be reduced to 1 mil by 100 successive drawings. The final process before mounting is to reheat the filament in hydrogen to remove oxide from its surface.

The resistivity of tungsten is about 40 microhm-cm. at the working temperature. Consequently the filament of a 60-watt high voltage (200—250 volts) lamp is about four feet long.

8. Life of Lamps: Variation with Efficiency and Voltage

The C.P. of a carbon filament lamp falls off during use, after an initial rise (see Fig. 15·07). This is due partly to blackening of the

inside of the bulb and partly to changes in the filament. Consequently the lamp will cease to give sufficient light before the filament actually breaks. Or, in other words, the lumens per watt will have decreased so much that it is cheaper to replace the lamp by a new one, than to continue to use the old one. The point at which this becomes true depends on the conditions of use, but it is usual to assume it to have been reached when the C.P. has fallen 20 per cent. below its original (not maximum) value. It is sometimes called the smashing point.

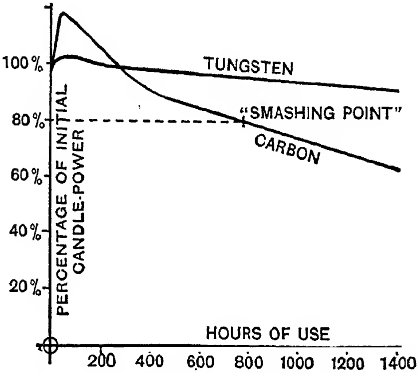


Fig. 15.07.—Variation of Candle-Power with Use.

The useful life of the lamp is the number of hours of light-giving before the smashing point is reached. But in testing for life the standard method (B.S.S., No. 161) is to take the time until the average lumens have fallen 10 per cent. below the initial value. If the fall were according to a straight line law the two methods would give the same result.

The length of life, both useful and total, depends greatly on the efficiency of the lamp: the higher the efficiency the shorter the life. The nature of the relation between these is shown in Fig. 15.08, but it varies for different makes of lamps.

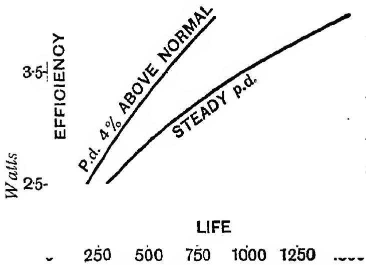


Fig. 15.08.—Effect of Variable P.D. on Life of Lamps.

The same figure shows the effect on lamp life of a variation of voltage. Any increase of voltage raises the working tem- and so shortens the life. Home Office regulations allow a 6 per

cent. variation from the declared pressure. This used to be 4 per cent., but has been increased owing to the general use of tungsten lamps (see Art 9).

Low voltage lamps, having shorter and thicker filaments, can be made with higher efficiencies than high voltage lamps with the same useful life. *E.g.* a 115-volt tungsten lamp taking 60 watts will give 710 lumens (57 M.S.C.P.), while a 230-volt lamp taking 60 watts gives only 551 lumens (44 M.S.C.P.), averaged during life.

Metallic filaments fall off in candle-power less rapidly than carbon filaments (see Fig. 15.07). In consequence of this and of their greater fragility and cost the smashing point is generally not reached before the filament breaks. Occasionally a badly made lamp will fall off in C.P. rapidly and cease to be useful while the filament is still intact. In other respects the above statements apply generally to metal filaments as well as to carbon ones (see further Art. 13).

9. Variation of Candle-Power with Voltage and Current

When the voltage applied to a lamp is varied the candle-power changes in a much larger proportion. The relation between the two can be expressed in the form

$$P = k.E^n,$$

where P = candle-power, E = applied P.D., and k and n are constants.

The value of k depends on the particular lamp in question, but for all carbon filaments n has a value of about 6 to 7. Taking the former value, this is equivalent to saying that a change of 1 per cent. in the voltage produces 6 per cent. change in the C.P., and a change of 10 per cent. in the voltage causes 77 per cent. change in the candle-power [since $(1{\cdot}10)^6 = 1{\cdot}77$].

For metal filaments n has a lower value (in the neighbourhood of 4) so that a given change of voltage causes considerably less change of C.P. than in the case of carbon filaments. The chief reason for this difference is that carbon has a negative temperature coefficient, while the metals have positive temperature coefficients. Consequently when the voltage increases, the current through a carbon filament will increase by a greater percentage, owing to the decrease of resistance due to the rise of temperature. With metal filaments, on the other hand, the percentage change of current is less than the percentage change of voltage. For 10 per cent. change of volts, the current changes by about 12 per cent. for carbon and 6½ per cent. for metal filaments.

Metal filament lamps therefore have the advantage of giving a less variable light than carbon ones on a varying voltage. The corresponding disadvantage is that on first switching on there is a momentary large current, because the cold resistance is less than $\frac{1}{8}$ of the resistance under working conditions, whereas a carbon lamp has a cold resistance $1\frac{1}{2}$ or more times its hot resistance. This disadvantage is not serious, since the temperature of the filament rises to its normal value very rapidly, with a corresponding decrease of the current.

The relation between candle-power and current can be expressed in a similar form to that between C.P. and voltage, and the value of the exponent is about 5 for both metal and carbon filaments, *i.e.*

$$P = k'.I^5,$$

where I = lamp current, and k' is a constant.

In the same way

$$P = k''.W^m,$$

where W = watts absorbed by lamp, k'' is a constant, and m is a constant whose value is about 2·8 for carbon and about 2·4 for metal filaments.

This last relation also shows that the watts per candle-power $\left(\frac{W}{P}\right)$ diminish as the watts increase. The cause of this is the increased temperature of the filament (see Art. 2).

10. Target Diagram

On every lamp is marked its voltage, its C.P. (mean horizontal) or the watts it takes (or both), and sometimes its efficiency. Carbon lamps usually have the C.P. marked, and metal lamps more often the watts. Owing to the fineness of the filaments and the difficulties of manufacture, considerable variations from the standard values must be expected. The results of the tests on a batch of lamps can be shown conveniently by means of a target diagram. This consists in plotting the mean horizontal C.P. of each lamp at its rated voltage, against the watts absorbed at this voltage. A number of points are thus obtained and their closeness to the standard shows how accurately the lamps have been made. Straight lines are drawn on the diagram representing the standard and other "efficiencies." These, together with lines marking the limiting values of permitted C.P. and watts, enclose the target.

Fig. 15·09 shows a target for a 16 C.P., 220 volt, short life (400 hours) carbon lamp with the allowable variations marked.

These are* approximately for individual lamps, 8 per cent above or below the standard watts, and 12½ per cent. above or below the standard candle-power. Further, the watts per C.P. must be within 16 per cent. of the standard value; the effect of this provision is to cut off two corners from the otherwise rectangular target. At least 90 per cent. of the lamps tested must hit the target, *i.e.* must lie within the stated limits. The xs in the figure show the results of a test on 27 lamps, of which only two fall outside the target. In addition the average of all the lamps tested (including those outside the target) must be on the smaller target. For this

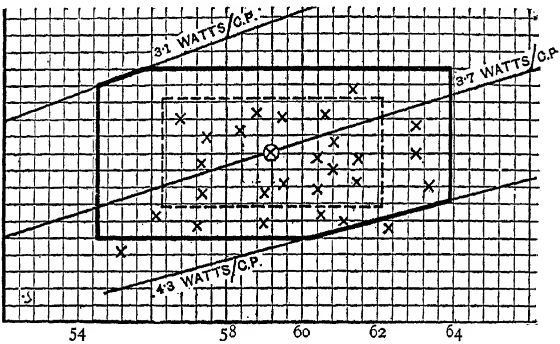


Fig. 15.09.—TARGET DIAGRAM FOR 16 C.P., 220-VOLT, CARBON LAMP.

——— Individual limits.

- - - - - - Limits for average values.

⊗ Standard values.

the allowable variation both of watts and of candle-power is $\frac{5}{8}$ of that allowed for individual lamps.

The corresponding limits for tungsten lamps were ± 12½ per cent. for the initial watts, and ± 15 per cent. for the initial lumens. These are measured after "ageing" the lamps by running them for 1 hour at their rated volts. The further proviso that the lumens per watt must lie within ± 10 per cent. of the standard value cuts off much larger parts of the rectangle than in the case of carbon lamps. No inner target was provided, and the number of lamps which might fall outside the target was 20 per cent. plus two lamps.

* See B.S.I. publication, B.S.S. No. 33, "Carbon Filament Glow Lamps."

These provisions have now been replaced by the following (see B.S.S. No. 161). The *average* watts must not vary from the standard by more than 10 per cent.; and the average lumens per watt must come within stated limits, which vary for different wattages between about 7 per cent. and 9 per cent. In addition the "coefficient of variation" of the lumens per watt must not exceed stated amounts ranging from 3 per cent. to 4½ per cent. This coefficient is the R.M.S. value of the deviation from the average.

11. Series Running

Although high voltage tungsten lamps can now be obtained down to 15 W. there still remains the advantage of obtaining stronger filaments by running two or more lamps of lower voltage in series. The disadvantages of series running are—

(*a*) The two (or more) lamps in series must be switched on and off together; they cannot be used separately.

(*b*) If one of a set breaks the other (or others) go out.

(*c*) The lamps must be chosen specially for series running.

This is known as "pairing," because usually two are connected in series.

The necessity of simultaneous switching (*a*), is of importance where only a small amount of light is required, but not where a large number of lamps is necessary.

Similarly (*b*), the extinguishing of a set by the breakage of any one lamp of it, does not matter much for two in series. It can be prevented by special arrangements where a large number are run in series.

Regarding "pairing" (*c*), the current is necessarily the same in each of a set of lamps in series. The necessary conditions are that the normal current of each lamp shall be the same, and that the sum of their normal voltages shall equal the total supply voltage; each lamp will then burn under normal conditions. As lamps in series are practically always all of the same voltage, the condition for proper series running is that their resistances shall be equal. If two lamps of unequal resistances are connected in series, the one with the higher resistance (and therefore the lower normal current) will have more than half the total voltage across its terminals, and will therefore give a high candle-power but have a short life. The other will have less than half the total voltage across it, and so will give only a dim light.

Example 2. *Two 110-volt lamps whose normal currents are 0·31 A. and 0·35 A. respectively are connected in series to a 220-volt supply. Find the current taken and the voltage across each lamp. [Neglect variations in resistances of lamps.]*

Call the lamps A and B respectively.

Then $$R_A = \frac{110}{0\cdot31} = 355 \text{ ohms},$$

and $$R_B = \frac{110}{0\cdot35} = 314 \text{ ohms};$$

$\therefore$ Total resistance with lamps in series $= 355 + 314 = 669$ ohms;

$$\therefore \text{Current} = \frac{220}{669} = 0\cdot329 \text{ amp.}$$

$$\text{P.D. across lamp A} = \frac{355}{669} \text{ of } 220 = 116\cdot7 \text{ volts.}$$

$$\text{P.D. ,, ,, B} = \frac{314}{669} \text{ of } 220 = 103\cdot3 \text{ ,,}$$

With carbon lamps the actual voltages will be closer than this, because the resistance of lamp A will be lower than normal owing to the increased current, while that of B will be higher than normal. With metal lamps the opposite takes place, so the voltages will differ by more than the calculated amount.

The calculated current will be nearly correct since the two changes of resistance almost balance.

In both cases lamp B will give a candle-power about

$$\left(\frac{\cdot329}{\cdot35}\right)^5 = 0\cdot73 \text{ of its normal value (see Art. 9),}$$

$$\text{and lamp A a C.P. of } \left(\frac{\cdot329}{\cdot31}\right)^5 = 1\cdot35 \text{ of normal.}$$

12. Gas-Filled Lamps

In 1913 tungsten filament lamps of the so-called "half-watt" type began to be supplied commercially. This name was used because their efficiency was roughly half a watt per C.P. (maximum). The improvement in efficiency has been effected in the following way. The ordinary tungsten filament can be run at this efficiency, but the bulb blackens so rapidly that the useful life is only a few hours. This is due to evaporation, though it is accelerated greatly by even a minute quantity of water vapour. By introducing nitrogen at atmospheric pressure the rate of evaporation at a given temperature is much reduced. Argon has been tried owing to its greater inertness, and the best results are obtained by using a mixture of nitrogen and argon.

The presence of gas in the bulb causes loss of heat by convection, consequently a higher temperature is needed for the same efficiency

as before. This loss is proportionately greater for thin filaments, and increases with rise of temperature much less rapidly than the energy radiated. By winding the filament in a close helix the convection loss is reduced. With filaments of this form the reduction of evaporation due to the presence of inert gas is greater, down to fairly small sizes, than the increase of evaporation which would occur without gas at the higher temperature necessary to compen-

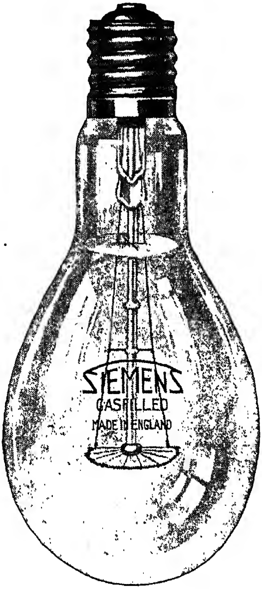


Fig. 15.10.—GAS-FILLED LAMP.

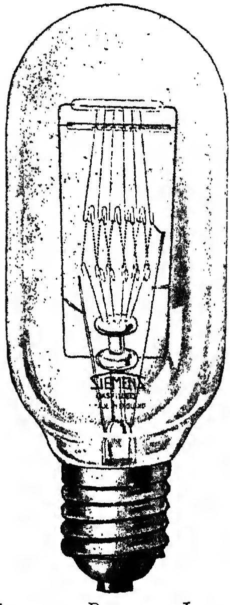


Fig. 15.11.—PROJECTOR LAMP.

sate for the convection loss. Hence the temperature can be raised still further, so obtaining a higher efficiency, without making the life of the lamp unduly short.

A lamp of this type to take 300 W. at 230 v. is shown in Fig. 15.10. It should be noted that the bulb has a long neck, the object of which is to cause any tungsten which evaporates to be deposited there by convection currents in the gas. The result is that the lower part of the bulb blackens very slowly, and so the

lamp has a useful life of about 1 000 hours. The seals of the leading-in wires are protected from the hot gas by a mica disc.

The same type of lamp is suitable for projection work of kinematographs, etc., if the filament is arranged to give as concentrated a source of light as practicable. Fig. 15.11 shows to the same scale as Fig. 15.10, viz. approximately $\frac{2}{5}$ths full size, a lamp to take 1 000 W. at 230 v. Similar lamps are made to take 20 A. or 30 A. at 30 v. to replace carbon arcs for this work.

The smallest sizes of gas-filled lamps made for ordinary lighting purposes are 15 W. for voltages up to 55 v.; 20 W. (altered to 25 W.

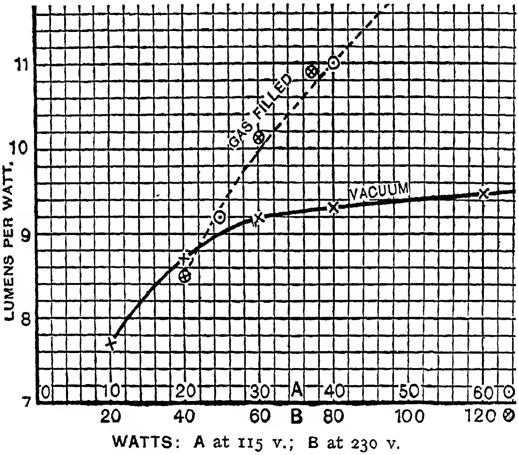


Fig. 15.12.—EFFICIENCIES OF VACUUM AND OF GAS-FILLED LAMPS.

in 1928) for 100 v. to 130 v.; and 40 W. for 200 v. to 260 v. In the smallest sizes, however, the convection losses more than wipe out the gain of the higher filament temperature. Consequently though the filament is brighter the total light is less than that of a vacuum lamp of the same consumption.

The standard average efficiencies during life of tungsten filament lamps are plotted in Fig. 15.12. This shows that as the current taken by the lamp—and therefore the diameter of the filament—increases, the convection loss becomes a smaller percentage of the input, and so the gas-filled lamp has an increasing advantage over the vacuum lamp.

This increase of efficiency continues up to the largest size made, viz. 1500 W., as is shown in Fig. 15.13. This is plotted in the same way as Fig. 15.12 except that the scale of watts is geometrical (or logarithmic) instead of arithmetical. The result is to space out more evenly the points for the standard lamp sizes.

The efficiencies of lower voltage lamps (*e.g.* 200 v. instead of 230 v., or 100 v. instead of 115 v.) are higher by a small percentage;

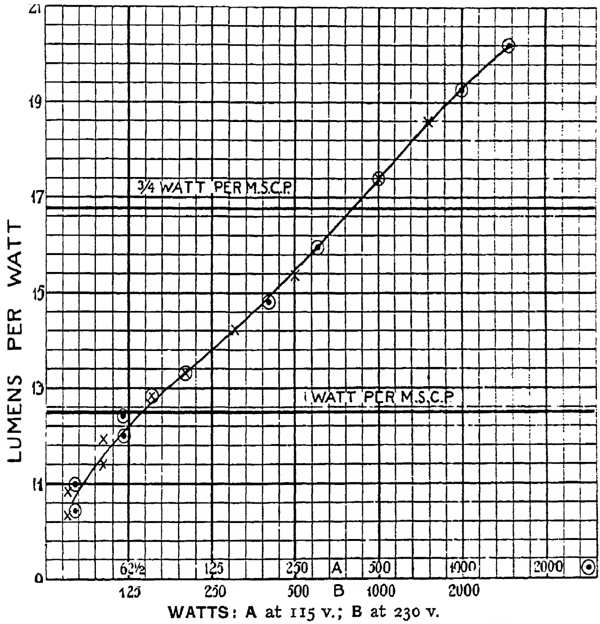


Fig. 15.13.—EFFICIENCIES OF LARGE GAS-FILLED LAMPS.

while those for higher voltages are a little lower. For details see B.S.S. No. 161.

The efficiency of the smaller sizes was improved by the introduction in 1935 of the "coiled-coil" lamp. In this, after the filament has been wound into a helix as usual, this helix is itself wound into a larger helix, hence the name. The result of concentrating the filament in this way is to reduce the convection losses still further, and so increase the efficiency. The lumens are thus increased in 230-volt lamps by the following percentages: 40-watt, 20 per cent.;

60-watt, 15 per cent.; 75-watt, $12\frac{1}{2}$ per cent.; and 100-watt, 10 per cent. For 115 v. lamps the gains are much less.

13. Comparative Costs

The cost of lighting by incandescent lamps consists partly of the cost of the electrical energy used, and partly of the cost of renewing lamps. With carbon lamps the latter is so small a fraction of the total that it is worth while to use lamps with a short life and comparatively high efficiency. Moreover cheap lamps are very false economy. With metal lamps the renewals form a considerable part of the total cost. Their amount depends largely on the treatment of the lamps, breakage of the filament being usually due to mechanical causes, though electrical deterioration assists.

The comparison of the cost of lighting by different lamps may be made on a "lamp-hour," or on a "candle-power hour" basis. The meanings of the terms will be evident from Example 3.

The former method is useful where a certain number of lamps are required, and the latter where a large room is to be lit by lamps whose number will depend on the candle-power selected.

Example 3. *Find the cost of lighting per lamp-hour and per candle-hour with the following three 230-volt lamps (including renewals).*

TYPE	C.P.	MEAN WATTS	COST OF LAMP	LIFE (HOURS)	ENERGY PER kWh.
			s. d.		
Carbon	16	66	1 4	800	3d.
Tungsten	50	60	1 9	500*	3d.
Gas-filled	70	75	2 6	500*	3d.

Carbon Lamp. Cost of energy per hour $= \frac{66}{1000} \times 3\text{d.} = 0{\cdot}198\text{d}$

$$\text{Cost of renewals per hour} = \frac{1}{800} \text{ of } 16\text{d.} = \cdot 020\text{d.};$$

$$\therefore \text{ Total cost per lamp-hour} = \cdot 198 + \cdot 020 = \cdot 218\text{d.}$$

The lamp gives 16 C.P.;

$$\therefore \text{ Total cost per candle-hour} \quad \frac{\cdot 218}{16} c$$

* This is taken small to allow for mechanical breakages, otherwise double this figure might be assumed.

Similar calculations for the other two lamps give the following results:—

Filament	Cost per Lamp-Hour in Pence			Cost per C.P. Hour Total (Pence)
	Energy.	Renewals.	Total	
Tungsten	·180	·042	·222	·0044
Gas-filled	·225	·060	·285	·0041
Carbon	·198	·020	·218	·0136

i.e. if a single lamp is needed the carbon is cheapest by a little, but if 200 C.P. or more is required tungsten is about a third the cost of carbon, and gas-filled a little cheaper still.

Owing to the variation in the meaning of candle-power it is preferable to make comparisons on the basis of lumen-hours. This is particularly the case when vacuum and gas-filled lamps are compared, owing to the great differences between their light-distribution curves. For the lamps in the above example the average lumens during life may be taken to be:—carbon, 150; tungsten, 500; gas-filled, 740. From this the total costs per 1 000 lumen-hours are found to be: carbon, 1·453d.; tungsten, 0·444d.; gas-filled, 0·385d

14. The Low Pressure Mercury Vapour Lamp

The low pressure mercury vapour lamp consists of a tube of special glass or of quartz, containing mercury and two electrodes sealed into the ends of the tube. The Bastian type is illustrated in Fig. 15.14. The tube is supported on a pivoted frame to which is attached a double iron core. This is attracted by a double coil connected in series with the tube, so that on closing the switch the tube is tilted quickly, causing the mercury column to break at *a*. The current continues to flow across the

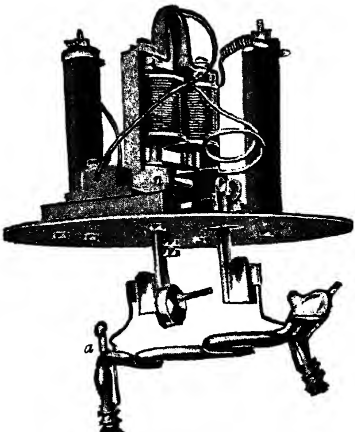


Fig. 15.14.—Bastian Mercury Vapour Lamp.

gap vaporising the mercury, and forcing the liquid mercury back till nearly the whole of the tube is filled with glowing mercury vapour. A steadying resistance is connected in series with the tube. The lamp is very efficient, but the greenish-blue colour of the light is very disagreeable, and alters greatly the appearance of colours, especially those containing red. It is employed therefore chiefly in printing, newspaper, and drawing offices where the work is mainly in black and white. The type illustrated gives 120 M.H.S.C.P. taking 0·4 ampere at 220 volts, *i.e.* an efficiency of 8·6 lumens per watt. The Cooper Hewitt lamps with straight tubes from $1\frac{1}{2}$ to 4 ft. long and 1 in. diam., taking about 400 watts, have efficiencies up to 10 lumens per watt.

The voltage for a given current and diameter $= 13 + k.l$,

where l = length of tube,

and k is a constant.

For varying diameters the voltage for a given current is found to vary inversely as the diameter. It does not vary as the inverse square of the diameter (as might be expected from the relation $R \propto l/A$), because the alteration of diameter alters the vapour pressure and the relative cooling surface.

15. Electric Discharge Lamps

The gas-filled lamp has superseded the low pressure mercury vapour lamp. But this discharge lamp, in its turn, has been improved by the use of a hot cathode of solid metal, and a higher vapour pressure. The result of this is a very large increase of efficiency, reaching approximately 40 lumens per watt in the 400-watt size. To obtain this the straight tube containing the electrodes and mercury is enclosed in an outer glass jacket, with a vacuum between tube and jacket. This reduces the convection losses, and obviates the risk of breakage of the hot tube by cold draughts or rain. Each electrode consists of a coil of thick tungsten wire surrounding a core of electron-emitting material.

A further improvement is the elimination of the necessity of tilting the tube to start the discharge. In the early patterns this was done by a heating wire wound outside the inner tube. It is now effected by auxiliary electrodes close to the main electrodes. On switching on, the discharge starts between auxiliary and main electrodes, and when the lamp has been warmed up by this the discharge transfers itself to the main electrodes, since this discharge path is then of lower resistance than that through the auxiliary

electrodes which have resistances in series with them. Ten minutes or so elapse before the lamp is burning normally.

The high efficiency of these lamps makes them very suitable for the lighting of streets and other large open spaces; and for large industrial interiors, such as foundries, if the colour drawback is not serious. They can be used on any A.C. pressure from 200 v. to 260 v.

The vertical distribution curve is approximately a circle touching the vertical line through the lamp. The corresponding Rousseau figure is the semicircle with vertical diameter, hence the reduction factor is about 0·79 ($\pi/4$).

To obtain the best results, special fittings designed for the distribution curve must be used. These may give either a distribution symmetrical about a vertical axis, for lighting large spaces; or a distribution stronger in two opposite directions than in directions at an angle with these, for street lighting. Moreover, in the latter case with lamps on both sides of the street the C.P. can be made less in the direction of approaching traffic than in the receding direction, so reducing the effect of " glare " which tends to diminish the *effective* illumination.

Another pattern has the tube horizontal: an electromagnet has to be used to prevent the heated vapour column from touching the side of the tube. This type gives almost uniform C.P. in a plane perpendicular to the tube; and thus gives a distribution suitable for street lighting, with less modification than the vertical tube.

Owing to the resistance of the tube diminishing with increase of current it is necessary to connect a choking coil in series with it to reduce current fluctuations. Moreover, it is usual to place in parallel with the lamp and choke a condenser of 20μF capacitance, thereby raising the combined power factor to 0·85 or more.

Various attempts to improve the colour of the light have been made by introducing other gases in addition to the mercury vapour. The difficulty is to ensure that all the gases shall be " excited " to become luminous simultaneously. One of the most successful of these is the Sieray W lamp, which gives very considerable addition of red rays at the expense of a small loss of efficiency.

The uncorrected colour lamps are made, too, in 250-watt and 150-watt sizes. These are less efficient than the 400-watt lamp, but retain about the same ratio of increase of output ($2\frac{1}{2}$ times) compared with tungsten lamps with the same input.

A similar lamp using sodium vapour is used too. This gives a monochromatic yellow light, and opinions differ as to the relative suitability of this and the mercury light.

QUESTIONS ON CHAPTER XV

1. How is the efficiency of a lamp stated, and what are the weak points of giving this in "watts per C.P." ?

2. How do the dimensions of a filament of a given material run at a stated efficiency depend on the candle-power and current respectively ?

3. Find the relative diameters and lengths of filaments for (*a*) a 16 C.P. 110-volt lamp, and (*b*) a 25 C.P. 220-volt lamp.

Assuming that strength $\propto \frac{d}{\sqrt{l}}$, which is stronger and by what per cent. ?

4. A 25 C.P. 115-volt and a 32 C.P. 230-volt lamp have filaments of the same material, run at the same efficiency. Find the relative diameters and lengths of the two filaments, and compare their strengths.

5. Describe recent improvements in the manufacture of metallic filament lamps. Find an expression connecting current and diameter of filament for a given maximum temperature. [Lond. Univ., El. Tech.

6. State the relative advantages of (*a*) carbon filament lamps, (*b*) tungsten filament vacuum lamps, (*c*) gas-filled lamps.

7. If the resistivity of tungsten is $\frac{1}{50}$th of that of carbon at the working temperature, compare the dimensions of filaments of the two materials for lamps of the same C.P., P.D., and efficiency.

In what way are these results modified by the higher efficiency at which tungsten lamps are run in practice ?

8. What is the relation between efficiency and "useful life" of an incandescent lamp ? Define the latter term.

9. A 220-volt 50 C.P. tungsten lamp and a 32 C.P. 220-volt carbon lamp are connected on a circuit whose voltage ranges between 212 volts and 228 volts. Between what values do their C.P.s vary ?

10. Compare the cost of light (*a*) per lamp-hour, (*b*) per C.P.-hour, for the following 220-volt lamps, including the cost of renewals.

Filament	C.P.	Consumption	Price
Carbon ..	16 ..	60 watts ..	9d.
Tungsten ..	48 ..	65 ,, ..	1s. 7d.
Gas-filled ..	85 ..	100 ,, ..	3s.

Price of electrical energy, 3d. per kilowatt-hour.

11. Explain why gas-filled lamps have higher efficiencies than vacuum lamps, and why this advantage is greater in high C.P. lamps.

12. Two lamps take 0·34 amp. and 0·37 amp. respectively when connected across 120-volt mains separately.

What current will they take and what is the P.D. across each when connected in series to 240-volt mains ? Neglect changes of resistance.

In what direction are the calculated values changed by the alterations of resistance that occur ?

13. Draw a target diagram for 100-watt, 230-volt., tungsten lamps, the standard initial lumens for which are 1160.

14. Describe the mercury vapour lamp with sketches of its working parts and a diagram of connexions.

Explain the object of each part and the action of the lamp.

State its advantages and disadvantages.

15. Draw a diagram of connexions for a hot-cathode mercury discharge lamp, including the auxiliary starting electrodes.

CHAPTER XVI

SPECIAL D.C. MACHINES

1. Motor-Generators

If power is required at a pressure differing considerably from that of the supply, it can be obtained by means of a motor-generator. This consists of two machines coupled together, preferably on the same bedplate.

One machine is a motor wound for the supply voltage, and the other a generator producing the desired voltage. The output is equal to the input less the losses in the two machines. For instance, for electro-chemical work a voltage of 10 (or less) is suitable. If the supply P.D. is 230 volts a motor-generator will deliver at the reduced pressure a current nearly 20 times as great as that which it takes from the mains.

Similarly, if a higher voltage is required than that of the supply a motor-generator may be used with the generator wound for this higher voltage. It is, however, better to use a booster in such a case (see Art. 8).

Occasionally **double-wound** machines are used as motor-generators. These machines have two independent windings on the same armature, connected up to separate commutators, one at each end of the armature. One winding acts as a motor and drives the armature, the other generates a voltage different from that of the supply. They have the following advantages compared with the ordinary type:—

(*a*) Less space occupied, and smaller weight.

(*b*) Only one set of field magnets, etc., therefore higher efficiency and

(*c*) Lower cost.

(*d*) Less armature reaction, since the two windings are magnetically opposed.

And the following disadvantages:—

(*a*) The armature must be considerably larger than that of either of the separate machines, so as to carry the two windings and avoid over-heating.

(*b*) A special design is required in place of using standard types of generator and motor, thus advantage (*c*) is diminished.

(*c*) The generated E.M.F. cannot be regulated by altering field strength, since any change in this causes the speed to alter in inverse proportion and so maintain the generated E.M.F. constant.

Because of these drawbacks, particularly the last, double-wound machines are used only in very special cases.

2. Feeder Booster

A *booster* is a generator whose voltage is added on to that of another generator or generators, thus increasing (or "boosting up") the total voltage.

One example is the feeder booster, whose connexions are as shown in Fig. 16.01. This consists of a shunt-wound motor connected to the bus-bars, and coupled mechanically to a series-wound generator connected between the positive bus-bar and positive feeder. The motor runs at a nearly constant speed and therefore the generator voltage is approximately proportional to the feeder current.

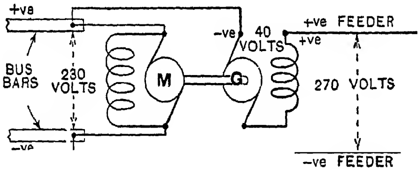


Fig. 16.01.—FEEDER BOOSTER.

The feeder drop is equal to this current multiplied by the sum of the resistances of the positive and negative feeders. Consequently the booster can be made to compensate automatically for the drop at all loads. The P.D. at the feeding point at the far end of the feeder is thus kept nearly constant. For instance, if the feeder current (Fig. 16.01) falls to half its former value the booster P.D. drops to about 20 volts, giving 250 volts across the feeders at the central station. The feeder drop is likewise halved, so that if it were 40 volts originally, it becomes 20 volts. Both in the original case and after the decrease of current the P.D. across the feeders at the far end is about 230 volts, and this value is maintained approximately for all values of the current.

The booster voltage can be adjusted either by a rheostat in the field circuit of the motor, or by a divertor in parallel with the generator's field winding (see Chapter X., Art. 8). The advantage of this over the use of overcompounded generators is that the drop

on *each* feeder is compensated. Whereas with the latter the rise of voltage depends on the load on the generators, and raises the station voltage equally for all the feeders; so that the compensation can be correct only if all of them have the same drop.

Other types of feeder booster in use are:—

(*a*) Shunt-wound motor driving a generator whose field current is supplied by an "exciter" (cf. Art. 6) whose field is connected in series with the feeder.

(*b*) As (*a*) together with a series winding on the generator field; *i.e.* the generator is compound-wound, the shunt winding being supplied by the exciter (see Fig. 16.02).

3. Negative Booster

Negative boosters are used in tramway work, where the rails

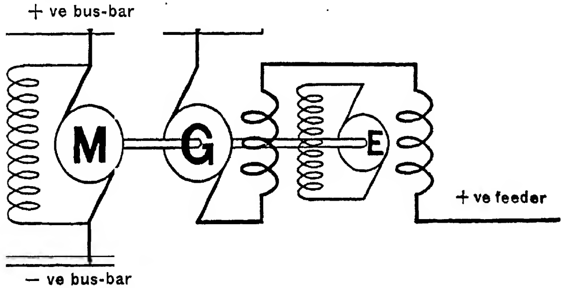


Fig. 16.02.—FEEDER BOOSTER WITH EXCITER.

form the return conductor (see Volume II.). To minimise the danger of electrolytic damage to gas or water mains by stray currents the Ministry of Transport limits the voltage at any point of the rails above that of pipes in their neighbourhood to 4·2 volts (or rather the E.M.F. of 3 Leclanché cells in series), and the P.D. between any two points on the rails to 7 volts. Consequently, except on short lines, the rails have to be supplemented by negative feeders. If these were merely connected in parallel with the rails the cross-section necessary would make their cost excessive. By using negative boosters the size of cable required can be reduced so much that the cost, inclusive of the machines, is less than that of feeders alone without boosters.

The negative booster is the same as a feeder booster except that the generator is connected between the negative bus-bar and feeder.

The connexions are shown in Fig. 16.03, and with the conditions as marked 105 volts are available for sending a current through the negative feeder. It will therefore carry 15 times as much current as it would if only the 7 volts potential difference allowed on the rails were available.

The rails themselves may not be connected to the booster because of the 7 volts limit; and because no point of them may be more than 1·4 volts (the E.M.F. of 1 Leclanché cell) below the potential of pipes near them. This last Ministry of Transport rule, like that quoted on p. 443, is to minimise electrolysis.

4. Battery Booster

Since an accumulator requires at least 2·5 volts when being

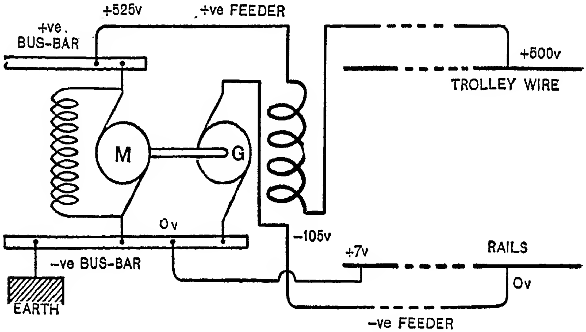


Fig. 16.03.—NEGATIVE BOOSTER.

charged, whereas it is discharged down to 1·85 volts or less (see Chapter XIII.), it is necessary when charging a battery to have a voltage 35 per cent. or more above the normal. It may be possible to obtain this by strengthening the field of one of the station generators, but it cannot then be used at the same time for ordinary supply purposes. The necessary increase of voltage can be obtained without interfering with the supply, by using a *battery-charging booster*. This consists of a shunt-wound motor (connected to the bus-bars) coupled mechanically to a shunt-wound generator which is connected between the positive bus-bar and the positive battery terminal as shown in Fig. 16.04. The generator produces the extra voltage required, and its amount can be regulated by a rheostat in its field circuit.

By reversing the field of the generator the booster may be used in discharging the battery. Its E.M.F. then assists that of the battery instead of opposing it, and the battery can thus deliver energy to the mains even when its E.M.F. is less than the P.D. across them. The use of regulating cells (see Chapter XVIII., Art. 16) can then be dispensed with, and the whole battery always charged and discharged together.

Other types of battery booster are:—

(*a*) Generator with field connected across the bus-bars.

(*b*) Series-wound generator driven by shunt-wound motor.

(*a*) has the advantage over an ordinary shunt-wound machine of maintaining a nearly constant boost when the current changes owing to a rise or fall in the battery E.M.F.

(*b*) is little used. It can boost in either direction without the use of a field reversing switch. It cannot, however, be made to charge the battery when its E.M.F. is above the bus-bar voltage, nor to discharge the battery when its E.M.F. is lower. The amount of boost can be regulated by a divertor.

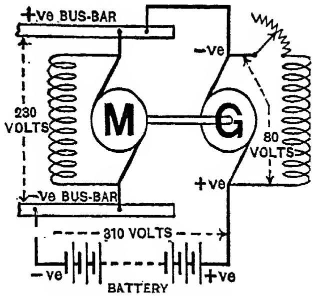


Fig. 16.04.—BATTERY BOOSTER.

5. Automatic Reversible Battery Boosters

These are a development of the reversible battery booster described in the last section. By the use of a booster of this type the battery is caused to discharge when the load is heavy and is charged when the load becomes light. The load on the generators is thus made much more uniform, resulting in diminished wear and tear on them and the engines driving them, and in a steadier voltage. They are used (as are compound-wound generators) mainly for traction supply in which large changes of load occur rapidly (see Figs. 16.05 and 16.06).

These machines can be divided into two main classes:—

(*a*) Differentially-wound. (*b*) Exciter-controlled.

The simplest form of the first class consists of a shunt-wound motor connected to the bus-bars and driving a generator with two

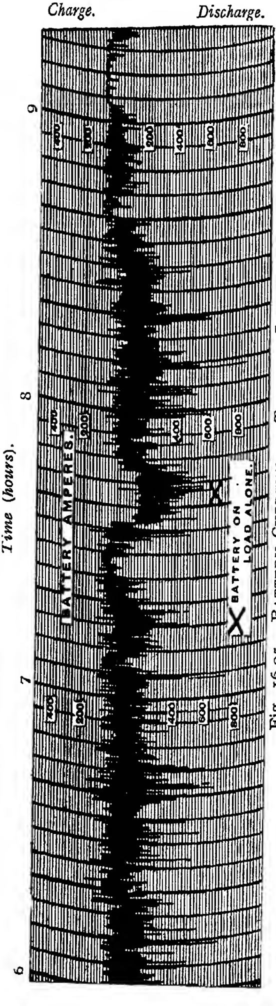


Fig. 16.05.—BATTERY CURRENT ON TRACTION LOAD.

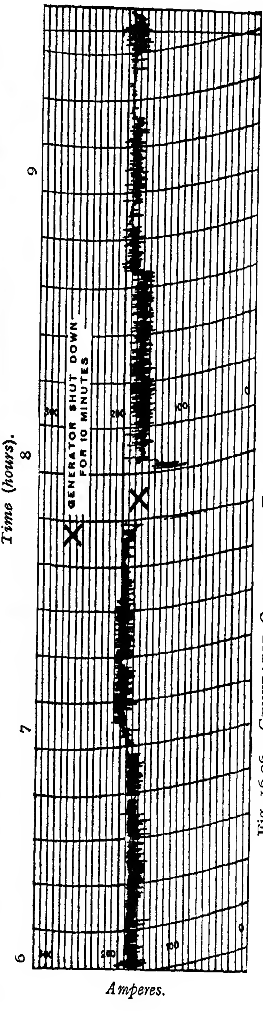


Fig. 16.06.—GENERATOR CURRENT ON TRACTION LOAD WITH BATTERY BOOSTER.

N.B.—Line current = Generator current + battery discharge current.

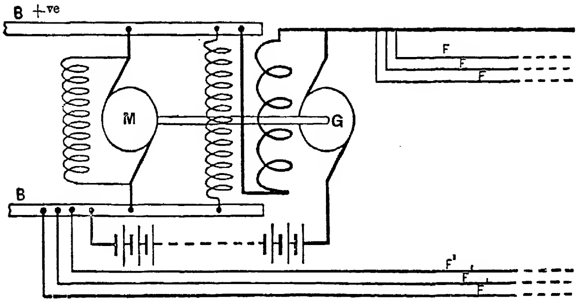


Fig. 16.07.—DIFFERENTIALLY-WOUND BATTERY BOOSTER.

B B, Bus-bars. F F F, Positive feeders. F′ F′ F′, Negative feeders.

separate field windings. One of these, the series winding, carries the load (or generator) current or a definite fraction of it, and tends to excite the generator so that it assists the battery to discharge. The other winding is connected as a shunt across the bus-bars or the battery (thus carrying a nearly constant current) and opposes the magnetising effect of the series winding (see Fig. 16.07).

With a certain current (which can, however, be adjusted to suit the conditions) the two windings have equal numbers of ampere-turns and the generator produces no E.M.F. If the load increases

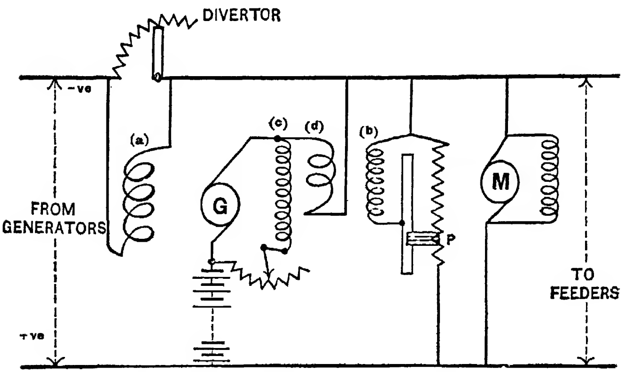


Fig. 16.08.—LANCASHIRE BATTERY BOOSTER.

(*a*) Series winding. (*b*) Shunt winding with "potentiometer" regulator (P). (*c*) Winding to compensate for variations in battery P.D.
(*d*) Winding to compensate for armature drop and reaction.

the series winding overpowers the shunt and the battery discharges, relieving the generators of a portion of the load. With a decreased load the reverse occurs, and the battery is charged by the generators assisted by the booster.

A more complicated form of the same class is the Lancashire booster. The fraction of the main generator current carried by (*a*) (Fig. 16.08) can be altered by means of the divertor, and the current in (*b*) can be altered by its regulator. Thus the load at which these windings neutralise each other, and the amount of boost for a given change of load can both be adjusted.

The whole of the magnetic circuit is laminated, which should always be done in battery boosters; otherwise the eddy currents induced by the changing magnetic flux retard this change and make the generator slow to respond to changes of load.

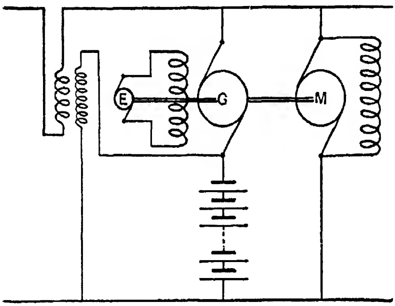


Fig. 16.09. SIEMENS AND HALSKE BATTERY BOOSTER.

6. Exciter-Controlled Battery Boosters

The disadvantage of the differentially-wound class is that a great deal of copper is required in the field windings, because the useful ampere-turns are the difference between those in the series and in the shunt turns. This is avoided in the exciter-controlled type by using a single field winding on the generator and supplying it with current from an exciter driven by the same motor. The exciter field windings are differential (or an equivalent), but since the exciter is much smaller than the generator the loss in the field windings is very much reduced. The main drawback is the extra complication of having three machines instead of two.

An example of this type is shown in Fig. 16.09. At a certain load the series and shunt windings of the exciter balance, and so the generator is unexcited. At higher loads the series ampere-turns are the greater, and the exciter sends a current through the generator field winding in the direction which causes the latter to aid the discharge of the battery. At lower loads these effects are all reversed.

Another example is the Highfield booster (see Fig. 16.10). In this the exciter produces a nearly constant voltage which is balanced against the battery P.D. When the battery discharges its terminal voltage falls, and so the current in the generator field winding is upwards in Fig. 16.10. This excites the generator so as to aid the discharge. When the battery is charged its terminal P.D. rises and reverses the generator field current. The generator therefore assists the charging in this case. The effect is thus to make the combined voltage of the generator and the battery nearly constant under all conditions. The series coil (S) tends to cause discharge.

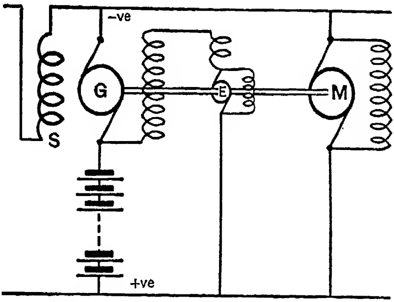


Fig. 16.10.—HIGHFIELD BATTERY BOOSTER.

The exciter is shunt-wound, with the addition of a few series-turns to compensate for its own armature reaction and drop.

7. The Entz Battery Booster

This is exciter-controlled, but differs in action from those mentioned above. Its action depends on the change of contact resistance between carbon plates when the pressure on them is varied. The connexions are as shown in Fig. 16.11.

The main current is passed through a solenoid which sucks downwards an iron core attached to one end of a pivoted lever.

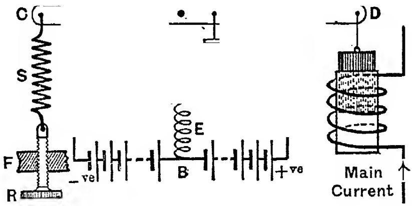


Fig. 16.11.— CONNEXIONS OF ENTZ BATTERY BOOSTER.

CD, Metal lever. E, Exciter field winding. F, Fixed frame. H, Pivot. I, Iron core. PP, Carbon rheostats. R, Regulating screw. S, Spring.

The other end of the lever is pulled down by a spring, attached to a screw by which its pull can be regulated. This lever presses on two rheostats at equal distances on each side of the pivot. Each rheostat consists of two columns of carbon discs, so that its resistance diminishes when the pressure is increased. The field winding, AB, of the exciter is connected from B, the middle point of the battery, to A, a point on the cable connecting the rheostats.

At a value of the main current dependent on the position of the regulating screw the pressures on the two rheostats will be equal, and so will their resistances. Consequently the cable between them will be at the same potential as the middle point of the battery, and no current will flow in the exciter field winding.

An increase of the main current will diminish the resistance of the right-hand rheostat (in Fig. 16.11) and increase that of the left-hand one, thus raising the potential of A nearer to that of the +ve battery terminal. A current will then flow from A to B, causing the exciter to magnetise the generator of the booster so that it causes the battery to discharge. A diminution of the main current will have the opposite effect on the rheostats, and bring the potential of A nearer to that of the *negative* terminal of the battery. Current then flows from B to A and the booster aids in charging the battery.

The advantage of this type is that a very small movement of the lever produces a large change in the resistance of the carbon rheostats so that it is prompt in its action. There is a loss in the rheostats even when no boosting is taking place, but this only corresponds with the similar loss in a differentially-wound exciter.

8. Comparison of Booster and Motor-Generator

A suitably wound motor-generator could perform the work of any of the above boosters. For instance, instead of the series-wound generator of a feeder booster (Art. 2) an over-compounded generator might be used directly connected to the +ve and −ve feeders. This would produce equally satisfactory results, but would be much more expensive. For instance, taking the voltages marked in Fig. 16·01 and a feeder current of 200 amperes, the output of the booster generator is $\dfrac{40^{\text{V}} \times 200^{\text{A}}}{1000} = 8$ kilowatts, whereas the motor-generator output would be $\dfrac{270^{\text{V}} \times 200^{\text{A}}}{1000} = 54$ kW. The motor would in each case have a slightly larger output than its generator to allow for losses. Thus the booster set consists

of a 12 B.H.P. motor and an 8 kW. generator, while the motor-generator set for the same work would require at least a 75 B.H.P. motor and a 54 kW. generator. Moreover, the efficiency of the booster arrangement is higher than that of a motor-generator, even allowing for the higher efficiency of the larger machines.

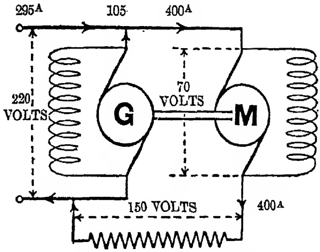


Fig. 16.12.—BACK-BOOSTER CONNEXIONS.

Similar reasoning applies to any case where the supply voltage requires raising. Even where it requires lowering a back-booster (or bucking booster) is cheaper than a motor-generator unless the required voltage is very low.

A back-booster consists of a motor connected in series with the load, and coupled to a generator which returns power to the mains. For instance, with the currents and voltages shown in Fig. 16.12, which are for a combined efficiency of 82·5 per cent. for the set, the generator output is $\frac{220^{V} \times 105^{A}}{1000} = 23{\cdot}1$ kW. With a motor-generator the output would be the power taken by the load, *i.e.* $\frac{150^{V} \times 400^{A}}{1000} = 60$ kW. The over-all efficiency of the back-booster arrangement is $\frac{150 \times 400}{220 \times 295} \times 100$ per cent. $= 92{\cdot}5$ per cent., *i.e.* much higher than that of the back-booster itself.

A somewhat similar arrangement suitable for greater voltage reductions is to connect the motor of a motor-generator set in

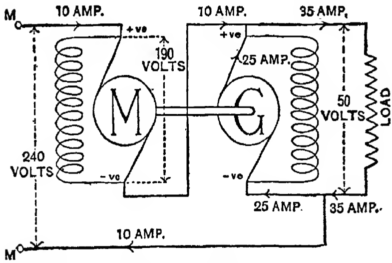


Fig. 16.13.—CONNEXIONS FOR LARGE REDUCTION OF VOLTAGE.

and the generator in parallel with the load (see Fig. 16·13). This may be called a "reducer," and has similar advantages to the above arrangement. Taking the conditions as marked

in Fig. 16.13, the generator output is 25 A. at 50 volts instead of 35 A. at the same P.D. The efficiency of the set is $\frac{50 \times 25}{190 \times 10^4} = 66$ per cent., but that of the arrangement is $(50 \times 35)/(240 \times 10) = 73$ per cent.

It can be shown that the latter arrangement is more efficient than the former when the required voltage is low. It always results in smaller machines. The motor field may be connected across the mains. This has some advantages in operation.

The following table gives average values for the efficiencies of motors and generators which may be assumed in working out the sizes of machines for the above arrangements.

MOTORS		GENERATORS	
B.H.P.	EFFICIENCY	OUTPUT kW.	EFFICIENCY
2	70 per cent.	2	74 per cent.
3	75 ,,	3	78 ,,
5	81 ,,	5	83 ,,
10	85 ,,	10	87½ ,,
15	88 ,,	15	90 ,,
20	89 ,,	20	91 ,,
50	90 ,,	50	92 ,,

9. Divided-Pole Machines

In the ordinary type of dynamo the brushes must be placed so that they short-circuit the armature coils when these are between the poles; otherwise excessive currents are produced in the short-circuited coils, and very bad sparking results (see Chapter IX.).

If however, each pole is divided in two, so as to leave a gap in which there is little or no magnetic flux, extra (or auxiliary) brushes may be placed on the commutator. Their position must be such that the coils they short-circuit are in the gaps, and thus sparking is avoided. The auxiliary brushes will have a potential intermediate between those of the positive and negative brushes. If a ring-wound armature is used the ratio in which the P.D. between the main brushes is divided by the auxiliary brushes will depend

on the relative numbers of lines of force coming from or passing to the two portions into which the poles are divided (cf. Chapter VIII., Art. 14).

E.g. in Fig. 16.14

$$\frac{\text{E.M.F. between +ve brush and auxiliary brushes}}{\text{E.M.F. between −ve brush and auxiliary brushes}} = \frac{\text{Flux from } N_1}{\text{Flux from } N_2}.$$

The values of the P.D.s can be obtained from the E.M.F.s by adding or subtracting the resistance drops in the respective circuits.

Note that the relative numbers of turns between the auxiliary brushes and the positive and negative brushes respectively do not directly affect the relative E.M.F.s. The pole-portion with the greater flux will as a rule have more turns under it, but this is not necessary.

With a drum-wound armature, since each conductor is connected to another a pole-pitch away, a change in the ratio of the fluxes would have no effect on the ratio of the voltages. This voltage ratio depends on the flux-distribution, as well as on the numbers of conductors in the two sections.

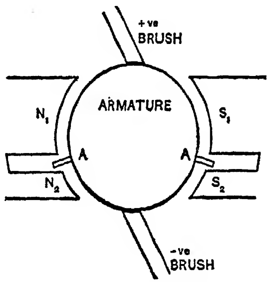


Fig. 16.14.—DIVIDED-POLE DYNAMO.

A A, Auxiliary brushes.

A change of the fluxes in N_2 and S_1 relative to the fluxes in N_1 and S_2 will alter the potential of the left-hand auxiliary brush in Fig. 16.14 in the same direction; and will at the same time alter the potential of the right-hand auxiliary brush in the opposite direction. (Cp. Art 12.)

10. The C.M.B. Auto-Converter

This (see Fig. 16.15) is one example of the divided-pole type of machine. It has four poles forming two divided poles. The two portions of each pole are separated magnetically across a horizontal plane, *i.e.* one part of the N. pole and the corresponding part of the S. pole form one magnetic circuit, and the remaining parts form another and quite independent circuit. The object of this construction is to enable the division of the voltage by the auxiliary brushes to be altered by altering the fluxes in the magnetic circuits. For the same reason the armature is ring-wound (see Art. 9).

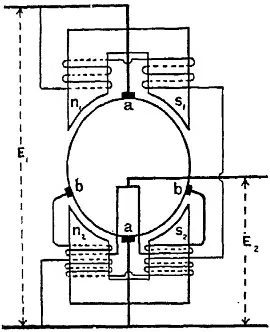


Fig. 16.15.—CONNEXIONS OF AUTO-CONVERTER.

a a, Main brushes.
b b, Auxiliary brushes.

For use as an auto-converter, *i.e.* to convert from the supply voltage to a lower secondary voltage, the connexions are as shown in Fig. 16.15. The windings on the poles are connected in series as a shunt across the supply mains. In addition each of the secondary poles (n_2, s_2) has a winding connected in series between an auxiliary brush and the independent secondary terminal. This is to counter-balance the armature reaction due to the secondary current and to any current flowing owing to a difference of potential between the auxiliary brushes.

11. Constant Current Generators

Constant current generators are those which give a nearly constant current over a wide range of speed, or a large variation in the external resistance. They are useful in train lighting, motor car battery charging, electric welding, and supply to a single arc (*e.g.* for search-lights). In the last case such generators have the advantage of doing away with the necessity of a steadying resistance, thus increasing the efficiency of the arrangement.

The simplest method is to use a series-wound generator with a drooping characteristic (see Chapter X., Art. 10). A large decrease of resistance in this case will produce only a small increase in the current supplied.

The C.M.B. Auto-converter (Art. 10) can be used as a constant current motor-generator (*i.e.* delivering constant current when supplied at constant P.D.) by modifying the windings.

12. Rosenberg's Constant Current Generator

This is another example of a divided-pole machine. The two halves of each pole are not magnetically separated, and there is only one field winding for the two halves (see Fig. 16.16). The brushes *a, a* are *short-circuited.* The short-circuit current sets up a cross magneto-motive force (see Chapter IX., Art. 2) causing a flux as shown in Fig. 16.16. This produces a difference of potential

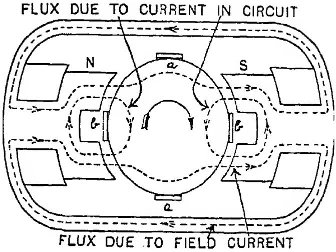


Fig. 16.16.—ROSENBERG GENERATOR.
a a, Short-circuited brushes. b b, Supply brushes.

between the brushes *b*, *b*, which are connected to the circuit to be supplied. The short-circuit current thus serves as an exciting current to produce the cross-flux which generates the working E.M.F. The yoke and pole-cores are therefore made of small cross-section so as to keep down the flux in them, and so prevent an excessive short-circuit current. The pole-pieces, on the other hand, are made very large, as they have to carry the cross-flux, which must be large so as to produce a sufficient E.M.F.

When used for supplying an arc lamp or welder, the field is connected in series with the arc to the brushes *b*, *b*, and the brushes *a*, *a* are short-circuited. This results in a characteristic (at constant speed) in which beyond a certain current the P.D. falls off rapidly with little increase of current.

When used as a train-lighting generator a battery is used to maintain the current when the generator (driven from one of the carriage axles) is at rest or running very slowly. A reverse current cut-out is fitted in the generator circuit, to prevent the battery discharging through it when the generator E.M.F. falls. The field winding is connected across the accumulator and thus receives a nearly constant current. The same method is used on motor cars.

In Mather and Platt's system using this generator the cut-out has two coils, one a shunt to the generator and the other in series with it. When the speed is high enough the current in the shunt coil raises the core and closes the switch, the series coil then increasing the pressure on the contacts. When the speed falls the current decreases and then reverses, whereupon the series coil opposes the shunt coil and allows the switch to open (see Fig. 16.18).

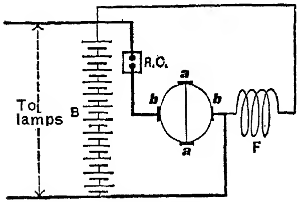


Fig. 16.17.—CONNEXIONS OF TRAIN-LIGHTING GENERATOR.
B, Battery. F, Field winding of generator.
R.C., Reverse current cut-out.

The field (connected as shown in Fig. 16.17) has a 3-section resistance (the "output adjuster") connected in series with it. This is altered by the barrel switch (Fig. 16.18), which at the same time throws lamps in and out of circuit. The generator is thus made to deliver a current suitable for no lights, half lights, or all lights, together with sufficient charging current to keep the battery in good condition and enable it to supply current during stoppages

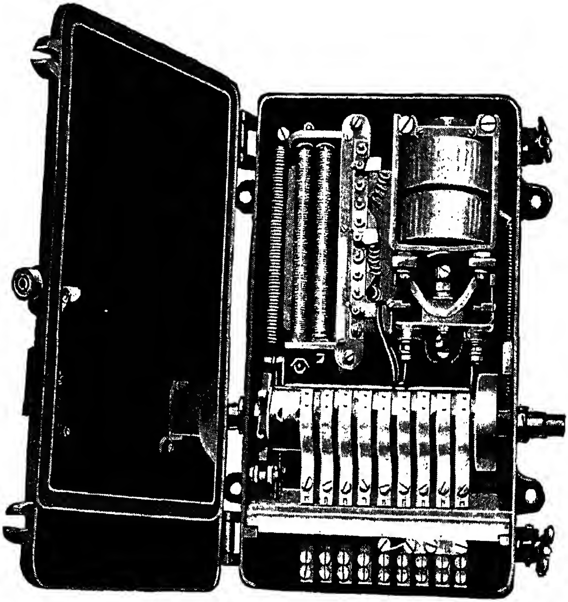


Fig. 16.18.—SOLENOID CUT-OUT, BARREL SWITCH, ETC.

The relation between speed and the current delivered by the generator is shown in Fig. 16.19. The reason for the approximate constancy of this current beyond a certain speed is that it exerts a great demagnetising effect on the field due to the magnet windings (see Chapter IX.). Consequently the current between the brushes *a, a* diminishes rapidly as the main current rises and so the working cross-flux produced by it diminishes. The limiting value of the main current is that which would make its demagnetising effect sufficient to wipe out the magnetising effect of the field windings.

Another advantage of this machine for use as a train-lighting generator is that no change of connexions is required when the direction of rotation is reversed (cf. Chapter X., Art. 11), *i.e.* when the carriage travels in the reverse direction. For a reversal of rotation with unchanged field current reverses the current between the short-circuited brushes, and therefore reverses the cross-flux set up by it. The E.M.F. developed between the brushes *b*, *b*, thus remains in the same direction as before, since both flux and rotation are reversed.

Another method is used in the "third-brush" generator, which has a wide application to motor cars for charging the car battery.

In this, as the name implies, a third small brush is provided in addition to the two main brushes. The field winding is connected between one main brush and the third brush: the latter being placed in advance of the other main brush. Thus armature reaction reduces the ratio of the field voltage to the total voltage, and so tends to keep the current constant.

In some cases this action is assisted by a series field-winding opposing the magnetic effect of the winding connected to the third brush. By suitable design a current-speed graph similar to that shown in Fig. 16·19 can be obtained. The maximum current can be increased or diminished by moving the third brush nearer to, or further from, the main brush to which the field is not connected.

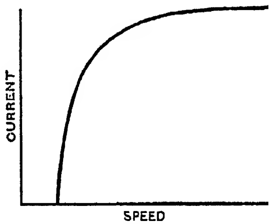


Fig. 16.19.—VARIATION OF CURRENT WITH SPEED.

The connexions to the battery and reverse-current cut-out are similar to those shown in Fig. 16·17.

For another type of constant current generator and its application see Chapter XVIII., Art. 25.

QUESTIONS ON CHAPTER XVI

1. Draw a diagram of connexions for a feeder booster, and explain its use and action.

2. Will the amount of boost of a feeder booster vary directly as the feeder current? Explain two methods of adjusting the amount of boost for any particular current.

3. Calculate the proper amount of boost for a feeder booster connected to a pair of 37/·064 in. feeders going to a point $1\frac{1}{2}$ ml. from the central station and carrying 90 amperes. Working temperature of feeder 85° F.

4. Describe the action of, and explain the need for, a negative booster in traction work.

5. State the advantages of using an automatic battery booster. Draw a diagram of connexions of one such booster and explain its action.

6. What are the functions of an automatic reversible booster? Explain fully with a diagram of connexions the action of one good example of this type of machine, using a separate exciter. [Lond. Univ., El. Eng.

7. How are the purposes of a reversible booster fulfilled by the three types of machine in common use? For what kind of load is each of these specially suitable? [Lond. Univ., El. Mach.

8. A "Lancashire" battery booster is adjusted so that the generator produces no E.M.F. when the main generators are giving 600 amperes, and produces 50 volts when they are giving 620 amperes. What is the effect of

(i) altering the divertor so that the current in coil (*a*) is reduced to $\frac{4}{5}$ of its previous value;

(ii) altering the potentiometer regulator so that the current in coil (*b*) is reduced to $\frac{4}{5}$ of its previous value;

(iii) making both the above alterations?

9. Explain the advantage of a booster over a simple motor-generator when power is required at a voltage higher than the supply P.D. Give a numerical example to illustrate this.

10. A current of 200 amperes at 150 volts is required from 110 volt mains. Calculate the sizes of machines and the overall efficiency if this is done (*a*) by a motor-generator, (*b*) by a booster. Assume reasonable values for the efficiencies of the machines.

11. A supply of 200 amperes at 65 volts is required from 110-volt mains. Find the sizes of machines required if this is got (*a*) by a motor-generator, (*b*) by a back-booster, (*c*) by motor and generator in series. Find also the efficiency of each arrangement.

12. Prove that the ratio of the E.M.F.s in a ring-wound divided-pole machine (Art. 9) is independent of the numbers of the turns under the two parts of a pole.

13. A motor-generator is used for giving power at a pressure higher than that of the supply available. What modification is needed to enable the *same* machines to be used as a booster, and what gain (if any) in efficiency results? Illustrate your answer with a numerical example.

14. Describe a generator to give approximately constant current over a wide range of speed, and explain its mode of action.

15. Explain, with diagrams, how the third-brush type of generator tends to keep constant the current delivered by it.

CHAPTER XVII

TRANSFORMERS

1. General Principle

A *transformer* consists essentially of two insulated coils surrounding a common magnetic circuit, and supplied with suitable terminals. It may be represented diagrammatically as shown in Fig. 17.01.

Alternating current power is supplied to one of the windings, called the *primary*, AA. An alternating flux is thereby produced in the magnetic circuit. This flux links with the other winding, which is called the *secondary*, BB. The alternations of the flux induce an alternating E.M.F. in BB, of the same frequency as the supply to AA. Therefore BB can deliver alternating power to any impedance or other apparatus connected to it.

Making certain assumptions (see Art. 2), it can be shown that the ratio of the voltages across the terminals of the two coils is equal to the ratio of the numbers of turns in the coils. In actual transformers these ratios are very nearly equal to each other.

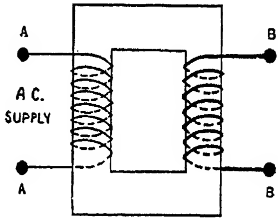


Fig. 17.01.—Transformer Circuits.

It is, therefore, possible by using suitable numbers of turns, to obtain any desired voltage across BB, from a supply at any fixed voltage. When the primary voltage is higher than that of the secondary the apparatus is called a *step-down transformer*, and when the secondary voltage is the higher the title becomes *step-up transformer*.

It might appear at first sight that a step-down transformer has the same effect as the simpler choking coil. This is true, with the very important exceptions, (*a*) that the reduction of voltage due to a choking coil varies greatly with the load in series with it (see Chapter V., Art. 14), whereas the voltage given by the secondary of a transformer is almost constant if the primary P.D. is constant; (*b*) the power factor is reduced by a choking coil to a considerable extent, whereas a transformer under normal conditions has an inappreciable effect on the power factor (see Art. 3, Ex. 2).

2. Relations of Turns, Voltages, and Currents

Neglecting the small effect of the resistance of the primary winding, the P.D. applied to the primary terminals is equal and opposite to the self-induced E.M.F. in the primary winding. This latter varies as the maximum flux, as the frequency (since the average rate of change of flux is proportional to this), and as the number of primary turns in series.

Similarly, the E.M.F. induced in the secondary coil varies as the maximum flux linking therewith, as its frequency, and as the number of secondary turns in series. The frequency is necessarily the same as that of the primary supply; hence, if the whole of the primary flux is assumed to link with the secondary (*i.e.* if magnetic leakage is neglected),

$$\frac{\text{Secondary E.M.F.}}{\text{Primary P.D.}} = \frac{\text{No. of secondary turns in series}}{\text{No. of primary turns in series}}.$$

And if the small effect of the resistance of the secondary is neglected, the secondary P.D. is equal to its E.M.F.

Let Φ_m = maximum flux,
f = frequency in cycles per sec.,
$\mathfrak{T}_1$ = No. of turns in series in primary,
$\mathfrak{T}_2$ = ,, ,, ,, secondary.

Then in half a period the flux is reversed, and so is changed by $2\phi_m$. The time of half a period is $\frac{1}{2}\cdot\frac{1}{f}$ sec.;

$$\text{Rate of change of } linkages \text{ in primary} = \frac{2\Phi_m \mathfrak{T}_1}{T} = 4f\Phi_m \mathfrak{T}_1,$$

$$\therefore \text{ Average E.M.F. induced in primary} = \frac{\mathfrak{T}_1}{} \text{ volts;}$$

$$\therefore \text{ Effective value of E.M.F. induced in primary} = \frac{\mathfrak{T}_1}{10^8} \text{ volts,}$$

where k = form factor of E.M.F. wave.

For sinoidal waves $k = 1{\cdot}11 = \frac{\pi}{2\sqrt{2}}$, and then

$$E_1 = 4{\cdot}44 f\Phi_m \mathfrak{T}_1 \times 10^{-8} \text{ volts.}$$

In exactly the same way it can be proved that the effective value of the E.M.F. induced in the secondary

$$(E_2) = 4kf\Phi_m \mathfrak{T}_2 \times 10^{-8} \text{ volts;}$$

which for sinoidal waves becomes—

$$E_2 = 4{\cdot}44 f \Phi_m \mho_2 \times 10^{-8} \text{ volts (R.M.S.)}.$$

If the transformer were of 100 per cent. efficiency, then the primary input ($V_1 I_1 \cos \phi_1$) would be equal to the secondary output ($V_2 I_2 \cos \phi_2$). Assuming that the two power factors ($\cos \phi_1$, $\cos \phi_2$) are equal, this gives $V_2 I_2 = V_1 I_1$,

whence $$\frac{I_2}{I_1} = \frac{V_1}{V_2} = \frac{\text{No. of primary turns in series}}{\text{No. of secondary turns in series}}.$$

And by cross-multiplying:—

Secondary ampere-turns = Primary ampere-turns.

Though a number of assumptions have been made in obtaining the above relations, they are nevertheless accurate within a few per cent. in commercial transformers. The voltage relationship is most nearly true when the secondary is unloaded, whereas the current relationship becomes more exact as the load is increased (see Ex. 2).

Example 1. *A transformer for a primary P.D. of 2000 volts and a secondary P.D. of 220 volts has a maximum output of 20 kVA. There are 65 secondary turns. Calculate the number of primary turns, and the primary and secondary full-load currents, neglecting losses.*

$$\frac{\text{No. of primary turns}}{65} = \frac{2000}{220};$$

$$\text{No. of primary turns} \quad 2000 \times 65 \quad \text{---}$$

$$\text{Secondary current} \quad \frac{20 \times 1000}{220} = 90{\cdot}9 \text{ amperes.}$$

$$\text{Primary current} = 90{\cdot}9 \times \frac{65}{591} \quad 10{\cdot}0 \text{ amperes.}$$

Or $$\text{Primary current} = \frac{20 \times 1000}{2000} \quad 10{\cdot}0 \text{ amperes.}$$

3. Effects of No-Load Current

When the secondary is on open circuit the primary acts like a choking coil, *i.e.* it carries a current which lags behind the applied P.D. by an angle approaching 90°. If this current is resolved into two components (see Chapter V., Art. 13), the one in phase with the P.D. depends on the power lost, partly in the primary resistance, but mainly in the core due to hysteresis and eddy currents. It may, therefore, be termed the *iron-loss-current*, since the resistance loss is usually negligible.

The other component, lagging 90°, is the true *magnetising current*, being required to produce the alternating flux, but representing no waste of power (see Ex. 2).

When a load is placed on the secondary, the current flowing in that winding produces a M.M.F. But with constant primary P.D., the formula of Art. 2 shows that the flux must remain constant. Hence an additional current flows in the primary to balance the secondary ampere-turns. The total primary current is, therefore, the vector sum of this balancing current and the original no-load current.

This effect may be represented by supposing an impedance to be connected in parallel with the primary winding, and by taking the current in the latter to be exactly opposite in phase to the secondary current, I_2, and exactly equal to $I_2 \times \frac{\mho_2}{\mho_1}$. The impedance is so chosen that it takes a current equal to the no-load current when the (constant) primary P.D. is applied, and with the same angle of lag (ϕ_0) as the actual no-load current.

This impedance (Z_0) is called the *open-circuit or no-load impedance* of the transformer. It is made up of a certain amount of reactance (X_0), the *open-circuit reactance*, and some resistance (R_0), the *iron-loss equivalent resistance*, connected in parallel, see Fig. 17.09 (*b*).

The following relations must, therefore, hold good:—

$$\overline{I_0 \sin \phi_0}, \qquad I_0 \cos$$

where ϕ_0 = angle of lag of no-load current (see Ex. 4, Art. 9).

In drawing vector diagrams for transformers it is convenient to use different scales for the primary and secondary currents, the sizes of the scales being in proportion to the respective numbers of turns. If there are more primary than secondary turns, the primary balancing current will be less than the secondary current in the inverse ratio of the turns. But on the above method the primary current will be drawn to a larger scale, and so the vectors representing the above two currents will be of the same length. Similarly, by using voltage scales inversely proportional to the numbers of turns, the two voltage vectors will be of equal lengths.

Fig. 17.02 gives vector diagrams showing the effects of the no-load current for non-inductive and for inductive loads on the secondary. In order to simplify matters the effects of the resistances of the windings are neglected until later (Art. 4).

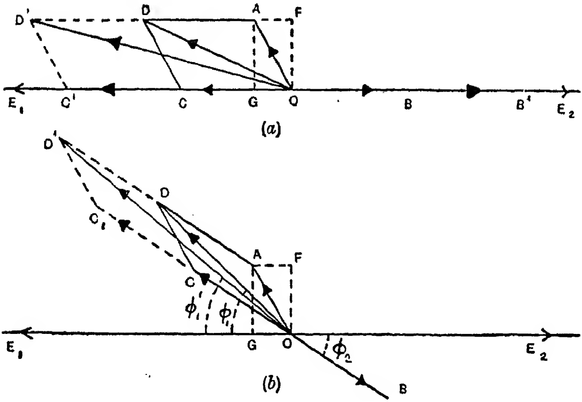


Fig. 17.02.—EFFECTS OF NO-LOAD CURRENT.
(a) Non-inductive loads. (b) Inductive loads.

OA = no-load primary current.
OB = secondary (load) current.
OB′ = doubled secondary current (due to increased load).
OC = primary current to balance OB.
OD = total primary current with current OB in secondary.
OC′ = primary current to balance OB′.
OD′ = total primary current with current OB′ in secondary.
OE_1 = primary applied P.D.
OE_2 = secondary induced E.M.F.
OF = true magnetising current.
OG = iron-loss current.

An examination of these diagrams shows the following points:—

(i) The primary current always exceeds the value $\left(I_2 \times \frac{\mho_2}{\mho_1}\right)$ given by the simple theory. But the amount of excess diminishes as the load increases, and the percentage excess diminishes still faster. The only exception is when the secondary load is so highly inductive that the angle of lag (ϕ_2) of the secondary current is equal to the angle of lag (E_1OA) of the no-load current. In this very unusual case the excess of the primary current is constant, and so the percentage excess still diminishes with increase of load.

(ii) The greater the value of ϕ_2 (up to equality with $\angle E_1OA$) the less rapid the diminution in the excess of the primary current.

(iii) The angle of lag of the primary current $(= \phi_1 = \angle E_1OD)$ is always greater than ϕ_2, the angle of lag of the secondary current. But the additional lag diminishes with increase of load, so that at large loads the two angles become nearly equal.

(iv) The greater the value of ϕ_2 the less the difference between the two angles of lag.

Example 2. *A* 40 *kVA. transformer,* 2000v *to* 200v, *takes a no-load current of* 0·54 *A. and then absorbs* 380 *watts.*

(*a*) *Find the components of the no-load current.*

(*b*) *Find the values of the primary current with* 2 *per cent.,* 4 *per cent.,* 6 *per cent., and* 10 *per cent. of full load on the secondary,* $\cos \phi = 1$; *and the corresponding values of the primary power factor.*

(*c*) *Repeat* (*b*), *but with* $\cos \phi = 0·6$.

[N.B.—The outputs of transformers and of alternators are stated in kilovolt-amperes, not in kilowatts, because the heating of the coils depends on the magnitude of the current, and not on its phase relative to the voltage.]

(*a*) Iron-loss current $= I_0 \cos \phi_0 = \dfrac{V_1 I_0 \cos \phi_0}{V_1} = \dfrac{380}{2000} = 0·19$ A.;

$$\therefore \text{ magnetising current} = \sqrt{\{·54^2 - ·19^2\}} = \sqrt{\{·73 \times ·35\}} = 0·51 \text{ A.}$$

(*b*) Full-load secondary current $= \frac{40000}{200} = 200$ A.

2 per cent. of full load $= 4$ A.

Primary balancing current (OC, Fig. 17.02*a*) $= 4 \times \frac{200}{2000} = 0·4$ A.

Total primary current (OD) $= \sqrt{\{(OC + OG)^2 + OF^2\}}$

$= 0·78$ ampere.

Similarly, at 4 per cent. of full load, OC $= 0·8$ A.

and OD $= \sqrt{\{(0·8 + 0·19)^2 + 0·51^2\}} = 1·11$ A.

At 6 per cent. of full load, OD $= \sqrt{\{(1·2 + 0·19)^2 + 0·51^2\}} = 1·48$ A.

At 10 per cent. of full load, OD $= \sqrt{\{(2·0 + 0·19)^2 + 0·51^2\}} = 2·25$ A.

Evidently as the load increases OD approximates to (OC + 0·19), see Fig. 17·03.

The values of $\cos \phi_1$ are $\left(\dfrac{0·59}{0·78} =\right) 0·76$; $\left(\dfrac{0·99}{1·11} =\right) 0·89$; 0·94; and 0·97.

(*c*) The values of OC are as in (*b*), but $\cos \angle E_1OC = \cos \phi_2 = 0·6$.
Further, $\sin \phi_2 = \sqrt{\{1 - \cos^2\phi_2\}} = 0·80$.
Thus at 2 per cent. of full load:—

Power component of OC $= 0·6 \times 0·4 = 0·24$ A.

Reactive component of OC $= 0·8 \times 0·4 = 0·32$ A.;

$$\therefore \text{ Total primary current} = \sqrt{\{(0·24 + 0·19)^2 + (0·32 + 0·51)^2\}} = \sqrt{\{0·43^2 + 0·83^2\}} = 0·93 \text{ A.}$$

Similarly, at 4 per cent. of full load:—

$$\text{Total primary current} = \sqrt{\{(0·48 + 0·19)^2 + (0·64 + 0·51)^2\}} = 1·33 \text{ A.}$$

At 6 per cent.:—

$$\text{Total primary current} = \sqrt{\{(0{\cdot}72 + 0{\cdot}19)^2 + (0{\cdot}96 + 0{\cdot}51)^2\}}$$
$$= 1{\cdot}73 \text{ A.}$$

At 10 per cent:—

$$\text{Total primary current} = \sqrt{\{(1{\cdot}20 + 0{\cdot}19)^2 + (1{\cdot}60 + 0{\cdot}51)^2\}}$$
$$= 2{\cdot}53 \text{ A.}$$

The values of $\cos \phi_1$ are

$$\left(\frac{0{\cdot}43}{0{\cdot}93} = \right) 0{\cdot}46; \ \left(\frac{0{\cdot}67}{1{\cdot}33} = \right) 0{\cdot}50; \ \left(\frac{0{\cdot}91}{1{\cdot}73}\right.$$

Note how $\cos \phi_1$ approaches the value of $\cos \phi_2$ as the load increases, and that the approximation is more rapid with non-inductive loads, though the angles differ more.

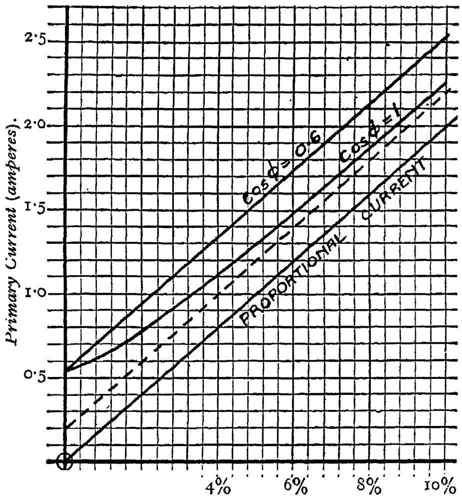


Fig. 17.03.—VARIATION OF PRIMARY CURRENT WITH LOAD.

4. Effects of Resistances of Windings

The resistance of the secondary winding absorbs part of the E.M.F. induced in the secondary, and so makes the P.D. at its terminals less than the E.M.F. With a non-inductive load the P.D. is equal to $(E_2 - I_2R_2)$ volts

where I_2 = secondary current in amperes,

and R_2 = resistance of secondary in ohms.

With an inductive load the same formula holds provided the *vector* difference is taken (see Fig. 17·04).

The primary resistance on the other hand causes the primary P.D. to exceed the primary E.M.F. because the P.D. must drive the current through the resistance in addition to balancing the E.M.F. If the primary current were in phase with the voltage (*i.e.* with non-inductive load and negligible no-load current) the primary P.D. would be equal to $(E_1 + I_1R_1)$ volts.

In the usual case when ϕ_1 is not zero this is true (as in the case of the secondary) if taken vectorially.

The two effects are similar to those occurring in a D.C. generator (for secondary) and motor (for primary) respectively (cf. Chapter X., Art. 7, and Chapter XI., Art. 2).

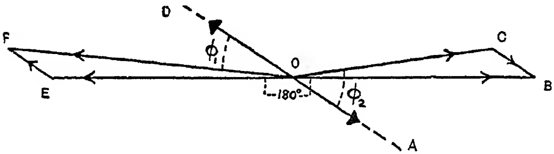


Fig. 17.04.—EFFECTS OF RESISTANCES OF WINDINGS.

Both resistances reduce the secondary P.D. for a given primary P.D. With both currents in phase with their respective P.D.s:—

$$\frac{\text{♉}_2}{\text{♉}_1} = \frac{E_2}{E_1} = \frac{V_2 + I_2R_2}{V_1 - I_1R_1};$$

and so the ratio of secondary P.D. to primary P.D. $\left(\frac{V_2}{V_1}\right)$ is less than the ratio of the number of turns in the respective windings.

With inductive loads the above must be taken vectorially, and so the effect is as shown in Fig. 17.04.

OA = secondary current.
OB = secondary E.M.F.
CB = I_2R_2, and is parallel to OA.
OC = secondary P.D.
OD = primary current.
OE = primary P.D. to balance induced E.M.F.
EF = I_1R_1, and is parallel to OD.
OF = total primary P.D.

In this diagram $$\frac{E_2\,(= OB)}{(= OE \text{ reversed})} = \frac{\quad}{\mho_1}.$$

and so the vectors OB and OE are equal in length, but are drawn to different scales (see Art. 3). Moreover, EOB is a straight line. BC is the resistance drop in the secondary, and OC, the vector difference of E_2 and I_2R_2, gives the secondary terminal P.D. (V_2). Similarly EF is the resistance drop in the primary, and OF, the vector sum of OE and I_1R_1, is the primary terminal P.D. Hence as before both resistances reduce the value of the ratio (V_2/V_1). But if the current is kept constant an increase in the angle of its lag diminishes the amount of this reduction (see further Art. 13 and Ex. 7).

5. Equivalent Resistance

A reduction in the primary E.M.F. equal to I_1R_1 causes a reduction in the secondary E.M.F. of $I_1R_1 \times \left(\frac{\mho_2}{\mho_1}\right)$. But I_1 is

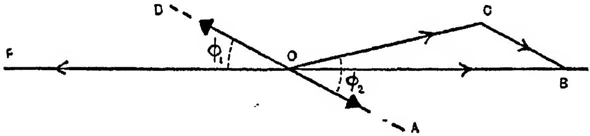


Fig. 17.05.—EFFECT OF EQUIVALENT SECONDARY RESISTANCE.

approximately equal to $I_2\left(\frac{\mho_2}{\mho_1}\right)$, the approximation becoming closer as the currents (and so the voltage reduction) increase (see Ex. 2). Therefore the reduction in the secondary E.M.F. is equal to $I_2R_1 \times (\mho_2/\mho_1)^2$. Consequently the drop in the secondary terminal P.D. due to the resistances of both the windings is

$$= I_2\{R_1\,(\mho_2/\mho_1)^2 + R_2\}.$$

The expression inside the brackets { } is called the equivalent secondary resistance, because a resistance of this amount in the secondary circuit, with no resistance in the primary circuit, gives very approximately the same drop as the two actual resistances.

This applies whether the currents are in phase with the voltages or not, except that in the latter case, as before, the drop must be taken vectorially. The vector diagram simplified in this way is shown in Fig. 17.05.

OA = secondary current.
OB = secondary E.M.F.
CB = $I_2 \times$ equivalent secondary resistance, and is parallel to DOA.
OC = secondary terminal P.D.
OD = primary current.
OF = primary applied P.D.
= BO.

Note that CB in Fig. 17·05 is equal to (CB + FE) in Fig. 17.04, and therefore the relative magnitudes and phases of OC and OF are the same in the two diagrams.

Another way of obtaining the equivalent secondary resistance is to consider the copper losses. These amount to $(I_1^2R_1 + I_2^2R_2)$ watts. But $I_1 = I_2 \times (\mho_2/\mho_1)$ approximately;

$$\therefore \text{ copper losses} = I_2^2 \{R_1(\mho_2/\mho_1)^2 + R_2\}$$
$$= (\text{secondary current})^2 \times \text{equivalent secondary resistance.}$$

Thus the copper losses are given approximately by the single equivalent resistance in place of the two actual resistances.

If preferred a single equivalent primary resistance $= R_1 + (\mho_1/\mho_2)^2 R_2$ may be used as an alternative in both cases.

Example 3. *A 30 kVA. transformer, 2000v to 230v, has a primary resistance of 1·05 ohm and a secondary resistance of 0·0130 ohm.*

Find the equivalent secondary resistance, the total resistance drop at full load, and the copper losses at full load.

$$\text{Equivalent secondary resistance} = 1{\cdot}05\left(\tfrac{230}{2000}\right)^2 + 0{\cdot}0130 \text{ ohm}$$
$$= {\cdot}0139 + {\cdot}0130 \text{ ohm}$$
$$= {\cdot}0269 \text{ ohm.}$$

[Note that this is roughly double the actual secondary resistance. A similar result will always be obtained because for equal current density, (cross-section of primary/cross-section of secondary) $= \mho_2/\mho_1$; and for equal lengths of mean turn, (length of primary/length of secondary) $= \dfrac{\mho_1}{\mho_2}$;

$$\therefore R_1/R_2 = \frac{l_1A_2}{l_2A_1} = \left(\frac{\mho_1}{\mho_2}\right)^2; \qquad \therefore R_1 \times \left(\frac{\mho_2}{\mho_1}\right)^2 = R_2 \text{ approximately.}]$$

Full load primary current, neglecting effect of no-load current $= \frac{30000}{2000} = 15$A;

$\therefore$ Drop in primary resistance $= 15 \times 1{\cdot}05 = 15{\cdot}8$ V.

This reduces the secondary E.M.F. by $15{\cdot}8 \times \frac{230}{2000} = 1{\cdot}82$ V.

Full load secondary current $= \frac{30000}{230} = 130$A;

$\therefore$ Drop in secondary resistance $= 130 \times 0{\cdot}0130 = 1{\cdot}69$ V.;

$\therefore$ Total resistance drop $= 1{\cdot}82 + 1{\cdot}69 = 3{\cdot}51$ volts.

Copper loss at full load $= 15^2 \times 1{\cdot}05 + 130^2 \times 0{\cdot}130$
$= 456$ watts.

Or, using the equivalent secondary resistance,

Total resistance drop $= 130 \times {\cdot}0269 = 3{\cdot}50$ volts.

Copper loss at full load $= 130^2 \times {\cdot}0269 = 455$ watts.

The small differences are due to approximations in the course of calculation.

6. Magnetic Leakage

Up to this point it has been assumed that the whole of the flux links with all the turns in both windings. This is not true, as some of the lines instead of passing round the main iron magnetic circuit will take various shorter paths, which, however, are of higher reluctance since a large part of these is through air or other non-magnetic materials.

The effects of this *magnetic leakage* can be studied conveniently by dividing the flux into three main components:

(*a*) The *common flux, i.e.* that which links with both primary and secondary windings.

(*b*) The *primary leakage flux, i.e.* that which links with the primary winding only.

(*c*) The *secondary leakage flux, i.e.* that which links with the secondary winding only.

The main paths of these three fluxes are shown diagrammatically in Fig. 17.06. In an actual transformer the two windings are placed on the same or neighbouring parts of the iron, but leakage still occurs in what is essentially the same manner.

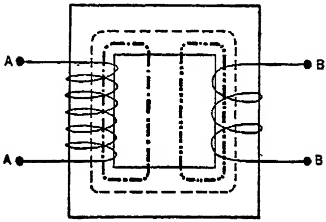


Fig. 17.06.—LEAKAGE FLUXES.

A A, Primary terminals.
B B, Secondary terminals.
------- Common flux.
—•—— Primary leakage flux.
——•–•–•—— Secondary leakage flux.

Evidently the actual primary flux is the vector sum of the primary leakage and common fluxes, and a similar relation holds for the actual secondary flux.

Each leakage flux is produced by the M.M.F. of the corresponding winding. As their paths are largely in air the reluctances are very nearly constant. Hence the primary leakage flux is proportional to and in phase with the primary current, and similarly for the secondary. Thus these fluxes increase with load, whereas the common flux remains nearly constant. Moreover, since the primary ampere-turns and the secondary ampere-turns are nearly equal the leakage fluxes will be so too, because the reluctances of their paths are necessarily fairly equal.

The common flux is due to the resultant M.M.F. of the primary and secondary ampere-turns, which are almost opposite in phase (see Art. 3).

The vector diagram as modified by magnetic leakage can now be constructed as follows:—

Let OA (Fig. 17·07) represent the common flux. Then OB, lagging 90°, represents the E.M.F. induced in the secondary by this flux. And OC, equal and opposite to OB, represents to a different scale the P.D. applied to the primary to balance the E.M.F. induced in it by this same flux.

Let OD represent the secondary current. Then the secondary leakage flux is given by AE, proportional and parallel to OD. Hence the actual secondary flux is OE, the vector sum of OA and AE.

The total E.M.F. induced in the secondary is OF, perpendicular and proportional to OE. And BF is the E.M.F. due to the leakage

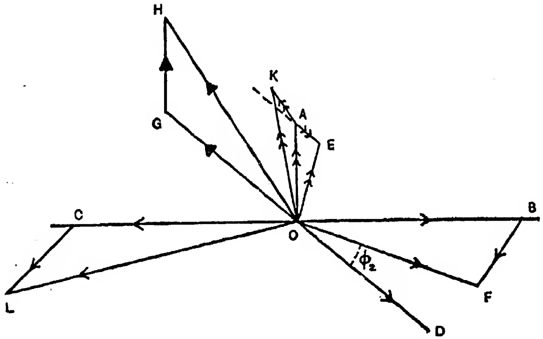


Fig. 17.07.—EFFECTS OF MAGNETIC LEAKAGE.

flux alone, and so is perpendicular to and proportional to AE, and therefore to OD. The triangle OBF has its sides perpendicular to, and proportional to, the corresponding sides of OAE; so they are similar.

OG, equal and opposite to OD, gives to a different scale the part of the primary current to balance the magnetic effect of the secondary current. GH, parallel to OA, represents the magnetising current required to produce the common flux OA. Then OH is the total primary current, neglecting iron losses.

The primary leakage flux AK is parallel and proportional to this, and the actual primary flux is OK.

The primary P.D. required is therefore changed from OC to OL, perpendicular to OK, by the addition of the component CL, perpendicular and proportional to AK and therefore to OH. The triangle OCL is similar to the triangle OAK.

7. Effects of Magnetic Leakage

The following deductions can be made from a study of the above diagram:—

(*a*) The two leakage fluxes produce similar effects. This is because (especially when large) they are almost opposite in phase considering the iron circuit, and therefore are almost in phase in the air portions of their paths (see Fig. 17·06). Therefore their effects may be dealt with together.

(*b*) One effect is an increase of the primary angle of lag (ϕ_1) compared with that of the secondary load (ϕ_2). For fixed values of the currents this effect is greatest when the angles of lag are small, since then BF and CL are nearly perpendicular to OB and OC.

(*c*) With lagging currents, magnetic leakage reduces the ratio of secondary P.D. to primary P.D. For fixed values of the currents

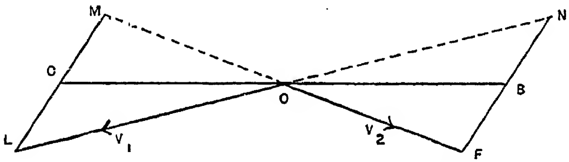


Fig. 17.08.—SIMPLIFIED DIAGRAM OF LEAKAGE EFFECTS.

O L = primary applied P.D. = NO. C L. = primary leakage P.D. = NB.
M C = secondary leakage E.M.F. = BF. M O = secondary terminal P.D. = O F.

this effect is greatest when $\phi_2 = 90°$, since then BF is in the opposite direction to OB, and CL is in the same direction as OC.

(*d*) The effects are the same as would be produced by suitable reactances in the primary and secondary circuits. For the voltages (CL and BF) are proportional to, and 90° out of phase with, the respective currents. These reactances are called the leakage reactances.

(*e*) Neglecting the influence of the no-load current, which becomes of less importance as the secondary current (and so the leakage) increases, the two leakage fluxes (AK, AE, Fig. 17.07) are opposite in phase, and the voltages (CL, BF) are in phase. They may therefore be replaced by a single leakage flux, represented by EK if taken in the primary or by KE if taken in the secondary.

Similarly the voltages may be replaced by a single voltage, either ML (Fig. 17.08) in the primary, or FN in the secondary.

These are equal in length, but drawn to the primary and secondary voltage scales respectively.

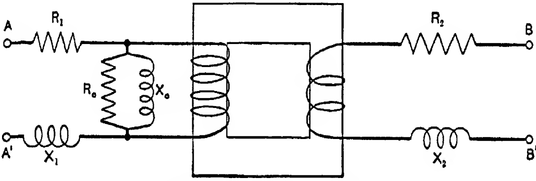
Hence the two leakage reactances may be replaced by a single equivalent leakage reactance, placed in either the primary or the secondary circuit.

If X_1 and X_2 are the primary and secondary leakage reactances then:—

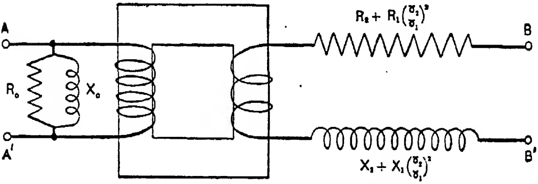
$$\text{Equivalent secondary leakage reactance} = X_2 + X_1\left(\frac{\delta_2}{\delta_1}\right)^2$$

and primary

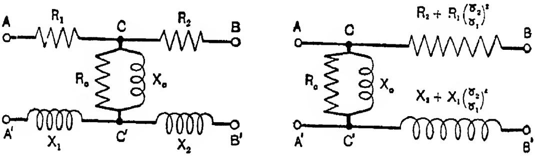
The above expressions can be proved true in the same way as in the first method of obtaining the equivalent resistances (Art. 5).



(*a*) EXACT REPRESENTATION.



(*b*) APPROXIMATE REPRESENTATION.



(*c*) (*d*)

Fig. 17.09.—EQUIVALENT CIRCUITS OF A TRANSFORMER.

8. Equivalent Circuits

Summarising the above results (Arts. 3-7) the behaviour of transformers can be represented by one of the following equivalent circuits (Fig. 17.09). Diagram (*a*) gives the exact representation. In diagram (*b*) a scheme is shown which gives results very nearly exact, sufficiently so for all practical purposes in power transformers.

A similar scheme can be made with the equivalent resistance and leakage reactance in the primary circuit instead of in the secondary. In all three cases the transformer itself is an ideal one, *i.e.* it alters the voltage in the exact ratio of the numbers of turns, and the current exactly in the inverse ratio. All the actual departures from these are given by the effects of the resistances and reactances.

A simpler equivalent circuit is shown in (*c*). The values of diagram (*a*) are treated in one of the two following ways:—

(i) The actual load current I_2 is replaced by an equivalent load current $I_2 \times \frac{\mho_2}{\mho_1}$.

Then
$$AC = R_1;\ A'C' = X_1$$
$$CC' = R_0 \text{ and } X_0 \text{ in parallel}$$
$$CB = R_2 \times \left(\frac{\mho_1}{\mho_2}\right)^2;\ C'B' = X_2 X \left(\frac{\mho_1}{\mho_2}\right)^f$$

and the P.D. across BB′ is then equal to

$$(\text{actual secondary P.D.}) \times \left(\frac{\mho_1}{\mho_2}\right).$$

Or:—(ii) The actual load current and secondary P.D. are retained; and $CB = R_2$; $C'B' = X_2$.

But $AC = R_1 \times \left(\frac{\mho_2}{\mho_1}\right)^2$; $A'C' = X_1 \times \left(\frac{\mho_2}{\mho_1}\right)^2$; and CC′ is made up of R_0 and X_0, each multiplied by $\left(\frac{\mho_2}{\mho_1}\right)^2$. Then the P.D. across $AA' = (\text{primary P.D.}) \times \frac{\mho_2}{\mho_1}$; and the current flowing at A and A′ $= I_1 \times \frac{\mho_1}{\mho_2}$.

As in the case of circuit (*a*) the latter arrangement may be replaced with little loss of accuracy by that of Fig. 17.09 (*d*), the impedance across CC′ being unchanged.

Similarly a simplified version of Fig. 17·09 (*c*) on method (i) can be made, using the equivalent primary resistance and leakage reactance.

9. Open Circuit Test

The values to be assigned to the various equivalent circuits can be obtained in an actual transformer by direct measurement.

One of the tests necessary for this purpose consists in applying to the primary winding a P.D. of the full magnitude, and of the normal frequency at which it is to work.

The secondary is left on open circuit (the test is known as the *open circuit test*), but a voltmeter may be connected to its terminals for testing the ratio of the numbers of turns. The current and power supplied to the primary are measured by an ammeter and a wattmeter. The latter gives the iron losses, since the copper losses under these conditions are negligible in the primary and nil in the secondary.

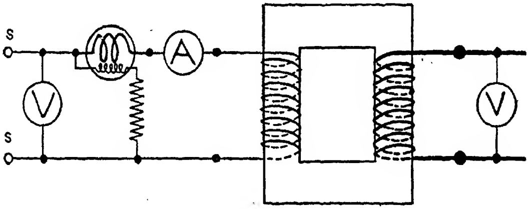


Fig. 17.10.—OPEN-CIRCUIT TEST.
S S, Supply at normal P.D. and frequency.

The ammeter reading gives the no-load current, and this can be split up into its two components by the method shown in Ex. 2, Art. 3.

Then, the equivalent open circuit impedance

$$= (\text{primary volts})/(\text{primary current}) = V_o/I_o = Z_o.$$

The resistance component (R_o) of this impedance

$$= \text{volts}/(\text{iron-loss current}).$$

And the reactance component (X_o) $= V_o/I_o \sin \Phi_o$.

Example 4. *From the figures given in Example 2, determine the equivalent open circuit impedance, etc.*

$$Z_o = \frac{2000}{0 \cdot 54}$$

$$R_o = \frac{2000}{0 \cdot 19} \quad 10\,530 \text{ ohms.}$$

$$X_o = \frac{2000}{0 \cdot 51} = 3920 \text{ ohms.}$$

10. Short-Circuit Test

The *short-circuit test* is the second of those by which the equivalent circuits can be determined. The secondary winding is short-circuited through an ammeter, which may be omitted if the ratio of the numbers of turns in the windings is known. The primary is supplied at normal frequency, but with only a small percentage of its normal P.D. This P.D. is adjusted until the full load current flows in the secondary (though other values may be used as a check). The P.D., current, and power supplied to the primary are then observed.

Since the P.D. required is small the flux is correspondingly small, and the iron losses are a still smaller percentage of their normal value and so may be neglected. The wattmeter reading therefore gives the full-load copper losses together with the power used in the secondary ammeter and its leads.

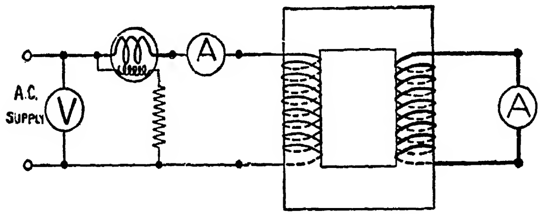


Fig. 17.11.—SHORT-CIRCUIT TEST.

On referring to Fig. 17·09 (*b*) or (*d*) it will be seen that:—

$$\text{Equivalent secondary resistance} + \text{ammeter resistance}$$

$$= \frac{\text{Watts supplied}}{(\text{Secondary current})^2}.$$

Further, owing to the reduced value of the no-load current at the reduced P.D., the angles of lag in the primary and secondary are very nearly equal. The value of the primary angle of lag can be obtained from the readings, and used to find the equivalent secondary leakage reactance.

Example 5. *The transformer of Example* 2 (40 *kVA*. 2000^V *to* 200^V) *with its secondary short-circuited through an ammeter of* ·0012 *ohm resistance, including that of the leads, carries* 200 *A. in its secondary when a P.D. of* 78 *volts is applied to its primary which takes* 20 *A. and* 370 *watts. Find its equivalent secondary resistance and leakage reactance.*

Equivalent secondary resistance + ammeter resistance

$$= \frac{370}{(200)^2} = \cdot 00925 \text{ ohm};$$

$\therefore$ Equivalent secondary resistance $= \cdot 00925 - \cdot 0012 = \cdot 00805$ ohm.

$$\text{Cos } \phi_1 = \frac{370}{78 \times 20} = 0{\cdot}237;$$

$$\therefore \phi_1 = 76{\cdot}3°; \qquad \therefore \tan \phi_1 = 4{\cdot}10$$

$$\text{(or directly:—}\tan \phi_1 = \frac{\sqrt{1 - \cos^2 \phi_1}}{\cos \phi_1} = \frac{\cdot 972}{\cdot 237} = 4{\cdot}10);$$

$\therefore$ Equivalent secondary leakage reactance $= 4{\cdot}10 \times \cdot 00925 = \cdot 0379$ ohm.

[N.B.—The equivalent primary values are $\left(\frac{2000}{200}\right)^2$ times as great, *i.e.* $\bar{R}_1 = \cdot 805$ ohm, $\bar{X}_1 = 3{\cdot}79$ ohms. The smallness of these compared with R_0 and X_0 (see Ex. 4, Art. 9) shows how little inaccuracy is introduced by neglecting the effects due to the no-load current flowing through the primary winding, *i.e.* by using the (*b*) or (*d*) equivalent circuits in place of the (*a*) or (*c*) ones in Fig. 17.09.]

11. Efficiencies

The losses in a transformer consist of (*a*) the copper losses in the two windings, and (*b*) the hysteresis and eddy current losses in the iron. The total under (*a*) is given by the short circuit test, and includes the loss due to eddy currents in the copper. This method of testing is, therefore, preferable to measuring the resistance by D.C. methods, since the additional loss due to these eddy currents frequently amounts to as much as 15 per cent. of the pure resistance loss.

The total iron loss is given by the open-circuit test, and remains practically constant at all loads when the primary supply is at constant P.D.

[N.B.—The approximate constancy of the iron losses arises from the fact that, with constant applied P.D., the E.M.F.s are constant apart from the small resistance drop, and hence the magnetic flux is very nearly constant (see Art. 2).]

The efficiency at all loads can, therefore, be calculated as shown in the following example:—

Example 6. *Find the efficiency at* 1, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, *and* $\frac{1}{10}$ *full load of a* 30 *kVA. transformer whose full load total copper losses are* 460 *watts, and total iron losses* 220 *watts,* (*a*) *on non-inductive loads,* (*b*) *on loads for which cos* $\phi = 0{\cdot}6$.

(*a*) The iron losses are constant, and the copper losses vary as the square of the load, thus:—

At	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$ full load
Copper losses =	460	259	115	29	5 watts
Total losses =	680	479	335	249	225 watts
Output	30 000	22 500	15 000	7 500	3 000 watts

$$\text{Efficiency} = \frac{\text{output}}{\text{output} + \text{losses}} = 1 - \frac{\text{losses}}{\text{output} + \text{losses}}$$

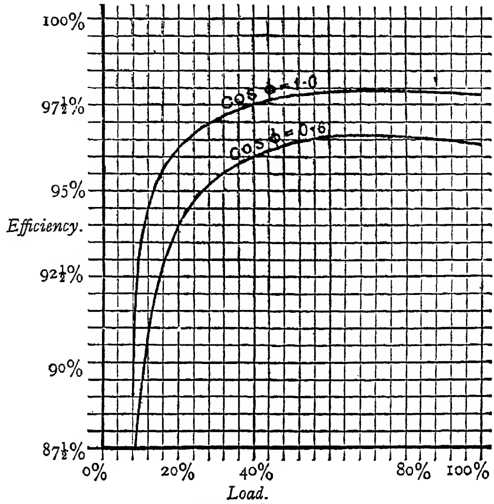


Fig. 17.12.—TRANSFORMER EFFICIENCIES.

giving:—97·8 per cent., 97·9 per cent., 97·8 per cent., 96·8 per cent., and 93·0 per cent. respectively.

(*b*) The losses at each fraction of full load remain as above, but the outputs in watts are reduced to 0·6 of former values, viz. to 18 000, 13 500, 9 000, 4 500, and 1 800 watts respectively;

∴ Efficiencies = 96·4 per cent., 96·6 per cent., 96·4 per cent., 94·8 per cent., and 88·9 per cent.

From these results the efficiency curves have been plotted in Fig. 17.12.

It will be noted that in the above example the maximum efficiency occurs at a load below full load. The efficiency of any transformer may be written:—

$$\eta = \frac{V_2I_2\cos\phi_2}{V_2I_2\cos\phi_2 + I_2^2\bar{R}_2 + W_0} \quad \text{(putting } W_0 \text{ for iron loss in watts)}$$

$$= 1 - \frac{I_2^2\bar{R}_2 + W_0}{V_2I_2\cos\phi_2 + I_2^2\bar{R}_2 + W_0}$$

$$= 1 - \frac{I_2\bar{R}_2 + \frac{W_0}{I_2}}{V_2\cos\phi_2 + I_2\bar{R}_2 + \frac{W_0}{I_2}}.$$

Since $\frac{W_0}{I_2} \times I_2\bar{R}_2 = W_0\bar{R}_2 =$ constant, the numerator of the fraction is a minimum when $\frac{W_0}{I_2} = I_2\bar{R}_2$ (cf. Chap. XVIII., Art. 10), *i.e.* when $W_0 = I_2^2\bar{R}_2$, *i.e.* when the copper losses are equal to the iron losses. And V_2 is a constant, therefore the maximum efficiency for a fixed value of the power-factor occurs nearly at the load which makes the copper losses equal to the iron losses.

In the above example the two losses are equal at the fraction ($\sqrt{\frac{220}{460}} =$) 0·69 of full load = 20·7 kVA. This gives an efficiency of

$$\frac{20700}{20700 + 220 + 220} = 97{\cdot}92$$

per cent. at unity power factor; *i.e.* very slightly greater than the efficiency at $\frac{3}{4}$-load.

12. Inherent Regulation of a Transformer

The *inherent regulation* of a transformer is the change in the secondary terminal P.D. between no load and full load, with constant P.D. applied to its primary terminals.

This is expressed usually as a percentage of the secondary P.D. at no load.

On considering the equivalent circuits shown in Fig. 17.09 (*b*), it will be seen that the regulation at any power factor can be obtained from a vector diagram as shown in Fig. 17.13. OA represents the full-load secondary current, OB the secondary terminal P.D.

BC $= I_2\overline{R}_2$ and is parallel to OA, and so represents the voltage drop across the equivalent secondary resistance. CD $= I_2\overline{X}_2$, and is perpendicular to OA, and so represents the voltage drop across the equivalent secondary reactance. OD, the vector sum of OB, BC, and CD, then gives the total E.M.F. in the secondary equivalent circuit. Since the primary P.D. is kept constant, OD has a constant known value. The regulation is the *arithmetical* difference between OD and OB.

If OA is kept fixed and ϕ_2 varied, D lies on a circle with O as centre. BC and CD remain constant in length and move parallel to themselves, consequently B lies on an equal circle with centre at O′, where OO′ is equal and parallel to DB.

These two circles are drawn in Fig. 17.14. If OB is produced to meet the other circle in E, then OE = OD, therefore BE = OD − OB = the regulation. Thus the variation of regulation with angle of lag can be found by drawing OBE in various positions.

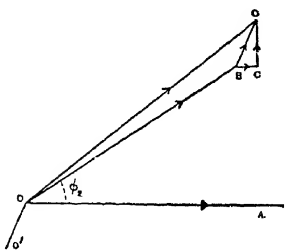


Fig. 17.13.—REGULATION DIAGRAM.

An inspection of the figure shows that as the angle of lag increases the regulation increases, reaching a maximum when OB is in line with O′O, *i.e.* when ϕ_2 = internal angle of lag (∠ DBC).

This maximum value is equal to O′O or BD. For still greater angles of lag the regulation diminishes again.

With leading currents the regulation diminishes as the angle of lead increases, and becomes zero when $\phi_2 = \angle$ AOF. Beyond this the regulation becomes *negative, i.e.* the terminal P.D. rises when the load is applied. This somewhat surprising result is due to the capacitance of the load counteracting the reactance effect of magnetic leakage, and so producing a certain amount of resonance. With a purely capacitance load the amount of this increase of P.D. is very nearly equal to DC.

The triangle OFO′ is isosceles and so the perpendicular from F on to OO′ bisects the latter. Therefore $\angle$ AOF is a little greater than $\angle$ CDB.

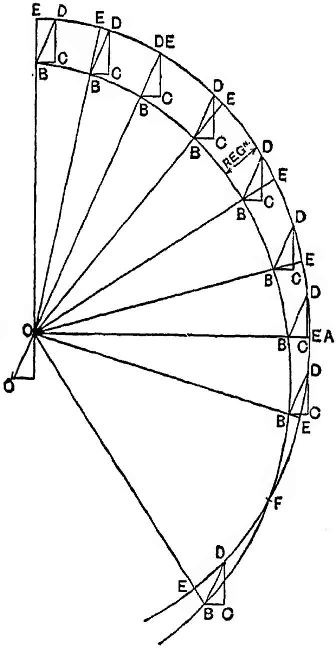


Fig. 17.14—TWO-CIRCLE DIAGRAM FOR REGULATION.

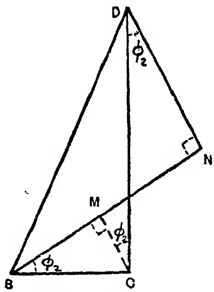


Fig. 17.15.—APPROXIMATE REGULATION.

I.e. the regulation is zero for an angle of lead which is equal to (90° − internal angle of lag), approximately.

13. Approximate Method for Regulation

The two-circle construction is valuable for showing the general way in which regulation varies with ϕ_2. But since in the usual type of commercial transformer BC and CD are very small compared

with OB, the construction is not suitable for obtaining the regulation accurately.

If BD is small in comparison with OD, the arc DE may be replaced with very little error by the perpendicular DN from D on to OB produced (Fig. 17.15). BN then gives the regulation. As ϕ_2 changes, N moves on the circle whose diameter is BD. By drawing this circle the regulation can be found readily for various angles of lag.

If CM is drawn perpendicular to BN,

$$\text{Regulation in volts} = \text{BN} = \text{BM} + \text{MN} = \text{BC} \cos \phi_2 + \text{CD} \sin \phi_2$$
$$= I_2 (\overline{R}_2 \cos \phi_2 + \overline{X}_2 \sin \phi_2).$$

Or- $\quad$ Percentage regulation $= r \cos \phi_2 + x \sin \phi_2$,

where $\quad r =$ percentage resistance drop $= \dfrac{I_2\overline{R}_2 \times 100}{V_2}$,

$x =$ percentage reactance drop $= \dfrac{(I_2\overline{X}_2 \times 100)}{V_2}$.

Example 7. *Plot the regulation against the power factor for a transformer with a resistance drop of $2\frac{1}{2}$ per cent., and a reactance drop of 4 per cent.*

Cos ϕ	Sin ϕ	2·5 cos ϕ	4 sin ϕ	Regulation Per Cent.
1·0	0	2·5	0	2·50
0·95	0·312	2·375	1·248	3·62
0·9	0·436	2·25	1·74	3·99
0·8	0·60	2·00	2·40	4·40
0·6	0·80	1·50	3·20	4·70
0·4	0·917	1·0	3·67	4·67
0·2	0·98	0·5	3·92	4·42
0	1·0	0	4	4·00

Sin ϕ can be obtained from a table of sines and cosines, or by using the relationship: $\sin \phi = \sqrt{1 - \cos^2 \phi}$.

The maximum regulation (see Art. 12) occurs when $\tan \phi = \dfrac{4}{2{\cdot}5} = 1{\cdot}60$.

This makes $\cos \phi = {\cdot}530$, and $\sin \phi = {\cdot}848$;

$$\therefore \text{ Regulation} = 2{\cdot}5 \times {\cdot}530 + 4 \times {\cdot}848$$
$$= 4{\cdot}72 \text{ per cent.}$$

Or alternatively: $\quad$ Regulation $= \sqrt{\{(2{\cdot}5)^2 + 4^2\}}$
$$= \sqrt{22{\cdot}25}$$
$$= 4{\cdot}72 \text{ per cent.}$$

These results are plotted in Fig. 17.16.

Since the reactance drop is always greater than the resistance drop, the maximum error due to the use of the approximate formula occurs when

$\phi = 0$. By drawing the vector diagram for this case, and using the Binomial Expansion, it can be shown that a closer approximation is given by:—

$$\text{Regulation at unity power-factor} = \left(r + \frac{x^2}{200}\right) \text{ per cent.}$$

E.g. in the above example the regulation for $\cos \phi = 1$, is 2·58 per cent.

14. Test for Transformers for Parallel Working

When two or more transformers are to be connected in parallel, it is necessary for satisfactory working that their ratios of transformation shall be the same; and, further, that their full load

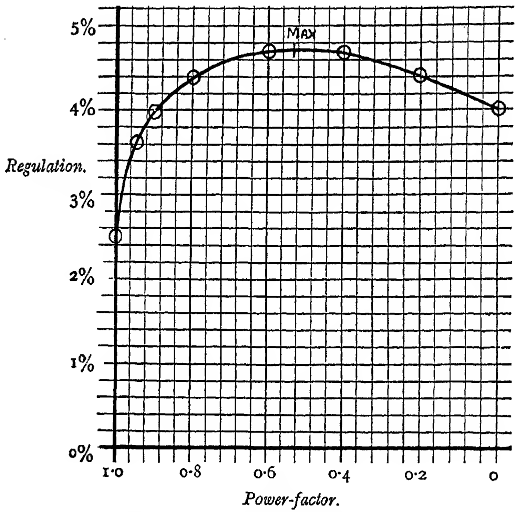


Fig. 17.16.—VARIATION OF REGULATION WITH POWER-FACTOR.

regulations shall be equal at all power-factors. If the latter condition is not satisfied the load will not be divided correctly between the transformers, those with large values of regulation taking less than their share if the no-load voltage ratios are equal.

If the no-load ratios differ, the transformer with the higher secondary voltage will supply power to the secondary winding of the other transformer, whose primary will return part of the additional power taken by the primary winding of the first transformer. These circulating currents cause additional copper losses.

The closeness with which the above conditions are satisfied in a pair of transformers, whether of the same or of different outputs, can be tested by connecting them as shown in Fig. 17.17.

Their primary windings, (P_1, P_2) are connected in parallel to a supply at normal P.D.; thus any small variations in this affect both transformers equally. The secondary windings (S_1, S_2) are connected to equal loads. One terminal of S_1 is connected to the corresponding terminal of S_2. The other two terminals are joined to a low-reading voltmeter, which thus gives directly the difference between the two secondary P.D.s. Care must be taken not to connect one of the secondaries the reverse way, as in this case the sum of the P.D.s will be applied to the voltmeter.

The reading of the voltmeter is taken with both secondary switches open. If the transformation ratios are equal the reading

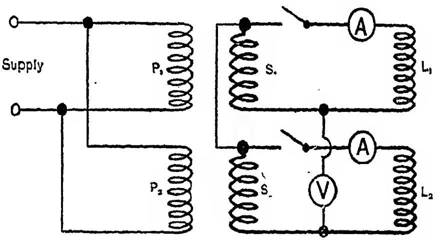


Fig. 17.17.—TEST FOR EQUALITY OF RATIOS.

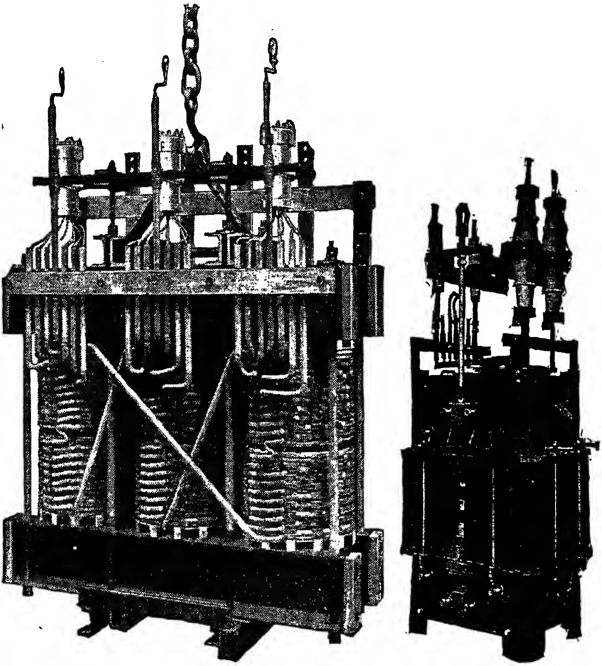
will be zero, and in other cases the reading shows the extent of the difference between the ratios.

The experiment is then repeated with the switches closed and equal loads (L_1, L_2) on the secondaries. Both non-inductive and highly-inductive loads should be used, and the voltmeter readings again show any differences in the transformation ratios.

The same connexions may be used to measure the amount of the regulation of either transformer. If the voltmeter reads zero with both switches open, then its reading with one only of the switches closed gives the regulation of the loaded transformer. It is important to notice, however, that the voltmeter gives the vector difference between the two secondary P.D.s, and so it is the maximum value of the regulation that is obtained, *i.e.* BD in Figs. 17.13 and 17.15.

If the open circuit reading is not zero, a correction must be made for the difference between the two open-circuit P.D.s. When the

loaded transformer gives initially a higher P.D. the correction is additive, and when it gives the lower open-circuit P.D. the voltmeter reading on load must be reduced to obtain the regulation. A simple way of testing which is to be done in any particular case is to apply the load gradually. If the voltmeter reading diminishes



(a) CORE TYPE BY FERRANTI. (b) SHELL TYPE (METROPOLITAN VICKERS).

Fig. 17.18.—TYPES OF TRANSFORMERS.

first and then increases, an addition must be made to the final reading; whereas if it steadily increases with the load, the final reading is higher than the regulation.

Since the open circuit P.D.s are in phase, whereas the P.D. of the loaded transformer is out of phase with the P.D. of the unloaded one, the correction must be made vectorially, not arithmetically.

This requires a knowledge of the angle of lag (ϕ) of the load current, and the internal angle of lag (θ), viz. $\angle$ BDC in Figs. 17.13 and 17.15. If these are known it can be shown that approximately:—

$$\text{maximum regulation} = V_L \pm V_0 \cos(\theta - \phi),$$

where V_L = voltmeter reading with one transformer on load,

V_0 = voltmeter reading with both transformers on open circuit.

The positive or negative sign is determined by the method given above.

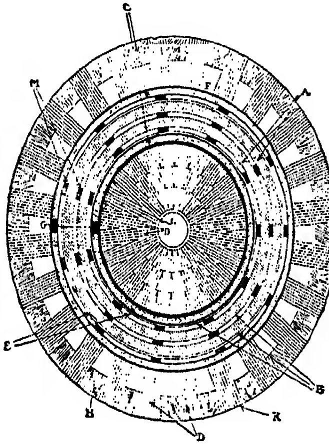


Fig. 17.19.—BERRY TRANSFORMER.

A, High pressure coils. B, Ventilating ducts. C, Low pressure coils.
DD, Iron stampings. E, "Earth" shields F, Intercore ventilating spaces.
G, Leads. H, Core spacing blocks. K, Core band.
L, Internal core-ring.

15. Types of Transformers

Transformers may be divided into two main types, according to their method of construction. In the core type the magnetic circuit is to a large extent covered by the coils, which are themselves mostly freely exposed [see Fig. 17.18 (*a*)].

In the shell type the reverse is the case, *i.e.* the iron of the magnetic circuit covers a large part of the coils, and is itself mostly freely exposed [see Fig. 17.18 (*b*)].

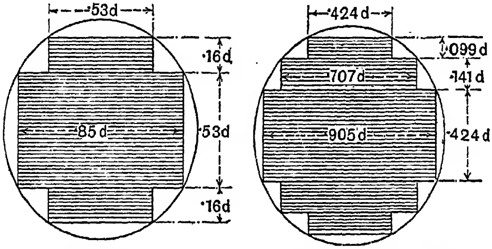
Further sub-divisions may be made according as the electric circuit is circular or rectangular, and according as the magnetic

circuit is circular or rectangular (this does not refer to the cross-sections of the circuits).

Thus Fig. 17.18 (*a*) is a rectangular core type with circular coils; and Fig. 17.18 (*b*) is a rectangular shell type with rectangular coils. Fig. 17.19 is a rectangular shell type with circular coils (Berry transformer).

A simple iron ring of rectangular section, with wires wound directly on it, is a circular core type with rectangular coils.

Another distinction may be made according to the arrangement of the coils. These may be either one inside the other ("concentric"), or one over the other so as to give the same mean length of turn. In this case they are often divided into several portions placed alternately, and are then called "pancake coils" from their shape.



Area occupied = ·62d^2. (*a*) CRUCIFORM.

Area occupied = ·67d^2. (*b*) THREE-STEPPED.

Fig. 17.20. –STEPPED CORES.

Even with concentric coils the high pressure winding is frequently divided into a number of sections with insulation between. The object of this method of construction is to reduce the voltage between successive layers, by reducing the numbers of turns in each layer. In this way the necessity that would otherwise arise of putting insulation between successive layers, in addition to the covering of the wires, is avoided, and in suitable cases space can be saved and a cheaper design produced.

16. Transformer Cores

The *core* is built up of thin laminated plates to reduce the eddy current loss. If these are of pure iron, the thickness for fifty cycles per sec. is about 14 mils (0·35 mm.), but when alloyed iron (see Chapter XII., Art. 3) is used, the thickness may be increased

to 20 mils (0·5 mm.). For 25 $\sim$ these may be increased somewhat, say to 18 mils (0·45 mm.), and 24 mils (0·6 mm.), respectively.

In the *core type* a rectangular section of core is most convenient, but circular coils are easier to wind, and give the shortest length of turn for a given area enclosed. But a rectangular section core inside a circular coil cannot occupy more than $\left(\frac{2}{\pi} =\right)$ ·637 of the space available, this being the fraction occupied by a square core.

A compromise often employed consists in building up the core of plates of two different widths, thus forming a *cruciform* core. Fig. 17.20 (*a*) shows the best dimensions to adopt for this; when these are used the space occupied (including that of the insulation between laminations) is ·79 of the total available, *i.e.* nearly 25 per cent. increase over that occupied by a square core.

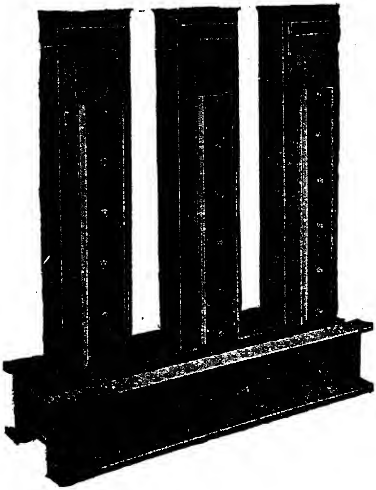


Fig. 17.21.—IMBRICATED THREE-PHASE CORE WITHOUT COILS AND YOKE.

A 3-stepped core [Fig. 17.20 (*b*)] of the dimensions shown occupies ·85 of the available space, so that usually the additional iron is not worth the further complication produced, except in very large transformers.

Generally the core is built in two parts, so that the coils can be wound separately and placed in position on the built-up core. The second part of the core is then added to complete the magnetic circuit, and the two parts clamped together. The joint may be a simple " butt " one, but is much better if " imbricated," *i.e.* with alternate plates overlapping (see Fig, 17.21 and 17.22).

In Fig. 17.21 the core is a 3-stepped one, and so there are three different heights of plates. But in addition each alternate plate rises only to the *bottom* of the yoke. Similarly the yoke is built up

alternately of strips going across the full breadth, and of two pieces fitting between the cores. In Fig. 17.22 an imbricated yoke is shown in position: the coils in this case are rectangular, but the core is stepped slightly to round off the corners of the coils.

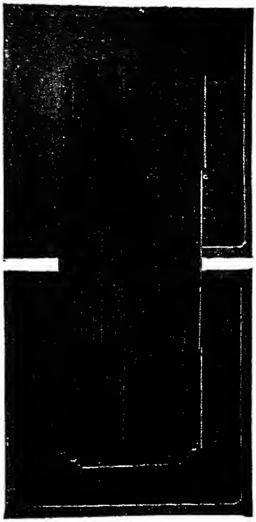
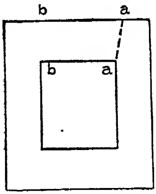


Fig. 17.22.—COMPLETED MONO-PHASE TRANSFORMER.

An alternative method sometimes used for small sizes is to cut through the core plates at *aa* (Fig. 17.23), and bend back the portion *aabb*. Each plate is then threaded through the coils, and the bent portion then straightened. The alternate plates are put through with their sides reversed, so that the joints *aa* come alternately right and left.

In some cases the yokes, *i.e.* the top and bottom portions of the iron, are made of larger cross-section than the limbs (*e.g.* double). This reduces the losses in the yokes, and enables the limbs to be run at a higher flux-density than with uniform cross-section without causing excessive iron loss or excessive magnetising current. The extra output thereby obtained makes the extra cost of the iron worth while in most cases.

The flux-densities usually employed, with oil-cooling and normal construction, are 12 kilolines to 14 kilolines per sq. cm. for 25 ~, and 10 kilolines to 12 kilolines per sq. cm. for 50 ~, the higher values being used for the larger sizes.



17. Details of Shells

The shape of the iron used for the *shell type* is shown in Fig. 17.24, thicknesses similar to those for cores being used.

The dimension *e* is half of *d* for equal flux-density, since the flux in the latter divides into two, half circulating round each "window." If *e* is made equal to *d* a result is obtained similar to that due to

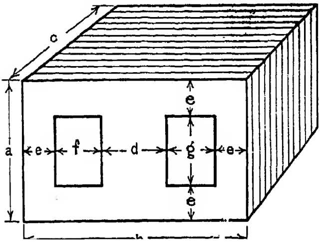


Fig. 17.24.—IRON OF SHELL TYPE.

using yokes of double the limb cross-section in the core type (Art. 16).

When $e = \frac{1}{2}d$ the following relationships between the dimensions hold good:—

Usually a is about $\frac{2}{3}b$, and b is between $3d$ and $4d$; f is $\frac{1}{2}d$ to d, and g is d to $1\frac{1}{2}d$, the larger "windows" being used with alloyed iron. The dimension c varies more in different designs but generally lies between $2d$ and $4d$.

By making the shell of E-shaped stampings and straight strips the coils can be wound separately. This gives two joints in each magnetic circuit, as in the core type with one detachable yoke. If the straight piece is put alternately at the top and at the bottom the joints are all imbricated, but it is difficult to detach the two portions of the shell without disassembling one part. An alternative is to make the plates as shown in Fig. 17.25 and to put the long leg of the E alternately left and right. The outer joints are then imbricated, leaving only the central one a butt joint, and the E's can be assembled before placing the coils in position, finishing by putting the assembled straight strips in position and clamping the two parts together.

Another method is to cut through the shell-plates in line with the sides of the central piece and to bend back the two portions above the windows. After placing the plates in position on the completed coils the bent parts are straightened. If imbricated joints are wanted the cuts must be put alternately at the top and bottom. This is easier to assemble than the corresponding construction for the core type since the bent portions do not have to be passed through the coils.

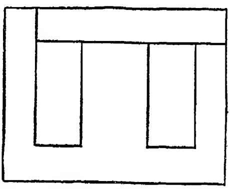


Fig. 17.25.—SHELL PLATES.

In both shell and core types, if the clamping bolts pass through holes in the iron the holes should be of considerably larger diameter than the bolts, to reduce eddy currents in the latter. For the same reason the clamps, if of magnetic material, should be magnetically insulated from the

core by non-magnetic packing pieces. Frequently the bolts are electrically insulated too as a further precaution.

18. Methods of Cooling

The maximum output of a transformer is limited almost entirely by the rate at which it can dissipate the heat produced by its losses. Now suppose all the linear dimensions of a certain transformer (A) to be increased in the ratio b, keeping the flux-density and current-density constant. The new transformer (B) thus produced will have its total flux and its currents increased to $b^2 \times$ former values. The voltage at the same frequency will be increased in this same ratio, and so the output of B is $b^4 \times$ output of A.

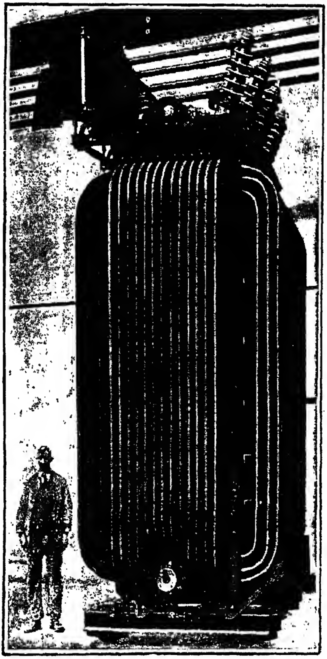


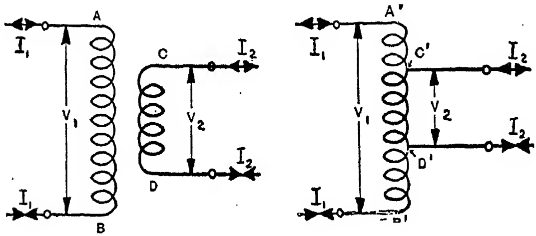
Fig. 17.26.—FERRANTI TRANSFORMER WITH COOLING PIPES.

But the iron losses, being proportional to the volume of the iron, will be only b^3 times as great as before. The windings are b times as long, and b^2 times greater in section and therefore $(1/b)$ of former resistance. So the I^2R losses are increased in the ratio $(b^2)^2 \times \frac{1}{b} =$ The efficiency of B is therefore higher than that of A, the percentage loss being only $(1/b)$ of former value.

On the other hand the surfaces through which the heat must be dissipated increase only as b^2. Therefore if the methods of cooling are the same the larger transformer will have a larger temperature rise. Hence as the size increases more elaborate methods of cooling become more valuable and desirable. The alternative is to use a larger core and windings, but this would be more costly.

For small transformers natural air cooling suffices. As the size increases, ducts are provided in both the core and the coils. Another step is to provide fan cooling, but the more usual method is to immerse the transformer in a tank filled with oil. This has the advantage of improving the insulation as well as increasing the cooling. With larger sizes still the case is ribbed, or provided with vertical pipes (see Fig. 17.26), to increase its dissipating surface. In this way self-cooled transformers can be made up to outputs of 3 000 kVA. where cooling water is not available or is too expensive. But the usual limit of output for this type is about 2 000 kVA.

For the largest sizes, either the oil is pumped through a series of pipes cooled by water, or by air; or water is circulated through a coil of pipes placed in the top of the tank, thus cooling the heated oil as it rises from the transformer. As an additional aid to several



(a)

Fig. 17.27.—COMPARISON OF ORDINARY AND AUTO-TRANSFORMER.

of these methods the circulation of the oil may be assisted by a pump or fan, so as to carry off the heat more rapidly than when the circulation depends only on the difference between the densities of the cold and the hot oil.

19. Auto-Transformers

By combining the secondary of a step-down transformer with part of the primary the arrangement shown diagrammatically in Fig. 17·27 (*b*) is reached. This is known as a one-coil transformer or auto-transformer.

It transforms the current and voltage in the same way as the two-coil arrangement, and effects a saving of material and a reduction of losses compared with the latter.

If transformers of the two types are used to effect the same transformation, and have cores of the same size, run at the same

flux-density, there will be no change in the iron losses. The voltage per turn will be the same, and so the total number of turns in the auto-transformer (A′B′) will be equal to the number of turns in the primary (AB) of the two-coil transformer; and the numbers of turns in the two secondaries (CD and C′D′) will likewise be equal.

Let $m = V_1/V_2 =$ (No. of turns in AB)/(No. of turns in CD).

Then the number of turns in the parts of the auto-transformer which act *only* as primary, viz. A′C′ and D′B′,

$$= \left(1 - \frac{1}{m}\right) \times \text{(No. of turns in AB).}$$

Since these carry equal currents the same cross-section of wire is suitable. Assuming that the mean length of turn is the same since the cores are equal:—

Volume of copper in A′C′ and D′B′

$= \left(1 - \frac{1}{m}\right) \times$ (vol. of copper in primary (AB) of two-coil transformer).

Again, the current carried by C′D′ is the vector sum of I_1 and I_2. But since these are nearly opposite in phase, the vector sum is nearly equal to the arithmetical difference $(I_2 - I_1)$.

Hence if equal current densities are used the ratio

(cross-section of C′D′/cross-section of CD) $= (I_2 - I_1)/I_2 = 1 - \frac{1}{m}$.

The numbers of turns in these two are equal and so:—

Volume of copper in C′D′

$= \left(1 - \frac{1}{m}\right) \times$ (vol. of copper in secondary (CD), of two-coil transformer).

Combining the above two results:—

Total volume of copper in auto-transformer

$$= \left(1 - \frac{1}{m}\right) \times \text{(total vol. of copper in two-coil transformer),}$$

i.e. saving of copper $= \frac{1}{m}$ of total for two-coil.

The copper losses in A′C′ and D′B′ are similarly equal to $\left(1 - \frac{1}{m}\right)$ of the losses in the primary (AB).

The resistance of C′D′ is greater than that of the secondary (CD) in the ratio $1\Big/\left(1 - \frac{1}{m}\right)$ owing to the smaller section of the

former. But as it carries only $\left(1 - \frac{1}{m}\right)$ of the current in CD its I^2R loss is the same fraction of the loss in CD.

Hence total copper loss in auto-transformer

$$= \left(1 - \frac{1}{m}\right) \times \text{total copper loss in two-coil transformer.}$$

Actually the saving of copper and the reduction in copper losses would be somewhat greater, owing to some diminution in the length of the mean turn. Instead, however, of confining the saving to the copper, it is better to reduce the iron and the iron losses and save less copper. In fact an auto-transformer should be designed as a two-coil transformer of primary P.D. equal to $(V_1 - V_2)$, and of secondary current equal to $(I_2 - I_1)$. Its cost therefore is the same as that of a transformer of an output equal to $\left(1 - \frac{1}{m}\right)$ of the actual output.

The auto-transformer principle can be used for step-up transformers too, and effects the same saving if m is in this case taken as the ratio (Secondary volts/Primary volts).

The one disadvantage of auto-transformers is that the high pressure primary supply is connected to the secondary, and so a dangerous shock may be given on the low pressure side in case of a breakdown, *e.g.* if a break occurs in the winding C′D′. The likelihood of this occurring in ordinary transformers is very much less; and it can be entirely obviated in them by the use of an " earth shield." This consists (see Fig. 17·19) of a metallic sheet placed between the primary and secondary windings. The shield is connected to the core, and so to earth. A breakdown in the insulation between the windings is thus prevented from raising the potential of the low pressure side under all conditions.

QUESTIONS ON CHAPTER XVII

1. What difference is there between the action of a transformer and of a choking coil, and what are their relative advantages?

2. The full-load output of a transformer is 80 kVA., its primary P.D. 6 000 volts, and its secondary P.D. 250 volts. If it has 35 secondary turns, calculate the number of primary turns, and the primary and secondary currents at full load, neglecting losses.

3. The sectional area of the core of a transformer is 25 sq. cm. of which 90 per cent. is iron. There are 200 turns in the secondary winding which is required to produce 102 volts at 50 cycles per second on open circuit, and there are 2 100 turns in the primary winding. Calculate (*a*) the P.D. applied to the primary terminals; (*b*) the maximum flux-density, assuming it uniform over the iron cross-section of the core.

4. A 20 kVA. transformer with a primary P.D. of 1 000 volts and a secondary P.D. of 200 volts has an iron loss of 250 watts, and takes 0·70 A. with the secondary on open circuit. Calculate the primary currents at 1 per cent., 3 per cent., 6 per cent., and 10 per cent. of full load (*a*) on non-inductive load, (*b*) on loads of power-factor 0·7.

Plot these currents and $\frac{1}{5}$ of the secondary currents against the load, and state any deductions which can be drawn from the curves.

5. Upon what factors does the no-load current of a transformer depend? Treat the magnetising and iron-loss currents separately, and state how the total no-load current depends on these two.

6. A 50 kVA. transformer supplied at 2 000 volts gives 230 volts at its secondary terminals on open circuit. The resistances of the windings are:—primary 0·62 ohm; secondary 0·0078 ohm.

Calculate the resistance drop in each winding at full load (neglecting the effect of the no-load current).

Find the regulation at full non-inductive load.

What resistance (*a*) in the secondary circuit, (*b*) in the primary circuit, would give the same regulation?

7. A 20 kVA. transformer, 2 000 volts to 230 volts, gave the following results with its secondary short-circuited:—Primary current = 10 A.; P.D. = 64 volts; power = 350 watts. Find the equivalent secondary resistance and reactance; and thence obtain the regulation at power-factors of 1 and 0·5.

8. Draw a diagram of connexions for a short-circuit test on a transformer.

The following readings were taken in such a test on a transformer for 4 kVA., 2 000/200 volts:—secondary current 20 amp.; primary current 2·1 amp.; primary P.D. 200 volts; primary power 150 watts.

The resistance of the secondary ammeter and its leads is 0·06 ohm. Find the equivalent secondary resistance and (leakage) reactance. Hence determine the regulation for power-factors of unity and 0·6.

9. What is the effect of magnetic leakage in transformers, and why is it more important when the load is inductive?

If a 2 000 volt, 20 kVA. transformer has an "equivalent primary resistance" of 4·2 ohms, and an "equivalent primary leakage reactance" of 7·6 ohms, calculate the percentage secondary drop at full load (*a*) $\cos\phi = 1$, (*b*) $\cos\phi = 0{\cdot}6$.

10. A 20 kVA. transformer, 2 000 to 220 volts, has a primary resistance of 2·1 ohms, and a secondary resistance of 0·026 ohm. The total iron losses are 200 watts.

Calculate the efficiency:—

(*a*) at full, $\frac{3}{4}$, $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{10}$ load, with a lighting (non-inductive) load;

(*b*) at the same loads with $\cos\phi = 0{\cdot}7$.

(*c*) Plot efficiency curves for the two cases.

11. A 40 kVA. 2 000 volts transformer took 600 watts, with the secondary on open circuit giving an E.M.F. of 115 volts. With the secondary short-circuited the primary took 20 A. at 60 volts and used 800 watts.

Find the regulation and the efficiency at full load with a power-factor of 0·8. Determine further the load which gives maximum efficiency at this power-factor, and the value of this efficiency.

12. Explain how the secondary drop of a transformer on any load can be found from tests which require little power.

13. A 10 kVA. transformer 6 000/400 volts with its secondary short-circuited takes 1·67 A. at a pressure of 285 volts and absorbs 195 watts. Plot the full-load regulation against the power-factor of the load.

14. Describe two tests from which the efficiency of a transformer can be obtained without putting it on a load. Explain how the following can be deduced from the results of the tests:—

(*a*) The efficiency at any load, $\cos \phi = 1$.

(*b*) The efficiency at any load, $\cos \phi$ less than 1.

(*c*) The load for which the efficiency is maximum.

(*d*) The secondary drop at full load, and any power-factor.

15. Describe the methods of performing three different tests on transformers, and the utility of the results of each test. Define the "equivalent secondary resistance" of a transformer, and point out its usefulness.

16. A transformer giving 120 volts at its secondary terminals on open circuit has an equivalent secondary reactance of 4 ohms, and an equivalent secondary resistance of 0·5 ohm. Find graphically or by calculation the current flowing with external resistances of from 4·0 ohms down to zero. Plot current against external resistance, and against external conductance.

Calculate the total power supplied to the transformer, neglecting iron losses, and plot in the same way.

[N.B.—This shows that a transformer with large magnetic leakage will give roughly constant current on a load of variable resistance.]

17. What is an auto-transformer? Explain clearly, with vector diagrams and sketches, the distribution of currents and potential differences in such a machine. [C. & G., II.

18. A single-phase transformer has one winding with twice as many turns as the other. How would you use it as a step-up transformer with a ratio of 1 : 3? Show by curves, approximately to scale, how the currents in the two windings would vary between no load and $\frac{1}{10}$th full load.

19. A 4 kVA. transformer, 150 volts to 50 volts, has a full-load efficiency of 95 per cent. on non-inductive load. If it is used as an auto-transformer for supplying 50-volt incandescent lamps from 200-volt mains, what output can it give and what will be its full-load efficiency?

Neglect the effects of the no-load current in calculating the above, but draw to scale a vector diagram for $\frac{1}{10}$th full load showing how this modifies the currents in the auto-transformer windings if its value in the two-coil transformer is 1·7 amp. with a power-factor of 0·4.

CHAPTER XVIII

TRANSMISSION AND DISTRIBUTION

1. Introductory

By the transmitting and distributing system (or network) is meant all the cables, connexions, insulation, etc., by which the electric power is conveyed from the switchboard in the central station to the places where it is to be utilised. These cost at least as much as the central station machinery.

The conditions to be satisfied by such a system are:—

(*a*) The maximum current required must not overheat the conductors and their insulation.

(*b*) The P.D. between the mains at all points where power is used must be maintained within certain limits.

(*c*) The power wasted in it must not exceed a moderate percentage of that transmitted.

(*d*) The insulation must be efficient.

(*e*) The cost of the network must not be excessive.

Conditions (*a*) and (*d*) have been considered in Chapter III.

Condition (*b*) depends to some extent on the arrangements at the central station, *e.g.* the use of feeder boosters (Chapter XVI., Art. 2) aids the maintenance of a constant P.D.

Conditions (*c*) and (*e*) are to a large extent contradictory. For by using cables of larger cross-section the amount of wasted power is reduced but the cost is increased. Some compromise must therefore be adopted (see Arts. 12, 13).

2. "Drop" and Efficiency

"Drop" is a very important quantity in connexion with transmission. In this connexion "drop" means the difference between the P.D. between the bus-bars and the P.D. between the mains at some other place, such as the main terminals of a house service. In the simplest case of transmission, viz. by two cables

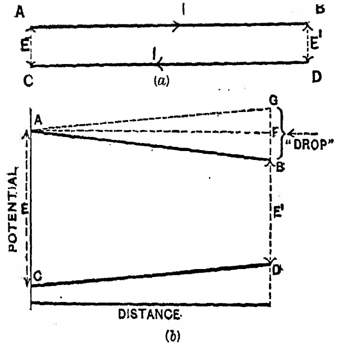


Fig. 18.01.—"Drop" in Cables.
(a) Cables, etc. (b) Potential-distance Graph.

E = P.D. at generating place.
E' = P.D. at receiving place.

AB, CD, each of R ohms resistance (Fig. 18.01) going to a single place of utilisation, the "drop" is 2IR, where I amperes is the current transmitted. For A is IR volts higher in potential than B, and C is IR volts *lower* in potential than D. Therefore E', the P.D. between B and D, is 2IR less than E, the P.D. between A and C (see Fig. 18·01). Similarly the drop at any smaller distance from AC is less in proportion to the distance.

The relation of drop to efficiency is obtained readily in such a case. For the efficiency of transmission

$$= \frac{\text{Power given out at BD}}{\text{Power taken in at AC}} = \frac{E'I}{EI} = \frac{E'}{E} = \frac{E - 2IR}{E}$$

$$= 1 - \frac{2IR}{E} = \left(100 - \frac{2IR}{E} \times 100\right) \text{ per cent.}$$

i.e. the percentage efficiency = 100 − percentage drop.

Example 1.—200 *kilowatts have to be delivered at a distance of one mile from the power station, and the pressure at the receiving station is* 1 000 *volts. What is the sectional area of the copper cable if the efficiency of transmission is* 90 *per cent.? The specific resistance of copper is* 0·69 *microhm per inch cube.*
(C. & G., I.)

The current (I) $= \frac{200 \times 1000}{1000} = 200$ amperes.

The received P.D. is 90 per cent., and therefore the "drop" is 10 per cent. of the P.D. at the power station;

$$\text{Drop} = \frac{10}{90} \text{ of } 1000 = 111 \text{ volts;}$$

$$\therefore \text{ Resistance of cable (lead and return)} = \frac{111}{200} = 0·555 \text{ ohm.}$$

Total length of cable = 2 miles;

$$\text{Resistance per inch} = \frac{0·555}{2 \times 1760 \times 36} \text{ ohm} = \frac{0·555 \times 10^6}{2 \times 1760 \times 36} \text{ microhms}$$
$$= 4·38 \text{ microhms.}$$

$$R \propto \frac{1}{\text{area}};$$

$$\text{Cross-sectional area} = \frac{0·69}{4·38} = 0·158 \text{ sq. in.}$$

3. Voltage and Distance of Transmission

If the distance of transmission is increased and no other change made, the drop is increased in the same proportion as the distance. The efficiency of transmission is reduced but not proportionately, *e.g.* doubling the distance would reduce the efficiency from 95 per cent. to 90 per cent., or from 90 per cent. to 80 per cent., etc. (cf. Art. 2).

If it is desired to maintain the efficiency at its former value two methods are available. One is to increase the cross-section of the cables in proportion to the increase of distance, thus keeping their resistance at its former value, since $R \propto \frac{l}{A}$. The drop for a given current is then the same as before, and therefore the efficiency is unaltered. If this method is adopted the amount of copper in the cables increases as the *square* of the distance, since both length and cross-section are increased.

The second method is to increase the voltage in proportion to the increase of distance. If the current and the cross-section of the cables are unaltered, then the following relations hold good:—

Drop $(= 2IR)$ varies as the distance (l).

Percentage drop $\left(= \frac{2IR}{E} \times 100 \text{ per cent.}\right)$ is unaltered, since both the numerator and the denominator are increased in proportion to l.

Power received by cables $= EI$, which is proportional to l.

Power wasted $= 2I^2R$, which varies as l;

$\therefore$ Power given out by cables $(= E'I)$ also varies as l;

$$\therefore \text{Efficiency} \left(= \frac{\text{Power given out}}{\text{Power received}}\right) \text{ is unaltered.}$$

Or alternatively:—

Efficiency $= (100 - \text{percentage drop})$ per cent.; $\therefore$ it is unaltered, since the percentage drop is unchanged.

Again:—Amount of copper in cables $= 2lA$, which varies as l.

Thus the power transmitted and the amount of copper required are both proportional to l. Or alternatively, by reducing the cross-section of the cables inversely as l, *the same amount of power can be transmitted by the use of the same amount of copper at the same efficiency* provided that, as stated above, the voltage is increased in proportion to the distance.

For a given distance and efficiency the copper required for transmitting a given power varies inversely as the square of the voltage.

This can be shown as follows:-

$$\text{Power } (= EI) \text{ is constant;}$$

Efficiency is constant; $\therefore$ percentage drop is constant;

$$\therefore \text{ actual drop} \propto E,$$

i.e. $$2IR \propto E,$$

i.e. $$R \propto E \times \frac{1}{I};$$

$$\therefore R \propto E^2.$$

But $$R \propto \frac{l}{A}, \text{ and } l \text{ is constant;}$$

$$\therefore \frac{1}{A} \propto E^2 \quad \text{or} \quad A \propto \frac{1}{E^2};$$

$\therefore$ Total amount of copper $\propto (l \times A) \propto A$, since l is constant,

i.e. ,, ,, ,, $\propto \frac{1}{E^2}$.

4. Disadvantages of High Pressure

The advantages of using high pressure for transmission are offset by the following disadvantages. The insulation must be capable of withstanding the higher voltage, and hence is more costly. The danger from a shock is increased, and so greater precautions have to be taken to prevent the possibility of such an occurrence. If the transmission pressure is more than moderate it must be reduced for distribution. With D.C. this must be done by motor-generators or other rotating machinery. But with A.C. transformers suffice, and this is the main reason for the extending use of A.C. for both transmission and distribution.

The pressure used for distribution is settled mainly by lighting considerations. In the United Kingdom 230 v. is the standard, and most systems lie within the range 200 v. to 260 v., though 300 v. has been used. In the U.S.A. about 110 v. is usual (range 100 v. to 130 v.), and a number of places in the U.K. use these too. Lower pressures are used occasionally, generally with transformers on the premises lighted. For lighting by incandescent lamps higher voltage necessitates more fragile filaments and a lower efficiency (see Chapter XV.).

With motors of low power the cost is increased, owing to the insulation occupying a greater proportion of the available winding.

space; another reason is the increased number of segments required in the commutator. For larger powers the latter does not apply, and the cost is scarcely affected by the voltage, unless this is unusually high (over 500 volts). Similarly the cost of motor starters is little affected by the voltage, the increase of resistance necessitated by a high voltage being counterbalanced by the reduction of current for a given power.

5. The Three-Wire System

In the *three-wire system* the distribution voltage is double that applied to the lamps. It thus combines to some extent the advantages of high pressure for distribution and of low pressure lamps, but naturally has certain disadvantages of its own.

There are three mains, the positive, the negative, and the *neutral*

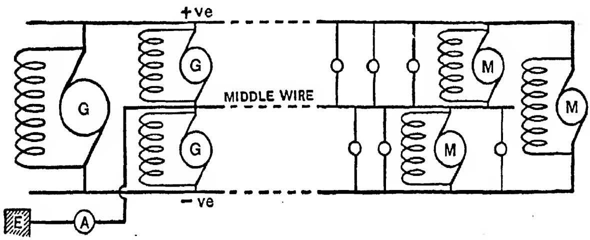


Fig. 18.02.—THE THREE-WIRE SYSTEM.

E, Earth. A, Earth ammeter. ——o—— Groups of lamps.

or *middle-wire*, instead of the usual two. The positive is maintained at a certain potential above the neutral, and the negative at the same potential below the neutral; the P.D. between the *outers* (*i.e.* the positive and negative) is therefore twice that between either of them and the middle wire. Lamps are connected between either outer and the neutral, while motors can be connected in the same way or directly across the two outers according to circumstances (see Fig. 18.02).

The two sides of the system are said to be balanced when the total current taken by the lamps, etc., connected to the positive main is the same as that taken by those connected to the negative main. In this case no current flows to or from the station by the middle wire.

But if there is a difference between the two loads they are called unbalanced, and the current in the middle wire is the

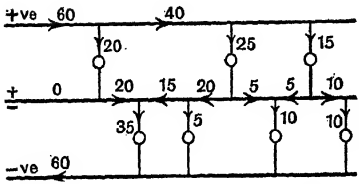


Fig. 18.03.—CURRENTS IN THREE-WIRE DISTRIBUTORS.

+ve = Positive distributor.
− ve = Negative distributor.
± = Middle-wire distributor.

difference between those in the outers, *i.e.* it is equal to the out-of-balance current.

Its direction is to or from the station according as the current in the positive is greater or less than that in the negative main. In other words, the total current away from the station is equal to the total current towards the station.

Even with balanced loads there will usually be a current in the distributing part of the middle wire; and this current may be in opposite directions in different parts of the middle wire. The truth of this can be seen by referring to Fig. 18.03, which shows a simple case of a 3-wire system with balanced loads. The figures and arrows indicate the amounts and directions of the currents in the various parts of the system.

6. Saving of Copper in 3-Wire System

Compare a 3-wire system with a 2-wire system transmitting the same power the same distance, with the same efficiency, and with the same P.D. at the lamps. If the 3-wire system is balanced the current in each outer is *half* that in each main of the 2-wire, for equal total power.

Let I = current in the 2-wire feeders in amperes,

R = resistance in ohms of each main in the 2-wire system,

and R' = resistance of each outer of the 3-wire system.

Then power wasted in 2-wire transmission $= 2I^2R$ watts.

3-wire

I^2R' watts.

For equal efficiency the above two losses must be equal;

$$\therefore\ R' = 4R,$$

or, each outer is $\frac{1}{4}$ cross-section of each of the 2-wire mains.

The middle wire is very frequently made half the cross-section of the outers. If this is the case and the *total* copper in the 2-wire is denoted by 100, the following are the sizes of the various mains:—

In the 2-wire	+ve main	50 per cent.
	−ve ,,	50 per cent.
	Total	100 per cent.

In the 3-wire	+ve main $\frac{1}{4}$ of 50 =	$12\frac{1}{2}$ per cent.
	−ve ,, $\frac{1}{4}$ of 50 =	$12\frac{1}{2}$ per cent.
Middle wire, half size of an outer =		$6\frac{1}{4}$ per cent.
	Total	$31\frac{1}{4}$ per cent.

i.e. the amount of copper required in the 3-wire system is only $31\frac{1}{4}$ per cent. (or $\frac{5}{16}$) of that needed in a 2-wire system for the same purpose.

The above assumes that the carrying capacity of the cables is not exceeded in the 3-wire system. This may occur, because the current is halved while the cross-section is diminished to one quarter. If the heating limit (see Chapter III.) comes in, the saving of copper will be diminished, but the efficiency of transmission will then be increased.

With *equal current densities,* the two outers of the 3-wire system together need 50 per cent. copper, and the middle wire $12\frac{1}{2}$ per cent., giving a total of $62\frac{1}{2}$ per cent. The percentage losses would then (with balanced loads) be half of those in the 2-wire system, and the efficiency of transmission correspondingly higher.

Example 2. *A 3-wire feeder is transmitting* 120 *A. in the positive main, and* 100 *A. in the negative. The terminal P.D. at the central station is* 460 *volts, and at the feeding point* 2 × 225 *volts.*

(*a*) *Calculate the power lost, and the efficiency of transmission.*

(*b*) *Find the resistance of a 2-wire feeder to transmit the same power the same distance, with the same efficiency and* 225 *volts at the feeding point.*

(*c*) *Compare the amounts of copper required in the two cases if the middle wire of the 3-wire feeder is half the cross-section of each outer.*

(*a*) Power given out by feeder = 120 × 225 + 100 × 225 = 49 500 watts.

$$\text{Power received by feeder} = \frac{120 + 100}{2} \times 460 = 50\,600 \text{ watts;}$$

$$\therefore \text{Power lost} = 1\,100 \text{ watts;}$$

$$\therefore \text{Efficiency of transmission} = \left(1 - \frac{1100}{50600}\right) \times 100 \text{ per cent.} = 97{\cdot}8 \text{ per cent.}$$

(*b*) Current in 2-wire feeder $= \frac{49500}{225} = 220$ amperes $(= 120 + 100)$.

For equal efficiency the power lost must be the same.

$$\therefore \text{ Resistance of each wire} = \frac{1100}{2 \times (220)^2} = 0{\cdot}01136 \text{ ohm.}$$

(*c*) Let R = resistance in ohms of each outer of the 3-wire feeder;

$\therefore$ 2R ohms = resistance of middle wire;

$$\therefore \text{ Power lost} = (120)^2 \times R + (100)^2 \times R + (20)^2 \times 2R = 1\,100 \text{ watts;}$$

$$\frac{1100}{25200} \quad {\cdot}0437 \text{ ohm.}$$

For equal lengths the amount of copper is inversely proportional to R.

Denote total copper in 2-wire feeder by 100 per cent., so that each of its wires contains 50 per cent. copper;

$$\therefore \text{ Each outer of 3-wire feeder contains } \frac{{\cdot}01136}{{\cdot}0437} \times 50 \text{ per cent.} \quad 13{\cdot}0 \text{ per cent.;}$$

$\therefore$ Middle wire contains $\frac{1}{2}$ of 13 per cent. = 6·5 per cent.;

$\therefore$ Total copper in 3-wire feeder = 13·0 + 13·0 + 6·5 = 32·5 per cent.

N.B.—This value is greater than that given in Art. 6, because the load is not balanced, but the increase is small.

7. Balancers

A 3-wire system may have its two sides supplied by independent generators; the voltage on each side can then be regulated in the ordinary way. It is, however, much more usual to connect the generators across the outers. This arrangement reduces the number of generators required for satisfactory working, though the total power is not altered. In this case means must be adopted for dealing with any difference between the loads on the two sides of the system, and this work is done by the balancer. This consists of two shunt-wound dynamos coupled together. One armature is connected between the positive and middle wires, and the other between the middle and negative mains.

In one arrangement the two field windings are connected in series across the outers (see Fig. 18.04). As long as the two sides of the system are equally loaded, both machines run as unloaded motors, taking only their no-load currents. If one side (say the positive) is more heavily loaded the P.D. on that side will fall, while that on the other side rises. The result is that the machine on

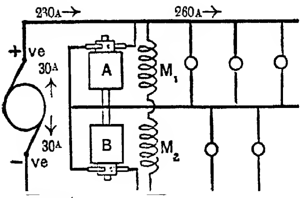


Fig. 18.04.—BALANCER FOR 3-WIRE SYSTEM.

M_1 = Field winding of A.
M_2 = Field winding of B.

the heavily loaded side becomes a generator driven by the motor action of the other machine. If there were no losses in the balancer each machine would carry half the out-of-balance (or middle wire) current, and the current in the main generators would be the mean of those in the two outers, as is shown in Fig. 18.04. At the same time the P.D.s across the two sides of the system would be maintained equal, or balanced. Owing to resistance and other losses in the balancer the above is only approximately true. *E.g.* in the above example the balancer currents might be 27 amperes in the generator and 33 amperes in the motor, and the main generators' current would then be 233 amperes. The P.D. would be lower on the positive side because the terminal P.D. of a generator is less than its E.M.F., while that of a motor is higher than its back E.M.F., and the two E.M.F.s are equal (assuming equal armature reaction effects).

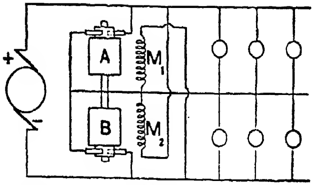


Fig. 18.05.—BALANCER WITH CROSS-CONNECTED FIELD WINDINGS.

M_1 = Field winding of A.
M_2 = Field winding of B.

8. Balancers: Booster Balancers

A better arrangement is to *cross-connect* the fields of the balancer as shown in Fig. 18.05. The effect of this is that the fall of voltage on the heavily loaded side weakens the field of the motor on the other side, and so increases its speed. Similarly the rise of voltage on the lightly loaded side strengthens the field of the generator. Both these actions increase the E.M.F. of the generator; or, more accurately, they enable the balancer to produce the necessary balancing effect with a smaller difference between the P.D.s on the two sides than with the previous method of connexion.

A third method is to use compound-wound dynamos for the balancer. The series windings* strengthen the field of the generator and weaken that of the motor. By using a suitable number of series-turns the voltages can be balanced exactly at full out-of-balance load. A larger number of series-turns will cause the P.D. on the heavily loaded side to be greater than on the other, thus

* These should be crossed, for then on heavy overload the motor field is not weakened to such an excessive extent. See further Carter, *The Electrician*, Vol. 78, p. 605.

compensating for the greater drop along the outer feeder to the heavily loaded side.

All these arrangements are reversible, *i.e.* they will balance equally well whether the heavier load is on the positive or the negative side. A balancer is in fact a motor-generator in which either machine may be the generator, according to circumstances.

When a battery is used in conjunction with a 3-wire system a **booster balancer** may be used. This consists of two reversible battery boosters each connected to half the battery. They are driven by a single motor and the middle wire is connected to the middle point of the battery. When the loads are unbalanced one half of the battery discharges and the other half is charged. The object of the boosters is to diminish the rise and fall of voltage required for a given charging or discharging current from the battery (see Chapter XVI., Art. 4).

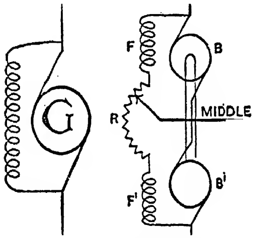


Fig. 18.06.—BALANCER WITH HAND REGULATION.

B, B′, Armatures of balancer.
F, F′, Field windings.
G, One of the main generators.

Where hand regulation (instead of automatic) can be used a convenient method of connexion is that shown in Fig. 18.06. By altering the position of the switch connecting the middle wire to the common regulating resistance one field is strengthened and the other weakened.

This may be combined with cross-connexion of the fields so as to give automatic regulation as well.

9. Comparison of 3-Wire and 2-Wire Systems

The advantage of the 3-wire system is the saving of copper effected, or alternatively the increase of efficiency and diminution of transmission drop (see Art. 6). The disadvantages are:—

(*a*) The third main increases the complication and cost of joints or other connexions.

(*b*) A balancer is required, increasing the cost and complication of the central station equipment.

(*c*) The loads on the two sides must be nearly balanced, not only over the whole system but on each section of it.

(*d*) The risk of breakdowns is somewhat increased by the doubling of the voltage.

For short distances and small powers the disadvantages outweigh the saving of copper. As the distance increases a high distribution voltage becomes more important, just as in the case of transmission (see Art. 3). As the power increases the saving effected becomes greater and makes the increased complication better worth while. No exact rules can be given as to the point at which a 3-wire system becomes advisable. A good deal depends on the ease of balancing the loads.

With 1-phase A.C. distribution, too, the 3-wire system may be used. The outers are no longer positive and negative since each is positive in turn. Compared with a 2-wire A.C. distribution it effects the same percentage saving as in the D.C. case. Compared with D.C. both A.C. systems have the disadvantage that if the load has a power factor below unity (which is usually the case with motor loads) the current for a given power is increased inversely as the power-factor. This increases the copper required for distribution and transmission, and is a disadvantage of the A.C. system.

On the other hand, balancing can be effected without rotating machinery. If the transmission is (as usual) at a higher voltage than the distribution the middle-wire can be connected to a centre-point tapping on the secondary of the step-down transformer. If the transmission and distribution (outer) voltages are the same, a balancing transformer effects the same result.

Such a balancing transformer may be described as a single-coil transformer with a centre-point tapping. It is better to consider it as a transformer with a one-to-one ratio, *i.e.* neither step-down nor step-up. The action is similar to that of the D.C. rotary balancer. When one side of the 3-wire system is more heavily loaded than the other its voltage falls and that on the other side rises. The part of the balancer winding connected across the lightly loaded side therefore acts as primary taking in power, and the part across the heavily loaded side acts as secondary giving out power. The loads are thus balanced with a difference of voltage between sides equal to the regulation of the balancing transformer with a load of about half the out-of-balance load. This action occurs automatically whichever side has the excess load.

10. Comparative Economy of Low-Pressure A.C. Systems

In the case of low-pressure transmission and distribution the P.D. on the incandescent lamps is the governing factor (see Art. 4). So that for the comparison of L.P. systems the lamp voltage will be taken to be constant.

The monophase two-wire system will be taken as the standard. If the copper in each wire is taken as 50 units this makes a total of 100, so that the copper for other cases will be expressed as a percentage of the total copper for the standard (cf. Art. 6).

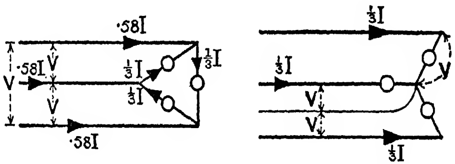
In the three-phase three-wire system with the lamps, etc., connected between two mains, *i.e.* in Δ [Fig. 18.07 (*a*)], the current in each phase of the load is I/3 for the same total power. Hence the line current is $\sqrt{3} \times I/3$, *i.e.* ·58I. For equal current densities the total copper is therefore $3 \times \cdot 58 \times 50$, *i.e.* 87.

For equal efficiencies let the copper in each of the three-phase mains be $50/n$, so that the resistance of each main is n times that of a monophase two-wire main. Hence by equating losses

$$3 \times (\cdot 58I)^2 \times n = 2I^2;$$

whence $$n = 2.$$

Therefore the copper in each three-phase main is 25, and the total copper 75.



(*a*) THREE-WIRE. (*b*) FOUR-WIRE.

Fig. 18.07.—LOW-PRESSURE THREE-PHASE SYSTEMS.

The three-phase four-wire [Fig. 18.07 (*b*)] need carry only I/3 in the outer lines if the loads connected between these and the fourth wire are balanced. This wire is needed only to carry any out-of-balance current there may be. It may therefore be smaller than the outers, say half as big in section. Then for equal current densities, the copper required is $3 \times (50/3)$ for the outers, and $\frac{1}{2} \times 50/3$ for the middle wire, *i.e.* $58\frac{1}{3}$ in all.

For equal efficiencies let the copper in each outer be $50/m$. Then equating losses

$$3 \times (I/3)^2 \times m = 2I^2; \text{ whence } m = 6.$$

Therefore total copper required is $3\frac{1}{2} \times (50/6)$, *i.e.* $29\frac{1}{6}$.

This explains why three-phase low pressure distribution is effected nearly always by the four-wire system.

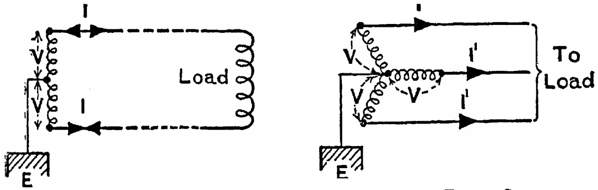
The two-phase three-wire system requires the same copper as the three-phase three-wire for equal current densities, viz. 87.

For equal efficiencies it requires 75 per cent. copper with the three equal mains, and 73 per cent. with a larger middle wire. The proof of these results is the same as for the H.P. system [Fig. 18.10 (*c*), Art. 11].

11. Comparative Economy of Systems of High-Pressure Transmission

The comparative advantages of different systems for the transmission of power by A.C., as regards economy in the line, depend on what factor limits the voltage employed. In the case of high pressure transmission either the maximum voltage between any line and earth, or the maximum voltage between two lines may be considered as constant in comparing two systems.

With overhead transmission the cost of the line depends partly on the copper in it, and partly on the voltage between line and



(*a*) MONOPHASE. (*b*) THREE-PHASE STAR.

Fig. 18.08.—COMPARISON OF SYSTEMS.

earth. If two lines are insulated satisfactorily from earth the same insulation will stand the voltage between the lines, since this cannot exceed the sum of the separate voltages to earth. The voltage between lines, therefore, affects only the distance between them, which must be sufficient to prevent a breakdown of the air space. On the other hand the insulation needed by transformer (or generator) windings connected to the lines depends partly on the voltage between lines.

Taking as the standard of comparison a monophase transmission with the middle point of the transformer earthed [Fig. 18.08 (*a*)], and a voltage V, between each line and earth, the power delivered to the line is $2VI \cos \phi$. For a three-phase system with transformer windings Y-connected and the star-point earthed, and the same voltage between each line and earth [Fig. 18.08 (*b*)] the power given to the line is $3VI' \cos \phi$. Therefore to transmit the same power at the same power-factor, $I' = \frac{2}{3}I$.

For equal current density the cross-section of each three-phase conductor is therefore $\frac{2}{3}$ of each monophase conductor, and so the total cross-section is the same in the two cases. Hence for equal distances of transmission the same total weight of copper is needed by the two systems. Moreover, if R is the resistance in ohms of each monophase conductor (so that $2I^2R$ is the transmission loss) the resistance of each three-phase conductor is $\frac{3}{2}R$. Hence the transmission loss in the latter system is $3I'^2 \times \frac{3}{2}R$, *i.e.* $3\left(\frac{2}{3}I\right)^2.\frac{3}{2}R$, *i.e.* $2I^2R$, so that the efficiencies of transmission are equal.

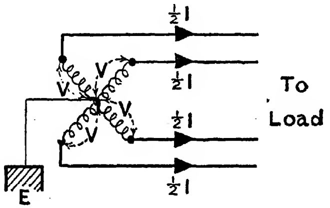
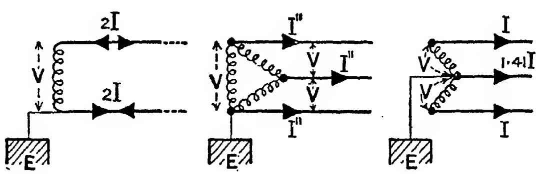


Fig. 18.09.—TWO-PHASE 4-WIRE SYSTEM.

These results are otherwise evident if each conductor is looked on as transmitting power amounting to $VI \cos \phi$ in the monophase system, and $VI' \cos \phi$ in the three-phase one. It is then seen that the only difference is that the power is divided equally between the three lines in the latter, and between the two in the monophase.

To obtain an equally good result in a two-phase system, a four-wire transmission with the centres of both phases earthed must be used (Fig. 18.09). Each conductor will then be half the cross-



(*a*) MONOPHASE. (*b*) THREE-PHASE DELTA. (*c*) TWO-PHASE 3-WIRE.

Fig. 18.10.—UNECONOMICAL SYSTEMS.

section of a monophase one for equal power, giving the same total copper as before, and the same transmission efficiency.

Examples of less favourable systems are (*a*) monophase with one terminal earthed; (*b*) three-phase Δ; and (*c*) two-phase three-wire (see Fig. 18.10). In the first case the current must be increased to double its former value for the same power. Hence for equal current densities the copper required is twice as great. But for

equal efficiency *four* times as much copper as before is necessary since the losses vary as (current)^2 (cf. Art. 3).

In the three-phase Δ system [Fig. 18·10(*b*)], if the line currents are I″, $\sqrt{3}VI'' = 2VI$, for equal power and equal power-factors. Hence $I'' = (2/\sqrt{3})\,I = 1{\cdot}155\,I$. Thus for equal current densities the total copper is increased to $(3 \times 1{\cdot}155)/2$, *i.e.* 1·73 times that needed for the three-phase Y, or for the monophase with middle point earthed.

For equal efficiencies let each line have a section n times that of each of the monophase lines [Fig. 18.08 (*a*)]. Then, equating losses in the two cases, $3 \times (2I/\sqrt{3})^2 \times R/n = 2I^2R$;

$$\therefore\ 4/n = 2,\ i.e.\ n = 2;$$

$\therefore$ the total copper in the Δ-system is *three* times the original amount.

The Δ-system can be improved by earthing the middle point of one phase. The highest voltage to earth is then that of the line not connected to either end of the earthed phase. It can be shown that the copper required is then $\frac{9}{4}$ of that needed for the Y-system for equal efficiency.

The following table summarises the above results:—

SYSTEMS WITH EQUAL VOLTAGES TO EARTH

No. of Phases	No. of Mains	Earthed Point	Relative Copper		Fig.
			(*a*)	(*b*)	
One	Two	One main	400	100	18·10*a*
,,	,,	Middle	100	25	18·08*a*
Three-Y	Three	Star	100	25	18·08*b*
Three-Δ	,,	One main	300	75	18·10*b*
,, ,,	,,	Middle of one phase	225	56	—
Two	,,	Middle	300 (291)	75 (73)	18·10*c*
,,	Four	Star	100	25	18·09

The figure for relative copper is stated in all cases for equal distances of transmission, equal power transmitted, equal power factors and equal efficiencies. The standard taken in column (*a*) is 1-phase with middle point earthed, and in column (*b*) 1-phase with one line earthed.

Similarly, the two-phase 3-wire system [Fig. 18.10 (*c*)] for equal current densities requires (3·41/2), *i.e.* 1·71 times the copper needed by the monophase with earthed middle, owing to the additional middle wire in the former. For equal efficiencies with three lines of the same section and resistance R′

$$2\,I^2R = R'\{I^2 + (1{\cdot}41\,I)^2 + I^2\} = 4\,.$$

∴ each line has twice as much copper as a monophase line, and so the total copper is three times as great, *i.e.* it is on a level with the three-phase △ in this respect.

By increasing the cross-section of the middle wire and reducing that of each outer the total copper can be reduced slightly. The best result is obtained by making the middle wire 1·41 times the size of the outers, so that the current densities are equal. The copper required is then 2·91 times that for monophase.

12. Kelvin's Rule

As mentioned earlier (Art. 1), the requirements of efficiency and cheapness are opposed to a large extent. When the voltage has been settled, the size of cable for a given transmission which will give the best results can be determined by means of Kelvin's rule. This is as follows:—

Make the annual cost of the energy wasted in transmission equal to the annual cost of interest and depreciation on the capital cost of the copper (or aluminium) used in the cables.

PROOF.—The annual cost of the lost energy $= I^2Rtp$ pence,

where I = average current transmitted in amperes,

R = total resistance of line (lead and return) in ohms,

t = time per annum during which transmission occurs, in *thousands* of hours,

and p = cost of generating energy, in pence per kWh.

And $R = \dfrac{2\rho l}{A}$, where ρ = resistivity of copper

A = cross-section of copper

and l = distance of transmission

in inch units.

$$\therefore \text{ Annual cost of lost energy} = \frac{2I^2\rho ltp}{A} \text{ pence,}$$

$$= £\frac{2I^2\rho ltp}{240A},$$

$$= £\frac{a}{A}, \text{ where } a \text{ is a constant.}$$

The annual cost of interest and depreciation

$$= £\frac{i}{100}.\frac{2A.l.k}{240} = £b.A,$$

where i = percentage for interest and depreciation together,
k = cost of copper in pence *per cubic inch*,
and b is a constant;

$$\therefore \text{ total cost} = £\left(\frac{a}{A} + b.A\right).$$

Now $\frac{a}{A} \times bA = ab$ = constant.

And it can be shown by algebra or by the differential calculus that when the product of two quantities is constant their sum is least when they are equal. *E.g.*

$$6 \times 6 = 4 \times 9 = 3 \times 12 = 2 \times 18 = 36;$$
$$6 + 6 = 12;\ 4 + 9 = 13;$$
$$3 + 12 = 15;\ 2 + 18 = 20, \text{ etc.}$$

Therefore the total cost is least when, as Kelvin's Rule states, the two components of it are equal.

13. Modification for Insulated Cables

With insulated cables the cost of the insulation must be considered. This increases with the size of the cables, but not in proportion to the area. It is possible to represent the total cost of such cables with sufficient accuracy, over the range of areas likely to be suitable, by a straight line, *i.e.* cost of cable $= c.A + d$, where c and d are constants.

> The minimum total cost will then be obtained by making the annual cost of the energy lost equal to the annual cost (interest and depreciation) on that part of the capital cost of the cables which varies as the area ($c.A$).

The original and modified rules are illustrated graphically in Figs. 18.11 and 18.12.

In Fig. 18.11 ABC represents the annual cost of the lost energy plotted against the area of the cable. Since the cost varies inversely as the area, ABC is a rectangular hyperbola. The annual cost of the cable (proportional to the area) is represented by the straight line OBD. The total annual cost is obtained by adding the ordinates of the two. It will be seen that the minimum value occurs vertically above B, *i.e.* for the area which makes the two portions of the annual cost equal.

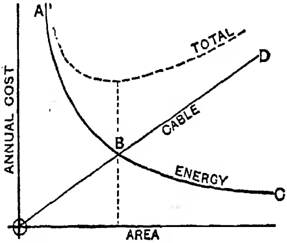


Fig. 18.11.—KELVIN'S RULE

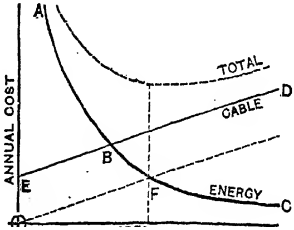


Fig. 18.12.—KELVIN'S RULE MODIFIED.

Fig. 18.12 shows the same for the modified case. The minimum total cost does not now occur for the area corresponding to B (*i.e.* equal cable and energy costs) but for that corresponding to F (OF is parallel to EBD and its ordinates represent the variable part of the cable cost).

Example 3. *Find the most economical size of cable to use for transmitting 500 kilowatts 15 ml. for 8 hr. per day with 300 working days in the year. P.D. at the receiving end 6000 volts.*

Cost of concentric cable for this pressure £(2·5A + 0·31) per yd., where A = cross-sectional area in sq. in.

Cost of generation ¼d. per kWh.

Interest and depreciation together = 9 per cent. per annum.

Take resistance of a single core cable at the working temperature as $\frac{\cdot 046}{A}$ *ohms per mile.*

Interest and depreciation on cable per year

$$= \frac{9}{100} \text{ of } £(2{\cdot}5A + 0{\cdot}31) \times 1760 \times 15$$
$$= £(5940A + 737).$$

$$\text{Current} = \frac{500 \times 1000}{6000} = 83{\cdot}3 \text{ amperes.}$$

$$\text{Watts lost in transmission} = 2 \times (83{\cdot}3)^2 \times 15 \times \frac{\cdot 046}{A} = \frac{9375}{A};$$

$$\therefore \text{ Annual cost of lost energy} = \frac{9375}{A} \times \frac{8 \times 300}{1000} \times \tfrac{1}{4}\text{d.}$$
$$= \frac{5625}{A}\text{ d.} \doteqdot £\frac{23{\cdot}44}{A};$$

$$\therefore \text{ For least total cost} \qquad 5940A = \frac{23{\cdot}44}{A};$$

$$\therefore A = \sqrt{\frac{23{\cdot}44}{5940}} = \cdot 0628,$$

i.e. the cable should be ·063 sq. in. cross-section.

With this size the current density is $\frac{83{\cdot}3}{\cdot 063} = 1320$ A. per sq. in., which will not cause overheating with this size of cable (see Chapter III.).

The annual cost of lost energy $= £\frac{23{\cdot}44}{{\cdot}063} = £372$

Annual cost of cable $= £(5940 \times {\cdot}063 + 737) = £1111$

Total $£1483$

14. Standing and Running Costs

It can be seen from Art. 12 that if the time (t) per annum during which power is required is increased and no other change is made, then the best cross-section of cable to use is increased as $\sqrt{t}$. Further, the total cost for transmission is increased in the same proportion: thus the cost *per kilowatt-hour* is reduced inversely as $\sqrt{t}$.

On the other hand, if the area is kept fixed the annual cost of the energy lost in transmission varies as the number of units supplied, while the interest and depreciation charge remains constant;

or $$\text{Total cost} = pW + q,$$

where W = number of units of energy supplied per annum

and p and q are constants.

The same formula applies to the case of insulated cables, but q then includes the interest and depreciation on the constant d (see Art. 13).

A similar formula holds for the cost of generation. For this can be divided into *standing* and *running costs*. The latter consist of the greater part of the expenditure on coal or other fuel, water, and lubricating oil, and of those parts of the labour and depreciation charges which vary as the number of units generated.

The standing costs comprise the interest on the capital cost of the power station and all its apparatus, and of those parts of the labour (particularly supervision) and depreciation charges which depend on the total kilowatt capacity of the station and are independent of the number of units generated. They further include part of the cost of coal, etc., for some is burned in keeping the boilers ready to supply steam whether steam is actually required or not. Moreover some steam is used in keeping at least one generator running in readiness to supply power when needed. Similar considerations apply when oil engines are used.

15. Maximum Demand System

This division of costs forms the basis and the justification of the *maximum demand system* of charging for electrical supply. In its simplest form this system comprises a certain sum per kilowatt of

maximum demand, plus a low charge per unit (kWh.) supplied, *e.g.* £5 per annum per kilowatt + 0·4d. per kilowatt-hour. In such a tariff the price per kilowatt is intended to cover the consumer's share of the standing costs, and the charge per unit is meant to pay for his share of the running costs. This method of charging is adopted frequently in bulk supply.

For charging separate consumers Wright's modification is sometimes used. This consists of a high first charge per unit up to a consumption equivalent to one or more hours per day at the maximum demand, and a lower charge for further units, *e.g.* 5d. per unit for $1\frac{1}{2}$ hours of maximum demand, and 2d. per unit afterwards (cf. Qu. 12, Chapter VII., page 230). The extra charge on the high-priced units corresponds to the kilowatt charge in the simpler system. The difference lies in the fact that the price per unit cannot exceed the first charge in the Wright system. The result is that a consumer with a very "short-hour" load, *i.e.* a small total consumption compared with his maximum demand, is treated more favourably than by the first system.

16. Load Factors

The **annual load factor** of a central station is the ratio—

$$\frac{\text{Total kilowatt-hours generated per annum}}{\text{Kilowatts of maximum demand} \times 8760} \times 100 \text{ per cent.},$$

8760 being the number of hours in a year. This ratio is evidently equal to $\dfrac{\text{Average load during year}}{\text{Maximum load during year}} \times 100$ per cent. It is often called *the* load factor.

The same ratio taken for a day or other period gives the daily, etc., load factor.

It can be seen from Art. 14 that the higher the annual load factor the lower the average cost of generation and transmission per kilowatt-hour, other things being equal. The load factor of a station with only a lighting load may be 10 per cent. to 15 per cent.; whereas if it supplies power as well it will have a value between the latter figure and 35 per cent. (or a little higher), varying according to the relative magnitudes of the two loads. A traction load will raise the load factor by another 5 per cent. or thereabouts. Cooking and heating loads are advantageous by increasing the power load. Restricted-hour loads, *i.e.* those excluded from times near to that of maximum total load, are specially helpful.

Two similar factors are: (*a*) the **station load factor**, which

$$= \frac{\text{Kilowatt-hours generated per day}}{\text{Kilowatts of plant installed} \times 24} \times 100 \text{ per cent.}$$

$$= \frac{\text{Average load}}{\text{Kilowatts installed}} \times 100 \text{ per cent.};$$

and (*b*) the **plant load factor**, which is the same as the above with "kilowatts of plant in use" substituted for that installed. In the former case spare plant is included. In both cases the kilowatts are taken at rated full load, *i.e.* without taking into account the overload capacity.

The plant load factor does not differ much in value from the daily load factor, and may exceed it if the maximum load is of short duration and the generators can carry an overload safely during this time: or if a battery is used (unless its capacity is reckoned in the total).

The station load factor must be less than the plant load factor, because some plant is required as a reserve and for future increases of load. This load factor is the one on which the standing charge must be based, so its value should be kept as high as possible consistent with ensuring continuous supply.

17. Diversity Factor

In obtaining the value of the standing charge, account must be taken of the fact that the maximum kilowatts supplied by the station is less than the sum of the separate maximum demands of the consumers. This is due to the different times at which their various demands occur. It is allowed for by means of the **diversity factor,** which is the ratio

$$\frac{\text{Sum of separate maximum demands}}{\text{Maximum demand on the station}}.$$

The value of this varies widely with the nature of the load.

It now becomes evident that the maximum demand system, though much fairer than a flat rate, is not completely fair, for different classes of consumers have loads of different diversity factors. The proper standing charge per kilowatt = (standing charge per kilowatt installed) ÷ diversity factor, while in the simple maximum demand system the rates are the same for all consumers.

A number of special tariffs have been introduced, partly with the aim of satisfying the above requirements. An alternative object is

to obtain additional loads which will improve the load factor and lower the cost of supply to the old load even if the new load does not bear its full share of the total cost. For details of these tariffs the reader should refer to the electrical journals.

18. Laying of Cables

The methods used for this can be divided into the direct, the solid, and drawing-in.

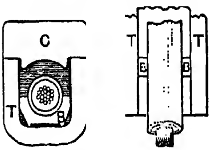


Fig. 18.13.—CABLE CONDUIT. (a) Sectional elevation. (b) Plan. B, Supporting bridge. C, Cover. T, Trough.

The **direct method** consists in laying the cable, which should be armoured, in a trench dug in the ground and covering it with the earth which has been dug out in making the trench. A few inches above the cable are placed tarred planks, or better, bricks or tiles, to protect the cable when the ground is opened again. The depth of the trench is generally about 3 feet.

In the **solid method** the cable is laid in a *conduit* or *trough*, with supporting bridges at intervals to keep it clear of the sides. An insulating compound is then run in, completely surrounding the cable, except at the bridges. Finally a cover is placed on top.

The compound is a mixture of waxes, etc., and is solid at ordinary temperatures, but is melted easily for pouring into the trough.

The trough and cover may be of earthenware or asphalt, the supporting bridges of porcelain, or of wood impregnated with insulating compound, or of asphalt.

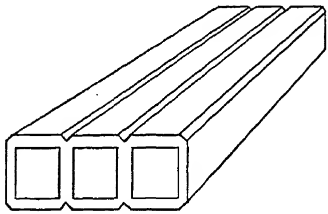


Fig. 18.14.—THREE-WAY CABLE CONDUIT.

For *drawing-in* cables, glazed earthenware conduits are laid down with a suitable number of " ways " for cables (see Fig. 18.14). These are made in lengths of about 4 ft. and with 1 to 6 ways. They are laid on concrete and then covered over with more concrete, about 6 in. thick in every direction. The joints between the lengths are made with cement, thus forming watertight channels. These run between a series of *cable pits* sunk in the ground at distances of about forty yards. *Drawing-in wires* are left in the conduits during laying, by means of which ropes

and then cables are drawn in when required. Sometimes cast-iron pipes are used for the same purpose.

The solid method is the most usual. Drawing-in is considerably more expensive, and is used chiefly in the centres of towns, where it is inadvisable to open the ground frequently. Moreover, it is unsuitable for distributors, since fresh connexions can be made only at the cable pits. The direct method is used where subsidences are likely, *e.g.* in mining districts, or for outlying places where its cheapness is of greater importance.

19. Mains Testing

Mains testing means the testing of the insulation of the mains, and sometimes of the apparatus connected to them in addition. The insulation resistance of an installation in a factory or other building is generally tested by some form of ohm-meter (see Chapter VII.). For mains, other methods are general, because often tests must be made when the mains are in use. A simple means of obtaining a continuous record of the insulation resistance of a whole system is to connect one point to earth at the central station, and place a recording ammeter in the earth connexion.

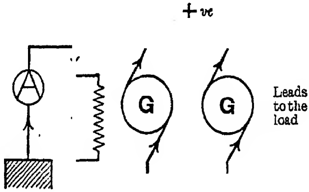


Fig. 18.15.—CONNEXIONS OF EARTH AMMETER.

Sometimes a resistance is placed in series with the ammeter. An alternative is a resistance shunted by a fuse, so that if the current increases sufficiently to blow the fuse, the earth connexion is not broken but the current flowing through it has to pass through the resistance (see Fig. 18.15). The amount of the resistance is chosen so as to limit the earth current to an amount which will not damage the ammeter, *e.g.* a 10-ohm resistance on a 250-volt supply with an ammeter recording up to 25 amperes.

In a 2-wire system, the negative main is practically always the point earthed. It can be seen from Fig. 18.15 that the earth ammeter records the current leaking from all the positive conductors to earth, independently of the load on the system. The insulation resistance to earth of the positive mains can then be obtained approximately by dividing the supply voltage by the earth (or leakage) current (see further Art. 20).

The Home Office rule is that the leakage current shall never exceed one thousandth of the maximum supply current in the case of power and lighting supply. In traction supply the leakage must not exceed 100 A. per mile of tramway, tested when the trams are not running and the line is fully charged.

In a 3-wire system, the middle wire is earthed nearly always, as shown in Fig. 18.02. This is compulsory when the pressure across each side exceeds 125 volts. The leakage current is then the difference between the leakages from the positive, and to the negative. It therefore shows only the difference in insulation resistance of the two outers, recording no current as long as they are equal, whether the insulation is good or bad. If the current in the earth connexion is from the earth the positive main has the worse insulation, while if it is to the earth the negative main has the worse insulation. The earth ammeter used is therefore of the moving coil type with central zero, the direction of the deflection indicating on which main the greater leakage is taking place.

20. Tests of Both Mains

Though the insulation of the positive main is more important than that of the earthed negative, the latter must be maintained and should be tested from time to time. The simplest method is to disconnect the negative from earth temporarily and to connect the positive to earth through the ammeter instead. The insulation resistance of the negative is then the supply voltage divided by the reading on the earth ammeter.

Another method is to remove the earth connexion temporarily, and to measure with a voltmeter of suitable resistance (*a*) the voltage between the mains, (*b*) the voltage of the positive main above earth, (*c*) the voltage of the negative main below earth.

The sum of the latter two voltages is *less* than the P.D. between the mains. The reason of this is that the voltmeter acts as a leak on the main to which it is connected, and so brings its potential temporarily nearer to that of the earth.

Let E = voltage between mains.
E_1 = voltage of positive above earth.
E_2 = " " negative below earth.
R = resistance of voltmeter.

Then it can be shown that the insulation resistance (R_1) of the positive main to earth is given by—

$$R_1 = \frac{E - (E_1 + E_2)}{E} \times R,$$

and the insulation resistance (R_2) of the negative main to earth is iven by—

$$R_2 = \frac{E - (E_1 + E_2)}{E_1} \times R.$$

And also from the above—

$$\frac{R_1}{R_2} = \frac{E_1}{E_2}$$

The resistance of the voltmeter should be fairly high so as to be comparable with the insulation resistance measured, otherwise the accuracy of the test suffers. A resistance of about 10 000 ohms is suitable for an average distribution system.

If an electrostatic voltmeter is used, ($E_1 + E_2$) will be equal to E, and only the ratio of the two insulation resistances can be obtained.

N.B.—These tests can be performed while the system is alive.

The proof of the above formulae is as follows:—

Fig. 18.16 shows the connexions for measuring E (which is assumed to remain constant) and E_1. While measuring the latter the voltage between earth and the −ve main is $E - E_1$.

The current which leaks from earth to the −ve main through R_2 comes from the +ve main to earth partly through R_1 and partly through the voltmeter.

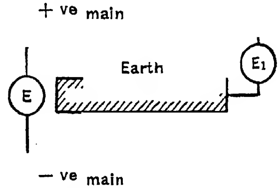


FIG. 18.16.—TEST OF INSULATION RESISTANCE OF LIVE MAINS.

E=Voltmeter connected to mains.
*E*1=Voltmeter connected between positive main and earth.

Therefore the voltages are directly proportional to the resistances and inversely proportional to the conductances;

$$\therefore\ E_1 : E - E_1 = \frac{1}{R_2} : \frac{1}{R_1} + \frac{1}{R};$$

$$\therefore\ E_1 : E = \frac{1}{R_2} : \frac{1}{R_1} + \frac{1}{R} + \frac{1}{R_2} \quad \ldots\ldots\ldots\ldots (1)$$

Similarly, when the −ve main is earthed through the voltmeter,

$$E_2 : E = \frac{1}{R_1} : \frac{1}{R_2} + \frac{1}{R} + \frac{1}{R_1} \quad \ldots\ldots\ldots\ldots (2)$$

From (1) and (2) $\qquad E_1 : E_2 = R_1 : R_2.$

Let $\qquad \frac{1}{R_2} = kE_1, \quad \text{then } \frac{1}{R_1} = kE_2,$

and $\qquad kE = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R}$, from (1) or (2);

$$\therefore\ k\{E - (E_1 + E_2)\} = \frac{1}{R} \quad \text{or} \quad R\{E - (E_1 + E_2)\} = \frac{1}{k};$$

$$\therefore\ R_1 = \frac{1}{kE_2} = \frac{E - (E_1 + E_2)}{E_2} R,$$

and $$R_2 = \frac{E - (E_1 + E_2)}{E_1} R. \qquad \text{Q.E.D.}$$

When the insulation resistance is obtained by means of earth ammeter readings the accurate values of the insulation resistances of the mains is given by the following formulae:—

$$R_1 = \frac{E - R_a(I_1 + I_2)}{I_2},$$

$$R_2 = \frac{E - R_a(I_1 + I_2)}{I_1},$$

where E = P.D. between mains in volts,
R_1 = insulation resistance of positive main in ohms,
R_2 = " " " negative main " "
R_a = resistance of ammeter in ohms,
I_1 = ammeter current (amperes) when connected between positive main and earth,
and I_2 = ammeter current (amperes) when connected between negative main and earth.

These values correspond with the simple expressions given in Art. 19, if $R_a(I_1 + I_2)$ can be neglected in comparison with E, which is usually the case unless R_a is increased by a series resistance.

The proof of the above follows the same lines as that given for the three voltmeter readings method.

Example 4. *A voltmeter of* 8 500 *ohms resistance gave the following readings:—*

Between mains 220 *volts,*
Positive main to earth 134 *volts,*
Negative main to earth — 49 *volts.*

(*a*) *Find the insulation resistance of each main.*

(*b*) *If the negative main is earthed through an ammeter what will be the reading on the instrument?*

(*c*) *If neither main is earthed what is the leakage current?*

(*a*) Apply the formulae of Art. 16,

$$\text{Resistance of positive main} = \frac{220 - (134 + 49)}{49} \times 8500$$

$$= \frac{37}{49} \times 8500 = 6420 \text{ ohms},$$

and

$$\text{Resistance of negative main} = \frac{37}{134} \times 8500$$

$$= 2350 \text{ ohms}.$$

(*b*) The ammeter reading gives the leakage through the insulation of the positive main;

$$\therefore \text{ earth current} = \frac{220}{6420} = \cdot 0343 \text{ A}.$$

(*c*) In this case the leakage takes place through the insulation resistances of the positive and negative mains in series;

$$\therefore \text{ leakage current} = \frac{220}{6420 + 2350} = \cdot 0251 \text{ A}.$$

21. Fault Location. Murray's Loop Test

A *fault* on an electric power main usually consists of a breakdown of the insulation between the conductor and earth. In the case of concentric or multicore cables the breakdown may be of the insulation between two of the conductors, and not to earth. In either case accurate location of the fault is important, so as to simplify the task of repairing it.

When the breakdown is to earth and a sound cable going to the same point as the faulty one is available, Murray's Loop Test is the best and simplest method. The faulty cable BD and the sound one AC are joined by a link at one end, thus forming a loop. At the other end are connected, as shown in Fig. 18.17, a galvanometer G, two known resistances P, R (one or both of which are variable), and a battery. Since one pole of the battery is earthed it is con-

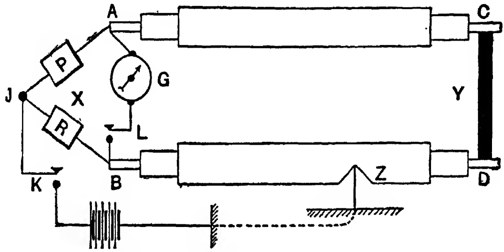


Fig. 18.17.—MURRAY'S LOOP TEST.

X=Testing point. Y=Point at which cables are linked. Z=Fault to earth.

nected through the fault to the cable at Z. Therefore when balance is obtained on the galvanometer, by adjusting P or R or both, it follows by the ordinary Wheatstone Bridge relation that:—

$$\frac{\text{Resistance of R}}{\text{Resistance of P}} = \frac{\text{Resistance of BZ}}{\text{Resistance of ACDZ}}.$$

If the two cables are of the same cross-section the latter resistances are proportional to the respective lengths;

$$\therefore \frac{R}{P} = \frac{x}{2l - x} \text{ or } \frac{R}{R + P} = \frac{x}{2l},$$

where R = resistance of JB,
P = resistance of AJ,
l = length of *each* cable = AC = BD,
x = distance to fault = BZ.

Thus x can be determined when R, P, and l are known. Only the ratio between R and P is required, not their actual values in ohms.

One great advantage of this test is that the resistance of the fault makes no difference to its accuracy. A high resistance fault merely reduces the current supplied by the battery, and this can be increased by using more cells in series.

[Note.—With a *very* high resistance fault, *i.e.* one whose resistance is comparable with the normal insulation resistance of the cable, the leakage through the insulation causes the test to give the position of the *resultant fault*. Such high resistance faults, however, usually can be left until they become worse.]

The galvanometer and battery might be interchanged. In this case, however, a high fault resistance cannot be overcome by an increase in the battery E.M.F. Moreover, an E.M.F. due to chemical action at the fault will destroy the accuracy of the test.

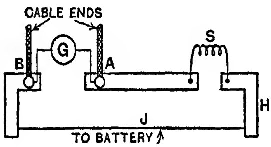


Fig. 18.18.—MANCHESTER FORM OF FAULT-LOCATING BRIDGE.

J, Sliding contact. S, Resistance equal to that of the wire connecting B to H.

22. Fault-Locating Bridges

The resistances P, R may conveniently consist of two portions of a single uniform wire of platinoid (or some similar alloy) stretched on a metre scale. The battery connexion J is made by a sliding contact, and $\frac{R}{P}$ is equal to the ratio of the lengths of the two parts of the wire on either side of this contact.

In the Manchester modification, a coil whose resistance is exactly equal to that of the wire is placed in series with it. The connexions are as shown in Fig. 18.18, B being connected to the faulty cable and A to the sound one.

$$\text{From Art. 21, } \frac{x}{l} = \frac{2R}{R+P} = \frac{R}{\frac{1}{2}(R+P)} = \frac{\text{resistance of BJ}}{\frac{1}{2}\text{ resistance of BJHA}}$$

$$= \frac{BJ}{BH}, \text{ since AH and HB are of equal resistance,}$$

i.e. J divides BH in the same ratio as the fault divides the length of the faulty cable. This modification doubles the accuracy of the bridge, without increasing its length.

Raphael's Direct Reading Fault-localising Bridge consists of a double wire on a scale 2 feet long with two movable connexions (see Fig. 18.19). One (S) is a cable connector which can be clamped in contact with any point of either portion of the wire. Its use is to make the length of wire employed a convenient multiple of the length of the "loop" under test; the other is the usual contact maker. Consequently when balance is obtained: since $\frac{x}{2l} = \frac{R}{R+P}$ (see Art. 21), the length to balance (PF) is a known multiple of the distance to the fault. *E.g.* if the length of each cable is 462 yards the cable connector is set at 924 scale-divisions. Then if balance is obtained at 342 divisions, the fault is 342 yards from the testing point. Hence this bridge is "direct reading," *i.e.* no calculation is required.

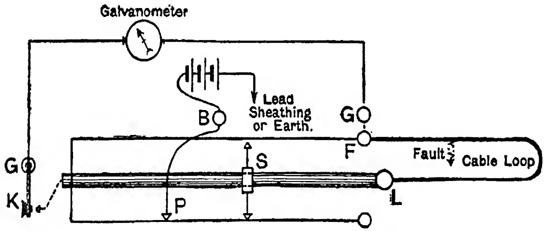


Fig. 18.19.—CONNEXIONS OF RAPHAEL BRIDGE.

B, Terminal for battery lead. F, Terminal for faulty cable. G G, Terminals for galvanometer leads. K, Key. L, Terminal for sound cable. P, Sliding contact maker. S, Contact maker for connecting L to any one point of the slide-wire.

In all loop tests, if the return part of the loop is made up of cables of a different size (or sizes) to that of the faulty cable, their equivalent length must be used instead of their actual length. The equivalent length is inversely proportional to the sectional area. *E.g.* if the faulty cable is 0·2 sq. in. section and 554 yd. long, and the return is of the same length but of 0·25 sq. in. section, its equivalent length is $554 \times \frac{\cdot 2}{\cdot 25} = 443$ yd. and the length of the loop is

$$554 + 443 = 997 \text{ yd.}$$

In such a case the distance to the fault may be more than half the length of the loop, in which case the Manchester pattern will not give a balance, unless S (Fig. 18.18) is short-circuited.

23. Other Methods

Most other methods of fault location depend on "fall of potential," or on "induction."

A test of the former class, which is used often, is shown in Fig. 18.20. The faulty cable (AC) is looped to a sound one (DB), and a large steady current is sent through the loop, preferably by a few large accumulators. An ammeter is inserted in the circuit to measure the current, and a variable resistance to adjust it to a constant value. A voltmeter or galvanometer is connected from earth to A and B in turn.

Then if E_1 = voltage of A above earth, and E_2 = voltage of B below earth, with the same current flowing,

$$\frac{\text{Distance to fault (AF)}}{\text{Length of loop (ACDB)}} = \frac{E_1}{E_1 + E_2}.$$

If a galvanometer is used the two deflexions may be substituted

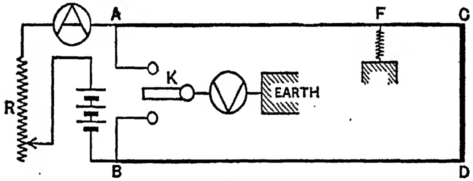


Fig. 18.20.—FALL OF POTENTIAL TEST FOR FAULT.
K, Two-way key. R. Variable resistance.

for E_1 and E_2, provided the deflexions are proportional to the voltages.

If a sound cable of the same size as the faulty one is not available, calculation of the equivalent length (Art. 22) can be avoided by running a pilot wire back from C to the voltmeter. If E_3 = voltage of C below earth, then—

$$\frac{AF}{AC} = \frac{E_1}{E_1 + E_3}.$$

This method is accurate only if the fault resistance is small compared with that of the voltmeter, and if there is no E.M.F. in the fault comparable with E_1.

A more accurate method is the following (Fig. 18.21). AB is the faulty cable, with a fault at C. AD is a sound cable, preferably of the same cross-section. If not, its equivalent length must be calculated.

BE and DF are testing leads which must be well insulated, but whose cross-section is immaterial.

A battery is connected between D and earth, with an ammeter and a rheostat in series so as to keep a steady current flowing.

A galvanometer with a high resistance in series is connected in turn between A and E, and between A and F, giving deflexions d_1 and d_2 respectively.

Then—
$$\frac{\text{Distance to fault (AC)}}{\text{Equivalent length of AD}} = \frac{d_1}{d_2},$$
since CB merely forms part of the connecting lead to the galvanometer.

The resistance in series with the galvanometer is adjusted until d_1 and d_2 are as large as possible with the same resistance.

The leads from A, B, and D should be arranged so that the

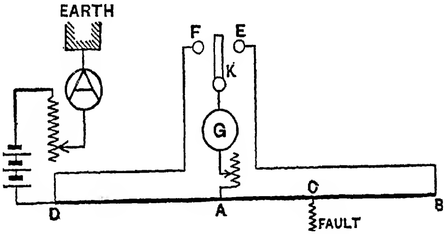


Fig. 18.21.—ACCURATE TEST FOR FAULT BY FALL OF POTENTIAL.
K, Two-way key.

galvanometer and ammeter are both at the same place, as this makes accuracy easier to obtain.

The sound and faulty cables should run alongside each other if possible (not as in the diagram), so that their temperatures will be equal. For the same reason the testing current must not be such as to raise the temperatures of either of the cables more than about 1° C., unless they are of the same cross-section.

Induction methods are employed on alternating current networks mainly. As their name implies, they depend on indications due to magnetic induction, and so require change of current in order to be used. To apply them to direct current circuits a changing current must be produced by using some form of "interrupter," *i.e.* an arrangement for repeatedly making and breaking the circuit. A large portable search coil connected to a telephone is used to locate the fault.

24. Location of Short-Circuits

If two cables (or two cores of a multicore cable) are short-circuited the position of the short-circuit can be obtained by the loop method (Art. 21) if a third cable is available. The connexions are as shown in Fig. 18.22.

AB and HK are the cables which are short-circuited at E.

CD is a sound cable, looped to one of the short-circuited cables at BD.

When balance is obtained on the galvanometer, G,

$$\frac{\text{Distance to short-circuit (AE)}}{\text{Equivalent length of loop (AEBDC)}} = \frac{R_1}{R_1 + R_2},$$

where R_1 = resistance of AF

and R_2 = resistance of FC.

It will be seen by comparing this with Fig. 18.17 that it is the

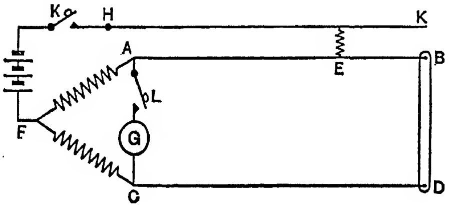


Fig. 18.22.—LOOP TEST FOR SHORT-CIRCUIT.

K, Battery key. G, Galvanometer. L, Key.

Murray Loop Test with one of the short-circuited cables substituted for "earth."

When a third cable is not available the short-circuit cannot be located by a loop test on the two cables which are short-circuited. It can be obtained by the Overlap method.

This consists of measuring the resistance between the cables from each end in turn.

Let R_1 = resistance between H and A *expressed in yards of cable*,

R_2 = resistance between K and B *expressed in yards of cable*,

and l = length of each cable in yards.

Then distance from A to short-circuit (AE) $= \dfrac{2l + R_1 - R_2}{4}$.

For $R_1 = AE + HE +$ short-circuit resistance (R_s)

$\qquad AE + R_s)$

and $R_2 = 2\,BE + R_s$

$\therefore$ by addition,

$$R_1 + R_2 = 2\,AB + 2R_s = 2\,l + 2\,R_s.$$

$$\text{But } AE = \frac{R_1 - R_s}{2} = \frac{1}{2}\left\{R_1 - \frac{R_1 + R_2 - 2\,l}{2}\right\}$$

$$= \frac{2l + R_1 - R_2}{4} \text{ as stated above.}$$

Example 5. *There is a fault in the insulation between the conductors of a concentric cable, AB, 500 yd. long. The resistance of the two conductors measured from A is 0·387 ohm, and measured from B 0·211 ohm. The actual resistance of each conductor is 0·242 ohm. Find the resistance of the fault and its position.*

$$\text{Fault resistance} = \frac{0{\cdot}387 + 0{\cdot}211 - 2 \times 0{\cdot}242}{2} \text{ ohm}$$

$$= {\cdot}057 \text{ ohm};$$

$\therefore$ Resistance of conductors alone between A and the fault $= {\cdot}387 - {\cdot}057 = {\cdot}330$ ohm;

$\therefore$ Resistance of each conductor between A and the fault $= {\cdot}165$ ohm;

$$\therefore \text{ Distance of fault from A} = \frac{0{\cdot}165}{0{\cdot}242} \times 500 \text{ yd.}$$

$$= 341 \text{ yd.}$$

Or directly from the formula (Art. 24): since 1 ohm $= \frac{500}{0{\cdot}242}$ yd. of the cable (single conductor),

$$\text{Distance of fault from A} = \frac{1}{4}\left\{1000 + ({\cdot}387 - {\cdot}211) \times \frac{500}{{\cdot}242}\right\} \text{ yd.}$$

$$= \frac{1}{4}\left\{100 + 364\right\} = 341 \text{ yd.}$$

25. The High Pressure Constant Current System

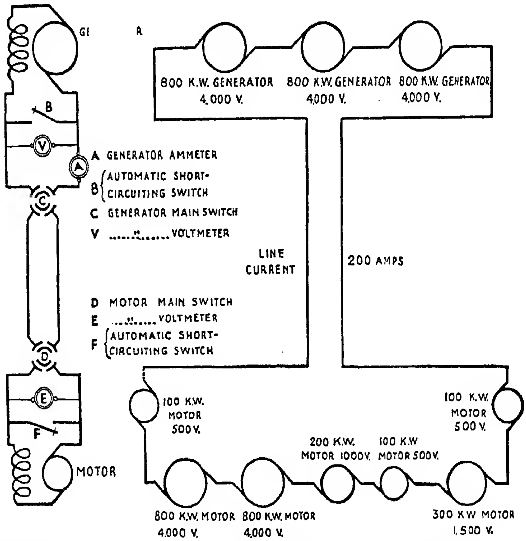
This system of transmission has been developed by M. Thury, and applied to a number of transmission systems in Switzerland and in France.

Series-wound generators are used, and these are all connected in series. The current is kept constant, and the power transmitted is varied by altering the voltage. This is done by varying the speed of the generators, or by altering the number of generators in series. When a generator is not required it is short-circuited. The system is thus the reverse of the usual system of constant voltage and variable current, with generators connected in parallel and open-circuited when not required.

An example is the transmission from Moutiers in Savoy to Lyons, a distance of 112 miles. The line current is kept constant at 75 A., and the pressure has a maximum value of over 60 000 volts, thus giving a maximum output of 4 500 kilowatts.

This is obtained from four groups of generators in series, each group driven by a water-turbine. There are four generators in each group, mounted in pairs on common bed-plates. Each armature generates about 3 800 volts, and ring-windings are used. (See Chapter VIII., Art. 16.) The two pairs forming a group are connected to each other and to their turbine by insulated friction couplings.

All the generating sets are insulated from earth and from each other by an insulated flooring. This consists of a concrete founda-



(*a*) Connexions of a generator and a motor.

(*b*) General arrangement of system.

Fig. 18.23.—THURY SYSTEM OF TRANSMISSION.

tion, on which is placed a layer of asphalt and small stones $1\frac{1}{2}$ cm. thick, and then a layer of pure asphalt 1 cm. thick. The bed-plates are supported on double-cup insulators resting on a foundation about 6 in. deep placed on top of the insulating floor.

The motors are likewise series-wound and connected in series. When not required to run a motor is short-circuited. They drive generators from which energy is distributed in the ordinary way, either by direct or by alternating current. The motors are insulated from earth by an insulating floor in the same way as the

Thury generators, and from the generators which they drive by an insulating coupling.

In the Moutiers-Lyons transmission the generators for distribution are 11 000 volt three-phase alternating ones. Each motor consists of two units on a common bed-plate. This arrangement, and the similar arrangement of the generators, is to allow for an increase of the maximum power transmitted. If this is required the line current will be doubled and the generators (or motors) on each bed-plate connected in parallel so as to be capable of carrying it. The voltage can then be restored to its original value by connecting new generators in series with the original ones. This is to overcome one of the drawbacks of the Thury system, viz. that an increase of maximum power necessitates an increase of voltage, since the current is constant. Consequently any increase of power beyond the original maximum will require fresh insulation for the line.

A greater drawback is that the line losses are constant at all loads, and therefore the efficiency is very poor at low loads.

The system has, however, a number of advantages over the high pressure alternating current system for long distance transmission. The main ones are:—(*a*) Power-factor does not reduce the watts to a value less than the volt-amperes; (*b*) the insulation needed for a given voltage is very much less than for an equal R.M.S. alternating voltage; (*c*) no rises of pressure due to resonance can occur.

The regulation, both of the generators to give constant current and of the motors to run at constant speed, is effected by shifting the brushes. This is effected automatically by regulators, whose action depends in the former case on the current in a solenoid, and in the latter on a centrifugal governor. For large changes of load the motor fields are altered by means of a divertor.

QUESTIONS ON CHAPTER XVIII.

1. What is meant by the term "drop of pressure" in electrical conductors? On what does it depend? Find the diameter necessary for a copper wire leading from a distribution board to a group of twenty 100-volt 50-watt lamps 40 yd. away in order that the drop may not exceed 1 per cent. Specific resistance of copper $\frac{2}{3}$ microhm per inch cube. [C. & G., I.

2. Six hundred H.P., at 600 volts, is to be delivered at a place two miles distant from an electrical generating station. Find the power and voltage of the generator required if the loss in transmission is to be 14 per cent. of power generated. Also determine the cross-section of the cable required. [C. & G., II.

3. Describe the 3-wire system of distribution, and compare it with the 2-wire.

4. A 3-wire feeder is transmitting 120 amperes along its positive and 160 amperes along its negative. The resistance of each outer is 0·05 ohm, and the cross-section of the middle wire is half that of either outer.

Calculate the power lost in the feeder.

Find the resistance of a 2-wire feeder with the same total loss when transmitting 280 amperes; and the relative amounts of copper in the two feeders if their lengths are the same.

Why does the latter result not equal 5 : 16?

5. Why is a balancer used in a 3-wire system? What is the advantage of cross-connecting the field windings?

6. Calculate the cross-section of conductor required for the economical transmission of 1 000 kilowatts 50 ml., delivered at a pressure of 40 000 volts by overhead lines, the price of copper being £70 per ton and the cost of energy at the generator 0·2 penny per unit. Resistivity of copper, 0·7 microhm per inch cube. Interest and depreciation may be assumed to be 6 per cent. per annum, and the working hours 12 per day. [Lond. Univ., El. Eng.

7. If the cost of 3 300-volt concentric cable is £$(2{\cdot}4A + 0{\cdot}19)$ per yard (A = area in square inches), what is the best size for transmitting 300 kVA. 10 ml. for 7 hr. per day; 3 000 volts P.D. at receiving end; cost of generation $\frac{1}{2}$d. per kWh.; interest and depreciation together = 8 per cent.?

Take resistance of a single cable $\dfrac{\cdot 046}{A}$ per mile at the working temperature.

8. (*a*) A 5 000-ohm voltmeter gave the following readings on a 2-wire distribution system:—

Positive main to negative main	220	volts.
„ „ „ earth	150	„
Negative „ „ „	− 45	„

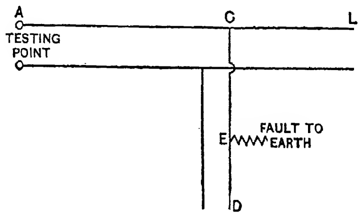
Calculate the insulation resistance of each main. [C. & G., II.

(*b*) Why is the difference of the last two readings (195 volts) less than the first reading?

9. Describe the Murray Loop Test for locating a fault.

What difference does the resistance of the fault make to the test?

10. Two similar cables lie side by side under a street. Both ends are accessible at manholes. One of the cables has a fault to earth. How would you find the position of the fault by means of a Wheatstone bridge? [C. & G., II.



11. If a distributor with two branches has one looped at L but has a fault on the other branch as shown, what will be the distance to the fault according to the test? How can the true distance ACE be found?

12. Prove that if the transmission line has the best area as given by Kelvin's Rule the current density is the same for all distances of transmission and for all values of the power transmitted.

Is the above still true when the modified rule is used?

13. Make a diagram of connexions showing the necessary machines in the supply station of a three-wire direct current system. What is the object of a balancer, and how does it act? If the supply voltage on each side of the three-wire system is kept at the same value at a feeding point, what is the effect on the voltage at terminals of lamps on each side, if the load on one side of the system is greater than on the other side? [C. & G., II.

14. A faulty 7/·080 in. feeder 628 yd. long is looped to a 19/·064 in. feeder of the same length. Balance is obtained at a point 61 per cent. of the total length of the slide-wire from the end to which the faulty feeder is attached. Find the position of the fault.

15. Give a diagram of connexions for a "fall of potential" method of fault location. Explain how the position of the fault is found, and any disadvantages from which this test suffers.

16. Calculate the H.P. transmitted and the efficiency of transmission by two lines each of 2·16 ohms resistance and carrying 200 A., the voltage at the receiving end being 10 000 and $\cos \phi = 0{\cdot}8$.

If the same H.P. is transmitted by three-phase currents the same distance and with the same power-factor using an equal total weight of line copper and with the same line voltage, find the line currents, the voltage at the generating end, and the efficiency.

17. Three lines each carry 120 A. and have a resistance of 3·25 ohms. The P.D. between any pair of lines is 10 000 volts at the receiving end and $\cos \phi = 0{\cdot}7$. Calculate the horse-power transmitted, the line P.D. at the generating end, and the efficiency of transmission.

What H.P. could be transmitted through two lines of the same size with the same P.D., power-factor, and efficiency?

18. Compare single-phase with three-phase transmission as regards line copper, (*a*) for high pressures, (*b*) for low pressures.

19. Compare the amounts of copper required for transmitting electrical energy with the same efficiency a given distance by (*a*) direct currents, (*b*) two-phase alternating currents, and (*c*) three-phase alternating currents, supposing, firstly, that the maximum volts to earth are the same in each case, and, secondly, that the maximum volts between conductors are the same.

20. Determine the necessary cross-section for each wire of a three-phase transmission line designed to convey 500 kW. over two miles at 6 000 volts, with an efficiency of 95 per cent. Specific resistance of copper 0·7 microhms per in. cube. [C. & G., II.

21. How many 60-watt 220-volt lamps can be supplied, (*a*) through two 0·5 sq. in. conductors; (*b*) through three 0·1 sq. in. and one 0·05 sq. in. conductors.

The drop must not exceed 10 per cent., and the distance of transmission is one-quarter mile.

State percentage saving of copper, and increase of power, in (*b*). If no copper were saved, what would be the percentage increase of power?

22. An ammeter of 0·2 ohm resistance in series with a rheostat of 11·8 ohms reads 4·7 amp. when connected between the positive of a 230 V. supply and earth; and 2·8 amp. when connected between earth and the negative. Find the insulation resistances of the two mains.

APPENDIX A

ELECTROMAGNETIC RELATIONS AND LENZ'S LAW

1. The strength of the field due to an isolated magnetic pole of strength m, at every point on the surface of a sphere of diameter r cm. having the pole as centre is $\frac{m}{r^2}$. This is therefore the number of magnetic lines passing through each sq. cm. of the sphere's surface (see Chapter IV., Art. 2), and everywhere perpendicular to it. Therefore the total number of lines coming from the pole

$$= \frac{m}{r^2} \times 4\pi r^2 = 4\pi m;$$

and in particular, from unit pole 4π ($= 12{\cdot}57$) lines issue.

2. When a magnetic pole is moved from a point to another of higher magnetic potential, the work done must appear in some other form, which may be either potential energy or electrical work in neighbouring circuits. If the pole is carried round a circuit to the point from which it started the work done (if any) must be converted into the latter form, since no potential energy is produced in this case.

Now, the work done on the pole

$$= m\int \mathsf{H}.dl = m\mathsf{H}l \text{ ergs (if } \mathsf{H} \text{ is uniform)},$$

where m = pole strength

and $\int \mathsf{H}.dl$ (or $\mathsf{H}.l$) = magneto-motive force in the circuit (see Chapter IV., Art. 12).

And the work performed in the electrical circuit

$= E.I.t$ ergs (see Chapter II., Art. 1),

where E = average E.M.F. in C.G.S. units due to the movement of the pole,

I = average current in the electrical circuit in C.G.S. units,

t = time in seconds taken in moving the pole round the magnetic circuit.

$= EIt.$

3. Thus if the current is *induced* by the movement of the pole, *i.e.* is caused by the E.M.F. due to this movement, it must produce

a force opposing the movement. This leads to *Lenz's Law*, which may be stated as follows:—

When a current is induced by the relative movement of a magnet and a conductor, its direction is such that it opposes the motion which produces it.

This law is therefore merely a particular case of the general law of the conservation of energy.

4. The strength of field at any point r cm. from a straight conductor carrying a current of I C.G.S. units is $\frac{2I}{r}$ (cf. Chapter IV., Art. 3) and is perpendicular to the radial line to the point from the axis of the conductor. Therefore if a pole of strength m is carried round the wire in a circle of radius r with its centre in the axis of the wire, the work done on this pole

$$= m . \frac{2I}{r} . 2\pi r = 4\pi I . m \text{ ergs.}$$

This is independent of the radius; and any movement along a radial line requires no work. Any circuit round the wire may be built up of a number of short arcs and portions of radii, and so for any path the work done $= 4\pi Im$.

Consequently the magneto-motive force $= 4\pi I$, the work done on unit pole (see Chapter IV., Art. 11).

5. In the above case the average rate of change of the flux (or rate of cutting of lines) $= \frac{4\pi m}{t}$, since each line is cut once but not more often, the return part of the electric circuit being at an infinite distance. But, from Art. 2,

$$EIt = m . \mathbf{H}l = 4\pi Im;$$

$$\therefore E = \frac{4\pi m}{t},$$

i.e. the E.M.F. is equal to the rate of cutting, or—

$$\text{Electromotive force in volts} = \frac{\text{rate of change of flux}}{10^8}$$

6. If the conductor, instead of being a long straight one, is a single turn of any shape, the number of " cuts " caused by moving the magnetic pole round any circuit linking with the conductor is still $4\pi m$. Therefore the induced E.M.F. is unaltered if t is unchanged, and in any case the product $E \times t$ remains equal to .

Thus, using again the equation of Art. 2, it follows that the magneto-motive force $= 4\pi I$, as in the special case of Art. 4.

7. If a coil of more than one turn is used the number of cuts is increased in the ratio of the number of turns (♉). Therefore the E.M.F., and so the M.M.F., are increased in the same proportion. Thus—

$$\text{Magneto-motive force} = 4\pi I \text{♉}$$

in all cases; or if the current is measured in amperes—

$$\text{M.M.F.} = \frac{4\pi I \text{♉}}{10} = \frac{4\pi \overline{A}}{10}.$$

(See Chapter II., Art. 2 and Chapter IV., Art. 13.)

APPENDIX B

THE METRE-KILOGRAM-SECOND SYSTEM OF UNITS

This system, devised by Giorgi, is known generally as the M.K.S. system. It differs from the C.G.S. system in using the metre as the unit of length, and the kilogram as the unit of mass. But the main difference is that the permeability of air is taken to be, not unity, but $1/10^7$. The following are the main resulting units.

Unit of force (M.K.S.) = mass $\times$ acceleration $= 10^3 \times 10^2 = 10^5$ dynes.

Unit pole is that which repels an equal pole at 1 metre (10^2 cm.) in a medium of unit permeability with unit force (M.K.S.). From the formula:—$F = mm'/\mu r^2$ it follows that two unit M.K.S. poles 1 cm. apart *in air* repel each other with a force of $10^7 \times (10^2)^2$ M.K.S. units, which is equal to $10^{11} \times 10^5$ dynes, *i.e.* 10^{16} dynes. Hence $\quad$ 1 M.K.S. pole $= 10^8$ C.G.S. unit poles.

The M.K.S. unit of E.M.F. is that due to cutting 1 M.K.S. line per sec., *i.e.* 10^8 C.G.S. lines per sec., and so is the same as the volt.

Again the M.K.S. unit of current is that which, flowing in a conductor curved to a radius of 1 metre, exerts on unit M.K.S. pole at the centre, unit M.K.S. force per metre of conductor. Hence unit M.K.S. current $= (10^2)^2 . 10^5/(10^8 . 10^2)$ C.G.S. units $= 1/10$ C.G.S. unit = 1 ampere.

The M.K.S. unit of work or energy = force $\times$ displacement $= 10^5 \times 10^2 = 10^7$ ergs = 1 joule.

The remaining electrical units depend on those dealt with already. Hence all the M.K.S. units coincide with the practical units. This abolition of the powers of 10, necessary when converting from C.G.S. units to practical units, makes the ultimate adoption of the M.K.S. system likely.

ANSWERS

Questions on Chapter II. (*Page* 21.)

2. 1185 ft. 3. 1·572 amperes.

Questions on Chapter III. (*Pages* 62, 63.)

1. 8·31 microhms per inch cube; 85·2 yd.
2. 50° C. or 51° C., according to initial temperature.
3. ·00395 per ° C. at 15° C., or ·00419 per ° C. at 0° C.; 11·04 ohms.
4. 98·5 per cent.; 15·49 ohms.
5. ·00532 sq. in.; ·01246 sq. in.; ·0337 sq. in.; ·0657 sq. in. (All with B.E.S.A. lay.) 6. 0·545 ohm.
7. 1·890 ohms per ml.; 350 megohm-ml. 8. ·0648 ohm; 443 megohms.
11. 1 076 megohm-ml. 13. 33·6 A.; 1 495 A. per sq. in.
14. (*a*) 5·43 A.; 12·7 A.; 2·33; (*b*) 10·4 A.; 20·9 A.; 2·01.
15. 12·75 A. (Preece) or 11·0 A. (Russell); $13·7 \times 10^3$ A. per sq. in.; $10·1 \times 10^3$ A. per sq. in. (Preece) or $8·75 \times 10^3$ A. per sq. in. (Russell).
17. 62·7 A. (Schwartz), or 61·4 A. (Maccall).
18. By Schwartz's formula, which does not apply to this size, 82·2 A.; by the author's formula, 73·6 A.
19. (*a*) 8 260 (Schwartz), 3 930 (Maccall); (*b*) 12 300 (Schwartz), 5 810 (Maccall).

Questions on Chapter IV. (*Pages* 112–114.)

2. 377. 4. 2 610. 7. 7 960. 8. 95; 239; 382; 668; 450.
9. 398, 796, 955, 1 114, and 995 extra; 493, 1 035, 1 337, 1 782, and 1 445 total.
10. 290. 11. 836. 12. 4·95 A.; 0·26 A.; 19 : 1.
13. 7·34 A.; 1·45 A.; 5·1 : 1.
17. 8 240 (neglecting fringing); 6 230 (with allowance).
18. 33½; 3 350 yd.; ·071 in. diam. (say 15 S.W.G.); 17·8 sq. in. (D.C.C.).
19. 1·09 mm. diam.; 1·21 kilograms. (N.B.—This requires a single layer only.)
20. 105 lb. wt. 22. 0·95 lb. with about 10 in. of rod inside coil.
23. 0·208 ton. 24. 4 710 lb. wt.; 1 065 joules.
25. Equal. 26. 0·727 sq. mm.; 12 220 turns.

Questions on Chapter V. (*Pages* 157-162.)

1. (*a*) 1·12 (*b*) 1·17; (*c*) 1·11; (*d*) 1·22. 2. 17·7; 16·4; 14·1; 1·16.
3. 8·28 A.; 1·17; 1·52; 11·71 A.
7. 11·1 A.; 167 volts; 111 volts; 0·83. 8 A.; 120 volts; 160 volts; 0·60.
8. 10·6 A. 9. 19·1 A. 10. 0·368 henry.
11. 0·82; 6·09 ohms; 4·24 ohms.
12. (*a*) 0; 1·0. (*b*) 332 ohms; 0·83. (*c*) 441 ohms, 0·75. 13. 22·7 volts.
14. 4 800 watts; 226 volts; 42·4 A. 9 600 watts; 0·50.
15. 334 volts; 6·20 A.; 1853 watts; 0·90; 25°.
16. 7·11 A.; 224 volts.; 1·56; 1·79. 1 100 watts. $\cos\psi = 0·71$; $\cos\phi$ 0·69.
18. 174 volts. 115 volts on coil; 199 volts on non-inductive part.

19. 29·2 ohms; 29·3 ohms; 0·102; 0·60. 17 per cent.
20. 8·89 ohms; ·0283 henry; 8·97 ohms; 0·133; 0·783.
21. 179 volts; 28·1 ohms. 139 volts.
22. 8·15 ohms; 8·18 ohms. 18·8 A.; 78 volts on coil, 66 volts on resistance.
23. ·042 A.
24. 0·193 microfarad.
25. 19·5 μF; 0·46 leading.
26. 4·48 A.; 7·54 A.
27. 7·07 A.; 16·7 A.
28. 7·90 A. $64\frac{1}{4}°$ lead.
29. $8 \cdot 76 \times 10^3$ cycles per sec.
30. 1570 volts; 1560 volts.
31. 0·61 A. at 50 ~; 4·81 A. at 100 ~, 0·66 A. at 200 ~. 103 cycles per sec.; 5 A.
32. (*a*) 0·75 henry. (*b*) 6·8 henries.
33. 175 ohms. 525 volts; 150 volts; 390 volts. 0·80; 0·38; 0·51.
34. 1·22 A. 305 volts on coil; 129 volts on condenser. 1·08 A. 270 volts; 114 volts on 1st condenser; 172 volts on 2nd; 286 volts across the two.
35. 0; 150 ohms; 400 ohms. 806 ohms; 743 ohms. 403 volts; $371\frac{1}{2}$ volts.
36. 20 A.; 25 A.; 40·3 A. 0·10; 0·125; 0·202.
37. 0·92; 255 microfarads; 37 A.
38. 26·3 A.; 0·90.
39. 19·6 A., power-factor 0·39; 4·0 A., power-factor 0·04; 23·4 A., power factor 0·34.
40. 1·26 A.; 1·73 A.; 1·55 A.
41. 14·9 A.
42. 5·56 A.; 6·46 A.; 7·78 A.; 9·34 A.; 11·05 A.
43. 0·50 A.; 2·49 A.; 2·12 A.; 1·00 A.; 1·48 A.; 0·66 A.
44. 16·7 A.; 17·9 A.; 16·7 A.; 33·0 A.
45. 10·7 A.; 0·26. 182 ohms; 0·19 henry.
46. 960 volts.
47. 3·11 μF; (*a*) 5 000 ohms. (*b*) 1 950 ohms.
48. 22·5 ohms; 34·6 ohms; 138 watts; 386 watts.
50. (*a*) 159 ohms. (*b*) 112 ohms; 75·7 ohms. (*c*) 204 watts.
51. 5 000 amperes per sec.

Questions on Chapter VI. (*Pages* 178-180.)

1. 4 670 volts. (*a*) 100 A. in both cases. (*b*) 130 A., or 57 A.
2. 208 A., 250 A. and $312\frac{1}{2}$ A.; 400 A., 455 A. and 488 A.
3. $562\frac{1}{2}$ A., 455 A. and 256 A.; or 315 A., 455 A. and 460 A.
5. 1st and 2nd waves:—332 volts, 1·17; 3rd wave:—470 volts, 1·08 : ratio 1·41.
6. 1st and 2nd waves:—166 volts, 1·17, 1·54; 3rd wave:—286 volts, 1·11, 1·50 : ratio 1·72.
7. 137 volts; 1·08; 1·33.
8. 1st and 2nd waves:—325 volts, 1·16, 1·60; 3rd wave:—337 volts, 1·06, 1·38 : ratio 1·04.
9. (*a*) 12·5 A. lagging 46°; (*b*) 7·32 A. lagging 77°; (*c*) 17·2 A. lagging $35\frac{1}{2}°$.
11. 6 500 H.P.
12. 80 kW. (*a*) 5·55 ohms. (*b*) 1·85 ohms; 5·78 ohms; 4·44 ohms.
14. 4·11 A. per core, or 7·12 A. equivalent current. 0·30 μF per mile.
15. 1·92 A. per core.

Questions on Chapter VII. (*Pages* 230, 231.)

1. (*a*) 97 ohms; (*b*) 0·01508 ohm.
2. 0·263 ohm; 0·0505 ohm; 0·01253 ohm: 0·25 ohm; 0·05 ohm; 0·01 ohm.
8. 802 kWh.; 17·4 A.
12. £5 10s.; 3·67d. per kWh.
21. 0·96 per cent.; 0·67d.
25. (*a*) 110 volts; (*b*) 80 volts.

Questions on Chapter VIII. (*Pages* 251, 252.)

3. $0{\cdot}862 \times 10^6$ lines. 9. 232 volts.
12. 6-pole:—206 (33 and 35), and 208 (35 and 35); 8-pole:—206 (25 and 27); 10-pole:—208 (21 and 21).
13. (*a*) 54; 13 and 13 (or 15); (*b*) 60; 15 and 13 (or 17); (*c*) 56; 13 and 13 (or 15 and 15).

Questions on Chapter IX. (*Pages* 286–288.)

2. 417 per pair of poles. About 2 per cent. 4. 2·19 volts.
8. 4·44 volts. 12. 0·78 volt. 16. 7 960.
17. ·0019 C.G.S. units; 4 210. 18. 7140 per pair of poles.
19. (*a*) 2·3 per cent.; (*b*) 300 per pair of poles.

Questions on Chapter X. (*Pages* 321, 322.)

6. (*a*) 230 volts; 20·7 kW.; (*b*) 230 volts; 20·7 kW.; (*c*) 460 volts; 41·4 kW.; (*d*) 230 volts; 41·4 kW. 7. 9 ft. diam. × 12 in. long.
15. 314 V., 287 V., 261 V.; 311 V. 16. 20 kW. and 30 kW.; 236 V.

Questions on Chapter XI. (*Pages* 349, 350.)

1. 35·2 lb.-ft. 2. 42 600. 3. 1078 lb.-ft. 10. 582; 570; and 559 r.p.m.
12. 3·54 ohms; (*a*) 28 r.p.m. increase; (*b*) 140 r.p.m. increase. 16. 586 r.p.m.
23. 34° C.; (*a*) 39½° C.; (*b*) 41° C.
24. 40·1° C. (40·9 from 1-hr. and 2-hr. readings); 2·02 hr.

Questions on Chapter XII. (*Pages* 371, 372.)

2. (*a*) 81·5 per cent. (*b*) 92·8 per cent.; (*c*) 96·6 per cent. (*d*) 13·0 kW.
3. 255·4 A.; 454·8 volts; 94·7 per cent.; 92·1 per cent.
4. 52·2 A.; 237·2 volts; 93·0 per cent. 5. 90·0 per cent.
6. 94·3 per cent.; 84·8 per cent.; 4210 watts. 11. 88·9 per cent.

Questions on Chapter XIII. (*Page* 392.)

2. 90 per cent.; 77·7 per cent. 4. 133. 13. ·00028 ohm.

Questions on Chapter XIV. (*Pages* 417, 418.)

6. 4·8 cm.; 9·1 cm.; 13·1 cm.; 16·8 cm.; 20·2 cm.; 27·8 cm.; 34·3 cm.
9. 20. 0·71. 10. 1 100. 11. 1 670.
15. (*a*) 2·30 ft.-candles; (*b*) 0·40 ft.-candle; (*c*) 0·75 ft.-candle.
16. 1 to 0·089; 24 ft. 8 in. apart; (i) 1 to 0·124; (ii) 1 to 0·617; assuming V.P. to be perpendicular to line joining the lamps.

Questions on Chapter XV. (*Pages* 439, 440.)

3. Diameters 1·18 to 1; lengths 0·542 to 1; (*a*) by 60 per cent.
4. Diameters 1·35 to 1; lengths 0·58 to 1; strengths 1·77 to 1.
9. 43·1 to 57·6 (assuming C.P. $\propto E^4$); 25·2 to 40·3 (assuming C.P. $\propto E^{6\cdot5}$).
10. For energy only:—(*a*) ·180d., ·195d., ·300d.; (*b*) ·0113d., ·0041d., ·0035d.
12. 0·354 A.; 125 volts and 115 volts.

Questions on Chapter XVI. (*Pages* 457, 458.)

3. 102 volts.
8. (1) Balance at 750 A., 50 volts boost at 775 A.; (2) Balance at 480, A. 50 volts boost at 500 A.; (3) Balance at 600 A., 50 volts boost at 625 A.

10. (*a*) 30 kW., 45 B.H.P., 82 per cent.; (*b*) 8 kW., 12·6 B.H.P., 91 per cent.
11. (*a*) 13 kW., 19·6 B.H.P., 79 per cent.; (*b*) 10·3 B.H.P., 6·4 kW., 84 per cent.; (*c*) 4 kW., 6·8 B.H.P., 86 per cent.

Questions on Chapter XVII. (*Pages* 492-494.)

2. 840 turns; 13·3 A.; 320 A.
3. (*a*) 1071 volts. (*b*) 10·2 kilolines per sq. cm.
4. (*a*) 0·79 A.; 1·07 A.; 1·59 A.; 2·34 A. (*b*) 0·89 A.; 1·28 A.; 1·88 A.; 2·68 A.
6. Primary, 15·5 volts; secondary, 1·70 volts. 3·48 volts. (*a*) ·0160 ohm. (*b*) 1·21 ohms. 7. ·0463 ohm; ·0709 ohm. 4·02 volts; 7·36 volts.
8. 0·315 ohm; 0·98 ohm. 6·3 volts; 19·5 volts.
9. (*a*) 2·1 per cent. (*b*) 4·3 per cent.
10. (*a*) 97·0 per cent.; 97·2 per cent.; 97·0 per cent.; 95·7 per cent.; 90·7 per cent. (*b*) 95·7 per cent.; 96·0 per cent.; 95·8 per cent.; 93·9 per cent. 87·3 per. cent.
11. 3·38 volts; 95·81 per cent. 35 kVA.; 95·85 per cent.
16. 19·9 A. to 29·8 A. 1 800 watts (with 3·5 ohms) to 440 watts.
19. 5·4 kW.; 96·2 per cent.

Questions on Chapter XVIII. (*Pages* 529-531.)

1. 0·157 inch. 2. 520 kW.; 698 volts; 1·28 sq. in.
4. 2 160 watts; ·0138 ohm for each wire; 0·345 : 1.
6. ·0467 sq. in. (with 300 working days per year).
7. 0·109 sq. in. (with 300 working days per year).
8. Positive 2780 ohms; negative 833 ohms.
14. 604 yd. from the testing point.
16. 2 145 H.P.; 90·3 per cent. 115 A.; 10 520 V.; 92·5 per cent.
17. 1 950 H.P.; 10 470 V.; 91·2 per cent. 1 390 H.P.
20. ·0238 sq. in.
21. (*a*) 2 080. (*b*) 2 500. 65 per cent. saving; 20 per cent. increase. 243 per cent. increase.
22. Positive 50 ohms; negative 30 ohms.

INDEX

References are to page numbers. When printed in heavy type this indicates that the subject is treated in a special section commencing on that page.